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# Synchronization Criterions between Two Identical or Different Fractional Order Chaotic Systems

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**Abstract.** This paper discusses chaos synchronization between two identical or different fractional order chaotic systems. Base on linear matrix inequality, two new synchronization criterions are constructed by which it is proved that two identical (Lü system) or different (Lü and Chen systems) fractional order chaotic systems are synchronized using the simple linear feedback control laws. Finally, simulations results show the method is effective.

**Keywords:** Synchronization, Fractional order, Linear matrix inequality

## 1. Introduction

Since Pecora and Carroll established a chaos synchronization scheme for two identical chaotic systems with different initial conditions[1], variety of method and techniques have been proposed for the control and synchronization of chaotic systems such as linear and nonlinear feedback synchronization[2-6], impulsive synchronization [7-8], adaptive synchronization [9-12], observer based control method[13-15], and etc.

Fractional calculus deals with derivatives and integration of arbitrary order [16–18] and has deep and natural connections with many fields of applied mathematics, engineering and physics. Fractional calculus has wide range of applications in control theory [19], Furthermore, recently, study of chaos synchronization in fractional order dynamical systems and related phenomena is receiving growing attention , some synchronization-based strategies have been devised to synchronize fractional chaotic systems [20-25]. In Ref. [26], the synchronization of fractional-order chaotic systems has been presented. In Refs. [27-28], nonlinear control are employed to synchronize two fractional-order chaotic systems. In Ref. [29], Synchronization of N-coupled fractional-order chaotic systems with ring connection has been reported. In Refs. [30-32], Synchronization of different fractional order chaotic systems using active control has been discussed. In Ref. [33], the stability of the fractional order unified chaotic system has been studied. In this paper, base on linear matrix inequality, two new synchronization criterions are constructed by which it is proved that two identical (Lü system) or different (Lü and Chen systems) fractional order chaotic systems are synchronized.

This work is presented as follows: Section 2 describes mathematical preliminaries and model. Chaos synchronization between two identical fractional order Lü systems in Section 3. Section 4 handles chaos synchronization between Lü and Chen systems of fractional order. Section 5 gives the conclusion of the paper.

## 2. Mathematical preliminaries and model

In this section, we give some useful mathematical preliminaries.

The mathematical definition of fractional derivatives and integrals has been the subject of several different approaches. The definition of fractional integrals by Grunwald–Letnikov and Riemann–Liouville

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is as follows [34]:

$$I_{\mu}^{\lambda}x(t)=\frac{1}{\Gamma(\lambda)}\int_{\mu}^{t}(t-\tau)^{\lambda-1}x(\tau)d\tau\,, \tag{1}$$

where $(\mu,t)\in R^2$, $\mu<t$, $0<\lambda<1$, $\Gamma(\cdot)$ is Gamma function, $\Gamma(\lambda)=\int_0^{\infty}\vartheta^{\lambda-1}e^{-\vartheta}d\vartheta$, and $\Gamma(z+1)=z\Gamma(z)$.

The definition of fractional derivatives is as follows:

$$D_{\mu}^{\lambda}x(t)=\frac{d}{dt}[I_{\mu}^{\lambda}x(t)]=\frac{1}{\Gamma(1-\lambda)}\frac{d}{dt}\int_{\mu}^{t}(t-\tau)^{-\lambda}x(\tau)d\tau\,. \tag{2}$$

The fractional order Lü system and Chen system [35-36] are described by (see Fig.1-2)

$$\begin{cases}\dfrac{d^{\alpha_1}x_1}{dt^{\alpha_1}}=36(x_2-x_1),\\ \dfrac{d^{\alpha_2}x_2}{dt^{\alpha_2}}=-x_1x_3+20x_2,\\ \dfrac{d^{\alpha_3}x_3}{dt^{\alpha_3}}=x_1x_2-3x_3,\end{cases} \tag{3}$$

and

$$\begin{cases}\dfrac{d^{\alpha_1}y_1}{dt^{\alpha_1}}=35(y_2-y_1),\\ \dfrac{d^{\alpha_2}y_2}{dt^{\alpha_2}}=(28-35)y_1+28y_2-y_1y_3,\\ \dfrac{d^{\alpha_3}y_3}{dt^{\alpha_3}}=y_1y_2-3y_3.\end{cases} \tag{4}$$

## 3. Chaos synchronization between two identical fractional order Lü systems

In this section we study the synchronization between two identical fractional order Lü systems, we define the drive (master) and response (slave) systems as follows:

$$\begin{cases}\dfrac{d^{\alpha_1}x_1}{dt^{\alpha_1}}=36(x_2-x_1),\\ \dfrac{d^{\alpha_2}x_2}{dt^{\alpha_2}}=-x_1x_3+20x_2,\\ \dfrac{d^{\alpha_3}x_3}{dt^{\alpha_3}}=x_1x_2-3x_3,\end{cases} \tag{5}$$

and

$$\begin{cases}\dfrac{d^{\alpha_1}y_1}{dt^{\alpha_1}}=36(y_2-y_1)+u_1,\\ \dfrac{d^{\alpha_2}y_2}{dt^{\alpha_2}}=-y_1y_3+20y_2+u_2,\\ \dfrac{d^{\alpha_3}y_3}{dt^{\alpha_3}}=y_1y_2-3y_3+u_3.\end{cases} \tag{6}$$

We define control functions $u_i$ as

$$u_1=-k_1e_1\,,u_2=-k_2e_2\,,u_3=-k_3e_3\,, \tag{7}$$

where $k_1>0\,,k_2>0\,,k_3>0$.

The error functions as

$$e_1 = y_1 - x_1,\ e_2 = y_2 - x_2,\ e_3 = y_3 - x_3. \tag{8}$$

Eq. (8) together with (5) and (6) yields the error system

$$\begin{cases} \dfrac{d^{\alpha_1} e_1}{dt^{\alpha_1}} = 36(e_2 - e_1) + u_1, \\ \dfrac{d^{\alpha_2} e_2}{dt^{\alpha_2}} = 20e_2 - x_3 e_1 - y_1 e_3 + u_2, \\ \dfrac{d^{\alpha_3} e_3}{dt^{\alpha_3}} = y_1 e_2 - 3e_3 + x_2 e_1 + u_3, \end{cases} \tag{9}$$

**Theorem 1.** The fractional order systems (5) and (6) can realize synchronization using the following linear matrix inequality:

$$\begin{pmatrix} -a(36+k_1) & (36a+b|X_3|)/2 & c|X_2|/2 \\ \Delta & -b(k_2-20) & (b+c)|Y_1|/2 \\ \Delta & \Delta & -c(3+k_3) \end{pmatrix} < 0, \tag{10}$$

where $|X_2|$, $|X_3|$ and $|Y_1|$ are the upper bounds of the absolute values of the states $x_2$, $x_3$ and $y_1$, $a,b,c>0$, $\Delta$ denotes the symmetric terms.

**Proof:** Let

$$\Phi = e^T P \frac{d^\alpha e}{dt^\alpha} = e_1 a \frac{d^{\alpha_1} e_1}{dt^{\alpha_1}} + e_2 b \frac{d^{\alpha_2} e_2}{dt^{\alpha_2}} + e_3 c \frac{d^{\alpha_3} e_3}{dt^{\alpha_3}}. \tag{11}$$

where $e = (e_1, e_2, e_3)^T$, $P = diag(a,b,c)$.

From Eqs. (9) and (11), we have

$$\begin{aligned} \Phi &= e_1[36a(e_2-e_1) - k_1 a e_1] + e_2[20be_2 - x_3 b e_1 - y_1 b e_3 - k_2 b e_2] + e_3(y_1 c e_2 + x_2 c e_1 - 3ce_3 - k_3 c e_3) \\ &= 36ae_1e_2 - 36ae_1^2 - k_1 a e_1^2 + 20be_2^2 - y_1 b e_2 e_3 - x_3 b e_1 e_2 - k_2 b e_2^2 + y_1 c e_2 e_3 + x_2 c e_1 e_3 - (3+k_3)ce_3^2 \\ &\le 36ae_1e_2 - 36ae_1^2 - k_1 a e_1^2 + 20be_2^2 + |y_1| b e_2 e_3 + |x_3| b e_1 e_2 - k_2 b e_2^2 + |y_1| c e_2 e_3 + |x_2| c e_1 e_3 - (3c + k_3 c)e_3^2 \\ &\le 36ae_1e_2 - 36ae_1^2 - k_1 a e_1^2 + 20be_2^2 + |X_3| b e_1 e_2 - k_2 b e_2^2 + |Y_1|(c+b) e_2 e_3 + |X_2| c e_1 e_3 - (3c + k_3 c)e_3^2 \\ &= e^T \begin{pmatrix} -a(36+k_1) & (36a+b|X_3|)/2 & c|X_2|/2 \\ \Delta & -b(k_2-20) & (b+c)|Y_1|/2 \\ \Delta & \Delta & -c(3+k_3) \end{pmatrix} e < 0 \end{aligned}$$

On the other hand, from Eqs. (9) we also have

$$\frac{d^\alpha e}{dt^\alpha} = \begin{pmatrix} -(36+k_1) & 36 & 0 \\ -x_3 & 20-k_2 & -y_1 \\ x_2 & y_1 & -(3-k_3) \end{pmatrix} e \overset{def}{=} Ae.$$

Supposing that $\lambda$ is one of eigenvalues of matrix A, and there should be a nonzero vector $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ being an eigenvector corresponding to the eigenvalue $\lambda$ i.e. $A\xi = \lambda\xi$, $PA\xi = P\lambda\xi$ One can gain $\xi^T PA\xi = \lambda \xi^T P\xi$, $\xi^T A^T P \xi = \bar{\lambda} \xi^T P \xi$. So,

$$\xi^T (PA + A^T P)\xi = (\lambda + \bar{\lambda})\xi^T P\xi, \text{ or } \lambda + \bar{\lambda} = \frac{\xi^T (PA + A^T P)\xi}{\xi^T P \xi}.$$

Since , $\Phi = e^T P \dfrac{d^\alpha e}{dt^\alpha} < 0$, so, $e^T P \dfrac{d^\alpha e}{dt^\alpha} + (\dfrac{d^\alpha e}{dt^\alpha})^T P^T e < 0$ , i.e.

$$e^T PAe + (Ae)^T P^T e = e^T PAe + e^T A^T Pe = e^T (PA + A^T P)e < 0,$$

that is, $PA+PA^T$ is a negative definite matrix, and then $\lambda+\overline{\lambda}=\dfrac{\xi^T(PA+A^TP)\xi}{\xi^TP\xi}<0$.

Formulation $|\arg(\lambda)|>\pi/2>\alpha\pi/2$ is obvious. According to the stability theory of fractional-order system [37], the system (9) is stable, therefore, the fractional order systems (5) and (6) can realize synchronization.

## 3.1. Simulation and results

In this section, computer simulations are used to verify and demonstrate the effectiveness of the above method. In all simulation, $\alpha_1=0.97,\alpha_2=0.98,\alpha_3=0.99$. The initial conditions of the master and slave systems are (-1 2 15) and (2 6 5), respectively. By estimating simulations, we let $|X_2|=30,\ |X_3|=50$ $|Y_1|=28$.

The conditions $a,b,c>0$ can be expressed as

$$\begin{pmatrix}-a & 0 & 0\\ 0 & -b & 0\\ 0 & 0 & -c\end{pmatrix}<0. \tag{12}$$

So, inequality (10) and (12) are linear matrix inequality in $a,b,c$, $ak_1$, $bk_2$, $ck_3$. By solving the LMI (10) and (12), the following solutions are obtained:

$$a=2.7649, b=1.3880, c=2.0564,\ ak_1=48.7577, bk_2=195.4376, ck_3=111.1031.$$

This yields $k_1=17.6345, k_2=140.8052, k_3=54.0280.$ Therefore, the controller (7) will drive the slave system (6) to synchronize the master system (5) as desired, the synchronous errors are shown in Figs. 3.

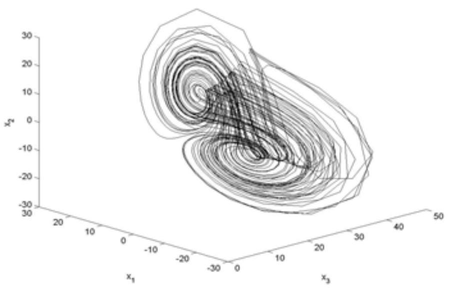

Fig.1. Chaotic attractor of the fractional-order Lü system (3)

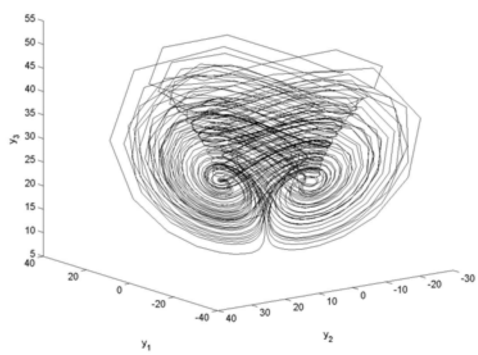

Fig.2. Chaotic attractor of the fractional-order Chen system (4)

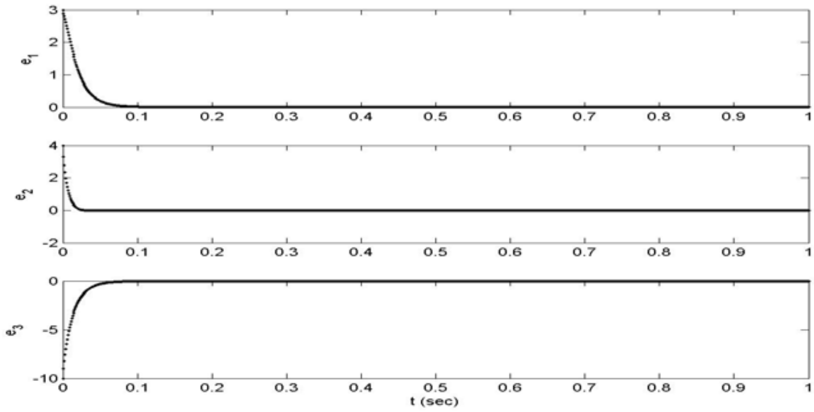


Fig.3. Synchronization errors between two identical fractional order Lü systems

## 4. Chaos synchronization between Lü and Chen systems of fractional order

In this section we study the synchronization between Lü and Chen systems of fractional order. Assuming that the Lü system drives the Chen system, we define the drive (master) and response (slave) systems as follows:

$$\begin{cases} \dfrac{d^{\alpha_1} x_1}{dt^{\alpha_1}} = 36(x_2 - x_1), \\ \dfrac{d^{\alpha_2} x_2}{dt^{\alpha_2}} = -x_1 x_3 + 20 x_2, \\ \dfrac{d^{\alpha_3} x_3}{dt^{\alpha_3}} = x_1 x_2 - 3x_3, \end{cases} \tag{13}$$

and

$$\begin{cases} \dfrac{d^{\alpha_1} y_1}{dt^{\alpha_1}} = 35(y_2 - y_1) + u_1, \\ \dfrac{d^{\alpha_2} y_2}{dt^{\alpha_2}} = (28-35) y_1 + 28 y_2 - y_1 y_3 + u_2, \\ \dfrac{d^{\alpha_3} y_3}{dt^{\alpha_3}} = y_1 y_2 - 3y_3. \end{cases} \tag{14}$$

Eq. (8) together with (13) and (14) yields the error system

$$\begin{cases} \dfrac{d^{\alpha_1} e_1}{dt^{\alpha_1}} = 35(e_2 - e_1) - x_2 + x_1 + u_1, \\ \dfrac{d^{\alpha_2} e_2}{dt^{\alpha_2}} = -7 y_1 + 20 e_2 + 8 y_2 - x_3 e_1 - y_1 e_3 + u_2, \\ \dfrac{d^{\alpha_3} e_3}{dt^{\alpha_3}} = y_1 e_2 - 3e_3 + x_2 e_1. \end{cases} \tag{15}$$

We define control functions $u_1, u_2$ as

$$u_1 = x_2 - x_1,\ u_2 = 7 y_1 - 8 y_2 - k_1 e_1 - k_2 e_2 - k_3 e_3, \tag{16}$$

where $k_1 > 0, k_2 > 0, k_3 > 0$.

**Theorem 2.** The fractional order systems (13) and (14) can realize synchronization using the following linear matrix inequality:

$$\begin{pmatrix} -35a & (35a+b|X_3|-k_1b)/2 & c|X_2|/2 \\ \Delta & -b(k_2-20) & [(b+c)|Y_1|-bk_3]/2 \\ \Delta & \Delta & -3c \end{pmatrix} < 0, \tag{17}$$

where $|X_2|$, $|X_3|$ and $|Y_1|$ are the upper bounds of the absolute values of the states $x_2$, $x_3$ and $y_1$, $a,b,c>0$, $\Delta$ denotes the symmetric terms.

**Proof:** Similar to proof of Theorem 1, let

$$\Phi = e^T P \frac{d^\alpha e}{dt^\alpha} = e_1 a \frac{d^{\alpha_1} e_1}{dt^{\alpha_1}} + e_2 b \frac{d^{\alpha_2} e_2}{dt^{\alpha_2}} + e_3 c \frac{d^{\alpha_3} e_3}{dt^{\alpha_3}}. \tag{18}$$

From Eqs. (15) and (18), we have

$$\begin{aligned}
\Phi &= 35ae_1(e_2-e_1) + e_2b[20e_2 - x_3e_1 - y_1e_3 - k_1e_1 - k_2e_2 - k_3e_3] + e_3c(y_1e_2 + x_2e_1 - 3e_3) \\
&= 35ae_1e_2 - 35ae_1^2 + (20-k_2)be_2^2 - x_3be_1e_2 - y_1be_2e_3 - k_1be_1e_2 - k_3be_2e_3 + y_1ce_2e_3 + x_2ce_1e_3 - 3ce_3^2 \\
&\le 35ae_1e_2 - 35ae_1^2 + (20-k_2)be_2^2 + |x_3|be_1e_2 + |y_1|be_2e_3 - k_1be_1e_2 - k_3be_2e_3 + |y_1|ce_2e_3 + |x_2|ce_1e_3 - 3ce_3^2 \\
&\le 35ae_1e_2 - 35ae_1^2 + (20-k_2)be_2^2 + |X_3|be_1e_2 + |Y_1|be_2e_3 - k_1be_1e_2 - k_3be_2e_3 + |Y_1|ce_2e_3 + |X_2|ce_1e_3 - 3ce_3^2 \\
&= e^T \begin{pmatrix} -35a & (35a+b|X_3|-k_1b)/2 & c|X_2|/2 \\ \Delta & -b(k_2-20) & [(b+c)|Y_1|-bk_3]/2 \\ \Delta & \Delta & -3c \end{pmatrix} e < 0
\end{aligned}$$

On the other hand, from Eqs. (15) we also have

$$\frac{d^\alpha e}{dt^\alpha} = \begin{pmatrix} -35 & 35 & 0 \\ -x_3-k_1 & 20-k_2 & -y_1-k_3 \\ x_2 & y_1 & -3 \end{pmatrix} e \overset{def}{=} Be.$$

The undone reasoning process is similar to proof of Theorem 1, thus we leave out the following proofs here.

## 4.1. Simulation and results

In this section, computer simulations are used to verify and demonstrate the effectiveness of the above method. The initial conditions of the master and slave systems are (-1 2 15) and (2 6 5), respectively. By estimating simulations, we also let $|X_2| = 30,\ |X_3| = 50\ \ |Y_1| = 28$. So, inequality (12) and (17) are linear matrix inequality in $a, b, c$, $ak_1$, $bk_2$, $ck_3$. By solving the LMI (12) and (17), the following solutions are obtained:

$$a = 0.0295, b = 1.0035, c = 0.0087,\ \ ak_1 = 7.7786, bk_2 = 23.7544, ck_3 = 15.0975.$$

This yields $k_1 = 26.7858, k_2 = 143.3579, k_3 = 55.3225$. Therefore, the controller (16) will drive the slave system (14) to synchronize the master system (13) as desired, the synchronous errors are shown in Figs. 4.

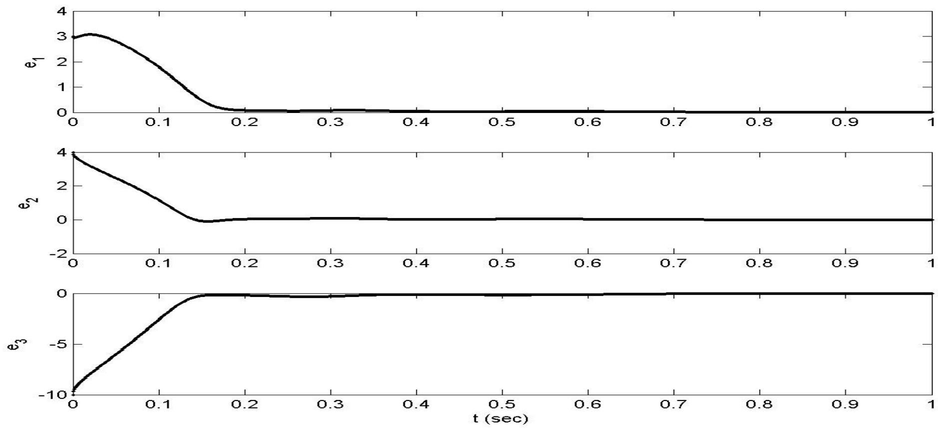


Fig.4. Synchronization errors between fractional order Lü and Chen systems

# 5. Conclusion

This paper discusses chaos synchronization between two identical or different fractional order chaotic systems. The simple state feedback controllers for fractional-order chaos synchronisation are obtained base on linear matrix inequality, simulation results are presented to demonstrate the application of theoretical results.

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# 7. References

[1] L. Pecora, T. Carroll. Synchronization in chaotic systems. *Phys. Rev. Lett*. 1990, **64**: 821–824.

[2] J. Lü, J. Lu. Controlling uncertain Lü system using linear feedback. *Chaos Solit Fract*. 2003, **17**: 127-132.

[3] J. Lu, X. Wu, X. Han, J. Lü. Adaptive feedback synchronization of a unified chaotic system. *Phys. Lett. A* 2004, **329**: 327–333.

[4] J. Lu, J. Cao. Adaptive Q-S (lag, anticipated, and complete) time-varying synchronization and parameters identification of uncertain delayed neural networks. *Chaos*. 2006, **16**: 023119.

[5] W. Yu, J. Cao. Adaptive synchronization and lag synchronization of uncertain dynamical systemwith time delay based on parameter identification. *Physica A.* 2007, **375**: 467–482.

[6] Sh.Chen, F.Wang, Ch. Wang. Synchronizing strict-feedback and general strict-feedback chaotic systems via a single controller. *Chaos Solit Fract.* 2004, **20**: 235–243.

[7] R. Luo. Impulsive control and synchronization of a new chaotic system. *Phys. Lett. A*. 2008, **372**: 648-653.

[8] Sh. Chen, Q. Yang. Impulsive control and synchronization of unified chaotic system. *Chaos Solit Fract*. 2004, **20**: 751–758.

[9] Sh. Chen, J. Lü. Parameters identification and synchronization of chaotic systems based upon adaptive control. *Phys. Lett. A*. 2002, **299**: 353–358.

[10] J. Zhou, J. Lu, J. Lü. Adaptive synchronization of an uncertain complex dynamical network. *IEEE Transactions on Automatic Control.* 2006, **51**(4): 652–656.

[11] W. Yu, J. Cao. Adaptive synchronization and lag synchronization of uncertain dynamical system with time delay based on parameter identification. *Physica A*. 2007, **375**: 467-482.

[12] Y. Tang, J. Fang. General methods for modified projective synchronization of hyperchaotic systems with known or unknown parameters. *Phys. Lett. A*. 2008, **372**: 1816–1826.

[13] G. Wen, D. Xu, Nonlinear observer control for full-state projective synchronization in chaotic continuous-time systems. *Chaos Solit Fract*. 2005, **26**: 71-77.

[14] J. Zhou, T. Chen, X. Li. Adaptive synchronization of coupled chaotic systems based on parameters identification and its applications. *Int. J. Bifur*. *Chaos*. 2006, **16**: 2923–2933.

[15] J. Zhou, X. Li, Z. Liu. Global synchronization in general complex delayed dynamical networks and its applications. *Physica A*. 2007, **385**: 729–742.

[16] I. Podlubny. *Fractional differential equations*. San Diego: Academic Press, 1999.

[17] S. Samko, A. Kilbas, O. Marichev. *Fractional integrals and derivatives: theory and applications.* Yverdon: Gordon and Breach, 1993.

[18] A. Kilbas, H. Srivastava, J. Trujillo. *Theory and applications of fractional differential equations*. Amsterdam: Elsevier, 2006.

[19] J. Sabatier, S. Poullain, P. Latteux, J. Thomas, A. Oustaloup. *Nonlinear Dyn*. 2004, **38**: 383–400.

[20] A. Kiani-B, K. Fallahi, N. Pariz, H. Leung. A chaotic secure communication scheme using fractional chaotic systems based on an extended fractional Kalman filter. *Commun Nonlinear Sci Numer Simulat*. 2009, **14**: 863–879.

[21] W. Deng, J. Lü. Generating multi-directional multi-scroll chaotic attractors via a fractional differential hysteresis system. *Phys. Lett. A.* 2007, **369**: 438–443.

[22] Y.Yu, H.Li. The synchronization of fractional-order Rossler hyperchaotic systems. *Physica A.* 2008, **387**: 1393–1403.

[23] G. Peng, Y. Jiang, F. Chen. Generalized projective synchronization of fractional order chaotic systems. *Physica A*. 2008, **387**: 3738–3746.

[24] X. Wu, J. Li. Chaos in the fractional order unified system and its synchronization. *Journal of the Franklin Institute*. 2008, **345**: 392–401.

[25] X. Wang, Y. He. Projective synchronization of fractional order chaotic system based on linear separation. *Phys. Lett*. *A*. 2008, **372**: 435–441.

[26] C. Li, X. Liao, J. Yu. Synchronization of fractional order chaotic systems. *Phys Rev E.* 2003, **68**: 067203.

[27] J. Lu. Chaotic dynamics of the fractional-order Lü system and its synchronization. *Phys. Lett. A*. 2006, **354**: 305–311.

[28] C. Li, J.Yan. The synchronization of three fractional differential systems. *Chaos Solit Fract.* 2007, **32**: 751–757.

[29] Y. Tang, J. Fang. Synchronization of N-coupled fractional-order chaotic systems with ring connection. *Commun Nonlinear Sci Numer Simulat.* 2010, **15**:401–412.

[30] Y. Tang, Z. Wang, J. Fang. Pinning control of fractional-order weighted complex networks. *Chaos*. 2009, **19**: 013112.

[31] X. Wang, J. Song. Synchronization of the fractional order hyperchaos Lorenz systems with activation feedback control. *Commun Nonlinear Sci Numer Simulat*. 2009, **14**: 3351–3357.

[32] S.Bhalekar, V. Daftardar. Synchronization of different fractional order chaotic systems using active control. *Commun Nonlinear Sci Numer Simulat*. (2010), doi:10.1016/j.cnsns.2009, **12**: 016.

[33] J. Yang, D. Qi. The feedback control of fractional order unied chaotic system. *Chin. Phys. B.* 2010, **19**: 020508.

[34] Caputo M. *Geophys J R Astron Soc.* 1967, **13**: 529.

[35] J. Lü, G. Chen. A new chaotic attractor coined. *Int. J. Bifur. Chaos.* 2002, **12**: 659–661.

[36] G. Chen, T. Ueta, Yet another chaotic attractor. *Int. J. Bifur. Chaos*. 1999, **9**: 1465–1466.

[37] D. Matignon. Stability result on fractional differential equations with applications to control processing. *IMACS-SMC Proceedings*. Lille, France. 1996, pp. 963–968.