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Global Exponential Stability of Cellular Neural Network with Discrete and Distributed Delays

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Abstract. This paper is concerned with analysis problem for the global exponential stability of a class of recurrent neural networks (RNNs) with mixed discrete and distributed delays. We give the sufficient condition of global exponential stability of cellular neural network with mixed discrete and distributed delays by employing the Lyapunov-Krasovskii functional and Young inequality, in addition, the example is provided to illustrate the applicability of the result.

Keywords: Global exponential stability; cellular neural network; discrete and distributed delays; Lyapunov–Krasovskii functional; Young inequality

1. Introduction

Cellular neural network (CNN) has become a new discipline branch since the Chua and Yang of California University proposed the CNN in 1988. The main function of CNN is about to change an input image into a corresponding output image, in order to accomplish this feature, we must first concern the stability of system. The various generalizations of neural networks have attracted attention of the scientific community due to their promising potential for tasks of classification, associative memory, parallel computation and the ability to difficult optimization [1-5]. Such applications rely on the existence of equilibrium points and the qualitative properties of neural networks. The time delay is commonly existed in various engineering systems such as chemical processes, hydraulic and rolling mill systems, etc[6-10]. These effects are unavoidably existed in the implementation of neural networks, and may cause undesirable dynamic network behaviors such as oscillation and instability. Therefore, it is important to investigate the stability of delayed neural networks. The stability analysis of neural networks plays an important role in the designs and applications. A large number of the criteria on the stability of neural networks have been derived in the literature. Neural network usually has a spatial nature due to the presence of various parallel pathways with a variety of axon sizes and lengths, so it is desirable to model them by introducing unbounded delays [11-15]. Thus, there will be a distribution of conduction velocities along these pathways and a distribution of propagation delays. In recent years there has been a growing research interest in study of neural networks with distributed delays. In fact, both discrete and distributed delays should be taken into account when modeling a realistic neural network [16-20]. Based on the above discussions, we consider a class of mixed discrete and distributed delays cellular neural network described by a neutral integro-differential equation. The main purpose of this paper is to study the global exponential stability for neutral-type delayed neural networks with unbounded distributed delays. The paper is organized as follows: In Section 2, System Description and Preli minaries are stated and some definitions and lemmas are listed. Based on the Lyapunov stability theory and Young inequality, theorems and corollary about global exponential stability of multi-delay and distributed delay cellular neural network in Section 3. We give the conclusion of this paper in Section 4.

2. Problem formulation

Consider the following multi-delay and distributed delay cellular neural network model：

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$$\dot{x}_i(t)=-d_i x_i(t)+\sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}c_{ij}\int_{-\infty}^{t}K_{ij}(t-s)h_j(x_j(s))\mathrm{d}s+\sum_{j=1}^{n}b_{ij}g_j(x_j(t-\tau_{ij}(t)))+I_i, i=1,2,\ldots,n \quad (2.1)$$

where $\phi_i(\theta)$ is bounded and continuous in the sub $[0,\infty)$, n is the number of the neurons in the neural network, , the constants a_{ij}, b_{ij} and c_{ij} denote, respectively, the connection weights, the discretely delayed connection weights and the distributively delayed connection weighted, of the jth neuron on the I neuron. $x_i(t)$ denotes the state of the ith neural neuron at time t, $f_j(x_j(t))$, $g_j(x_j(t))$ and $h_j(x_j(t))$ are the activation functions of the jth neuron at time t , I_i is the external bias on the ith neuron, d_i denotes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs. $\tau_{ij} \geq 0$ is a bounded time-varying delay, the kernel function K_{ij}: $[0,\infty)\rightarrow[0,\infty)$ is continuous in the sub$[0,\infty)$, and satisfies $\int_0^{\infty}K_{ij}(s)\mathrm{d}s=1, i, j=1,2,\ldots,n$, the initial situation is $x_i(\theta)= \phi_i(\theta)$, $\rho=\max(\tau_{ij}(t))$, $-\rho\leq\theta\leq0$.

Definition 1. $x_i^* \ (i=1,2,\ldots,n)$ is the equilibrium point of (2.1) associated with a given I_i (i=1,2,…,n) is said to be globally exponentially stable, if there are positive constants $k>0$and $\mu>0$such that every solution $x_i^* \ (i=1,2,\ldots,n)$ of (2.1) satisfies as follows

$$\left|x_i(t)-x_i^*\right| \leq \mu e^{-kt}\sup_{-\rho\leq\theta\leq0}\left|\phi_i(\theta)-x_i^*\right|, \forall t>0.$$

Definition 2. $\forall\ \phi(\theta)\in C\left([-\rho,0],R^n\right)$, we definite

$|\phi|=\max\left\{\left\|\phi(\theta)\right\|:\theta\in[-\rho,0]\right\}$, then we can get as follows

$$\left\|\phi-x^*\right\|_r^r=\sup_{-\infty\leq\theta\leq0}\sum_{i=1}^{n}\left|\phi_i(\theta)-x_i^*\right|^r, r>1$$

Assumption 1. (A1) For $i=1,2,\ldots,n$, the neuron activation functions in (2.1) satisfy

$$\left|f_j(s_1)-f_j(s_2)\right|\leq\alpha_j^+\left|s_1-s_2\right|, \left|g_j(s_1)-g_j(s_2)\right|\leq\beta_j^+\left|s_1-s_2\right|, \left|h_j(s_1)-h_j(s_2)\right|\leq\gamma_j^+\left|s_1-s_2\right|, \forall s_1\neq s_2$$

where $\alpha_j^+, \beta_j^+, \gamma_j^+$ are constants.

Assumption 2. (A2)

The neuron activation functions $f_j(x_j(t))$, $g_j(x_j(t))$, $h_j(x_j(t))$ j =1,2,…,n are bounded.

Lemma 1 [21] (Rogers-Holder Inequality)

if $p>1, \frac{1}{p}+\frac{1}{q}=1,$ and $a_k>0, b_k>0\ (k=1,2,...,n)$, Then

$$\sum_{k=1}^{n}a_k b_k \leq \left(\sum_{k=1}^{n}a_k^p\right)^{\frac{1}{p}}\left(\sum_{k=1}^{n}b_k^q\right)^{\frac{1}{q}}$$

Lemma 2 [22] (Young Inequality) if $e>0, h>0, P>1, \frac{1}{P}+\frac{1}{q}=1,$ then we can get

$$eh\leq\frac{1}{p}e^p+\frac{1}{q}h^q=\frac{1}{p}e^p+\frac{p-1}{p}h^{\frac{p}{p-1}}$$

3. Main results and proofs

Theorm 3.1. f_j, g_j, h_j are Lipschitz continuous and $\dot{\tau}_{ij}(t)<0$, if there are constants $r, \omega_i, q_{ij}, n_{ij}, h_{ij}, j_{ij}, l_{ij}, p_{ij}, q_{ji}, n_{ji}, h_{ji}, j_{ji}, l_{ji}, p_{ji}\in R\ (i=1,2,...,n)$, $\omega_i>0, r\geq1$ (when r =1, we must let $q_{ij}=n_{ij}=h_{ij}=j_{ij}=l_{ij}=p_{ij}=q_{ji}=n_{ji}=h_{ji}=j_{ji}=l_{ji}=p_{ji}=1$),

$$-\omega_i r d_i + (r-1)\sum_{j=1}^{n}\omega_i \left|a_{ij}\right|^{\frac{r-q_{ij}}{r-1}}\left|\alpha_j^+\right|^{\frac{r-n_{ij}}{r-1}} + \sum_{j=1}^{n}\omega_i\left|a_{ij}\right|^{q_{ij}}\left|\alpha_j^+\right|^{n_{ij}} + (r-1)\sum_{j=1}^{n}\omega_i\left|b_{ij}\right|^{\frac{r-h_{ij}}{r-1}}\left|\beta_j^+\right|^{\frac{r-j_{ij}}{r-1}}$$

$$+(r-1)\sum_{j=1}^{n}\omega_i\left|c_{ij}\right|^{\frac{r-l_{ij}}{r-1}}\left|\gamma_j^+\right|^{\frac{r-p_{ij}}{r-1}} + \sum_{j=1}^{n}\omega_i\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}} + \sum_{j=1}^{n}\omega_i\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}} + \sum_{j=1}^{n}\omega_i\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^{r} < 0$$

Then, the equilibrium point of multi-delay and distributed delay cellular neural network x* is global exponential stability.

Proof. We shift the equilibrium point $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ of (2.1) to the equation

$$u(t) = x(t) - x^* = [u_1(t), u_2(t), \dots, u_n(t)]^T$$

Thus we can get as follows

$$\dot{u}_i(t) = -d_i u_i(t) + \sum_{j=1}^{n} a_{ij} f_j^{\varepsilon}(u_j(t)) + \sum_{j=1}^{n} b_{ij} g_j^{\varepsilon}(u_j(t-\tau_{ij}(t))) + \sum_{j=1}^{n} c_{ij}\int_{-\infty}^{t} K_{ij}(t-s) h_j^{\varepsilon}(u_j(s))\mathrm{d}s \tag{3.1}$$

where

$$\begin{aligned} &f_j^{\varepsilon}(u_j(t)) = f_j(u_j(t)+x_j^*) - f_j(x_j^*), g_j^{\varepsilon}(u_j(t)) = g_j(u_j(t)+x_j^*) - g_j(x_j^*),\\ &h_j^{\varepsilon}(u_j(t)) = h_j(u_j(t)+x_j^*) - h_j(x_j^*), \end{aligned} \tag{3.2}$$

We design the following Lyapunov functional

$$\begin{aligned} V(u,t) = \sum_{i=1}^{n}\omega_i \Bigg\{ &\left|u_i(t)\right|^r e^{\varepsilon t} + \sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\int_{t-\tau_{ij}}^{t}\left|u_j(s)\right|^r e^{\varepsilon(s+\tau_{ij}(t))}\mathrm{d}s \\ &+ \sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\int_{t-\tau_{ij}}^{t}\left|g_j(u_j(\xi))\right|^r e^{\varepsilon(\xi+\tau_{ij}(t))}\mathrm{d}\xi + \sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^{r}\int_0^t\left|u_j(s-\tau_{ij})\right|^r e^{\varepsilon s}\mathrm{d}s \Bigg\} \end{aligned}$$

By (3.1), we calculate the Dini upper right derivative of the solution $V(u, t)$,

$$\begin{aligned} D^+V(u,t) = \sum_{i=1}^{n}\omega_i \Big\{ &e^{\varepsilon t}[\varepsilon\left|u_i(t)\right|^r + r\left|u_i(t)\right|^{r-1} sign(u_i(t))\,\dot{u}_i(t)] \\ &+ e^{\varepsilon t}\left[\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t)\right|^r e^{\varepsilon\tau_{ij}(t)} - \sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t-\tau_{ij})\right|^r\left(1-\dot{\tau}_{ij}(t)\right)\right] \\ &+ e^{\varepsilon t}\left[\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t))\right|^r e^{\varepsilon\tau_{ij}(t)} - \sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t-\tau_{ij}))\right|^r\left(1-\dot{\tau}_{ij}(t)\right)\right] \\ &+ \sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^{r} e^{\varepsilon t}\left|u_j(t-\tau_{ij})\right|^r + \sum_{j=1}^{n}\left|u_j(-\tau_{ij})\right|^r\dot{\tau}_{ij}(t) \Big\} \end{aligned}$$

By $\dot{\tau}_{ij}(t) \le 0$,

$$D^{+}V(u,t)\le \sum_{i=1}^{n}\omega_i\Big\{e^{\varepsilon t}[\varepsilon|u_i(t)|^r+r|u_i(t)|^{r-1}sign(u_i(t))\dot{u}_i(t)]$$
$$+e^{\varepsilon t}\left[\sum_{j=1}^{n}|b_{ij}|^{h_{ij}}|\beta_j^+|^{j_{ij}}|u_j(t)|^r e^{\varepsilon\tau_{ij}(t)}-\sum_{j=1}^{n}|b_{ij}|^{h_{ij}}|\beta_j^+|^{j_{ij}}|u_j(t-\tau_{ij})|^r\right]$$
$$+e^{\varepsilon t}\left[\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|g_j(u_j(t))|^r e^{\varepsilon\tau_{ij}(t)}-\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|g_j(u_j(t-\tau_{ij}))|^r\right]$$
$$+\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|\beta_j^+|^r e^{\varepsilon t}|u_j(t-\tau_{ij})|^r\Big\}$$
$$\le \sum_{i=1}^{n}\omega_i\Big\{e^{\varepsilon t}[\varepsilon|u_i(t)|^r+r|u_i(t)|^{r-1}sign(u_i(t))(-d_iu_i(t)+\sum_{j=1}^{n}a_{ij}f_j^{\varepsilon}(u_j(t))$$
$$+\sum_{j=1}^{n}b_{ij}g_j^{\varepsilon}(u_j(t-\tau_{ij}(t)))+\sum_{j=1}^{n}c_{ij}\int_{-\infty}^{t}K_{ij}(t-s)h_j^{\varepsilon}(u_j(s))\mathrm{d}s)]$$
$$+e^{\varepsilon t}\left[\sum_{j=1}^{n}|b_{ij}|^{h_{ij}}|\beta_j^+|^{j_{ij}}|u_j(t)|^r e^{\varepsilon\tau_{ij}(t)}-\sum_{j=1}^{n}|b_{ij}|^{h_{ij}}|\beta_j^+|^{j_{ij}}|u_j(t-\tau_{ij})|^r\right]$$
$$+e^{\varepsilon t}\left[\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|g_j(u_j(t))|^r e^{\varepsilon\tau_{ij}(t)}-\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|g_j(u_j(t-\tau_{ij}))|^r\right]$$
$$+\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|\beta_j^+|^r e^{\varepsilon t}|u_j(t-\tau_{ij})|^r\Big\}$$
$$\le e^{\varepsilon t}\sum_{i=1}^{n}\omega_i\Big\{(\varepsilon-rd_i)|u_i(t)|^r+r\sum_{j=1}^{n}|a_{ij}||\alpha_j^+||u_i(t)|^{r-1}|u_j(t)|$$
$$+r\sum_{j=1}^{n}|b_{ij}||\beta_j^+||u_i(t)|^{r-1}|u_j(t-\tau_{ij}(t))|+r\sum_{j=1}^{n}|c_{ij}||\gamma_j^+||u_i(t)|^{r-1}|u_j(t)|$$
$$+\sum_{j=1}^{n}|b_{ij}|^{h_{ij}}|\beta_j^+|^{j_{ij}}|u_j(t)|^r e^{\varepsilon\tau_{ij}(t)}-\sum_{j=1}^{n}|b_{ij}|^{h_{ij}}|\beta_j^+|^{j_{ij}}|u_j(t-\tau_{ij})|^r$$
$$+\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|g_j(u_j(t))|^r e^{\varepsilon\tau_{ij}(t)}-\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|g_j(u_j(t-\tau_{ij}))|^r$$
$$+\sum_{j=1}^{n}|\gamma_j^+|^{p_{ij}}|\beta_j^+|^r|u_j(t-\tau_{ij})|^r\Big\}$$

1.when $r>1$, by Young inequality

$$r\sum_{j=1}^{n}|a_{ij}||\alpha_j^+||u_i(t)|^{r-1}|u_j(t)|$$
$$=r\sum_{j=1}^{n}[|a_{ij}|^{\frac{r-q_{ij}}{r-1}}|\alpha_j^+|^{\frac{r-n_{ij}}{r-1}}|u_i(t)|^r]^{\frac{r-1}{r}}[|a_{ij}|^{q_{ij}}|\alpha_j^+|^{n_{ij}}|u_j(t)|^r]^{\frac{1}{r}}$$
$$\le r\sum_{j=1}^{n}[|a_{ij}|^{\frac{r-q_{ij}}{r-1}}|\alpha_j^+|^{\frac{r-n_{ij}}{r-1}}|u_i(t)|^r]^{\frac{r-1}{r}}[|a_{ij}|^{q_{ij}}|\alpha_j^+|^{n_{ij}}|u_j(t)|^r]^{\frac{1}{r}}$$
$$\le (r-1)\sum_{j=1}^{n}[|a_{ij}|^{\frac{r-q_{ij}}{r-1}}|\alpha_j^+|^{\frac{r-n_{ij}}{r-1}}|u_i(t)|^r]+\sum_{j=1}^{n}[|a_{ij}|^{q_{ij}}|\alpha_j^+|^{n_{ij}}|u_j(t)|^r] \tag{3.3}$$

$$
\begin{aligned}
& r\sum_{j=1}^{n}\left|b_{ij}\right|\left|\beta_j^+\right|\left|u_i(t)\right|^{r-1}\left|u_j(t-\tau_{ij}(t))\right| \\
& = r\sum_{j=1}^{n}[\left|b_{ij}\right|^{\frac{r-h_{ij}}{r-1}}\left|\beta_j^+\right|^{\frac{r-j_{ij}}{r-1}}\left|u_i(t)\right|^r]^{\frac{r-1}{r}}[\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t-\tau_{ij}(t))\right|^r]^{\frac{1}{r}} \\
& \le r\sum_{j=1}^{n}[\left|b_{ij}\right|^{\frac{r-h_{ij}}{r-1}}\left|\beta_j^+\right|^{\frac{r-j_{ij}}{r-1}}\left|u_i(t)\right|^r]^{\frac{r-1}{r}}[\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t-\tau_{ij}(t))\right|^r]^{\frac{1}{r}} \\
& \le (r-1)\sum_{j=1}^{n}[\left|b_{ij}\right|^{\frac{r-h_{ij}}{r-1}}\left|\beta_j^+\right|^{\frac{r-j_{ij}}{r-1}}\left|u_i(t)\right|^r]+\sum_{j=1}^{n}[\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t-\tau_{ij}(t))\right|^r]
\end{aligned}
\tag{3.4}
$$

and

$$
\begin{aligned}
& r\sum_{j=1}^{n}\left|c_{ij}\right|\left|\gamma_j^+\right|\left|u_i(t)\right|^{r-1}\left|u_j(t)\right| \\
& = r\sum_{j=1}^{n}[\left|c_{ij}\right|^{\frac{r-l_{ij}}{r-1}}\left|\gamma_j^+\right|^{\frac{r-p_{ij}}{r-1}}\left|u_i(t)\right|^r]^{\frac{r-1}{r}}[\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\left|u_j(t)\right|^r]^{\frac{1}{r}} \\
& \le r\sum_{j=1}^{n}[\left|c_{ij}\right|^{\frac{r-l_{ij}}{r-1}}\left|\gamma_j^+\right|^{\frac{r-p_{ij}}{r-1}}\left|u_i(t)\right|^r]^{\frac{r-1}{r}}[\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\left|u_j(t)\right|^r]^{\frac{1}{r}} \\
& \le (r-1)\sum_{j=1}^{n}[\left|c_{ij}\right|^{\frac{r-l_{ij}}{r-1}}\left|\gamma_j^+\right|^{\frac{r-p_{ij}}{r-1}}\left|u_i(t)\right|^r]+\sum_{j=1}^{n}[\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\left|u_j(t)\right|^r]
\end{aligned}
\tag{3.5}
$$

By (3.3), (3.4) and (3.5), we can obtain

$$
\begin{aligned}
& D^+V(u,t) \\
& \le e^{\varepsilon t}\sum_{i=1}^{n}\omega_i\Big\{(\varepsilon-rd_i)\left|u_i(t)\right|^r+(r-1)\sum_{j=1}^{n}\left|a_{ij}\right|^{\frac{r-q_{ij}}{r-1}}\left|\alpha_j^+\right|^{\frac{r-n_{ij}}{r-1}}\left|u_i(t)\right|^r \\
& +\sum_{j=1}^{n}\left|a_{ij}\right|^{q_{ij}}\left|\alpha_j^+\right|^{n_{ij}}\left|u_j(t)\right|^r+(r-1)\sum_{j=1}^{n}\left|b_{ij}\right|^{\frac{r-h_{ij}}{r-1}}\left|\beta_j^+\right|^{\frac{r-j_{ij}}{r-1}}\left|u_i(t)\right|^r \\
& +\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t-\tau_{ij}(t))\right|^r+(r-1)\sum_{j=1}^{n}\left|c_{ij}\right|^{\frac{r-l_{ij}}{r-1}}\left|\gamma_j^+\right|^{\frac{r-p_{ij}}{r-1}}\left|u_i(t)\right|^r \\
& +\sum_{j=1}^{n}\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\left|u_j(t)\right|^r+\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t)\right|^r e^{\varepsilon\rho} \\
& -\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t-\tau_{ij})\right|^r+\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t))\right|^r e^{\varepsilon\rho} \\
& -\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t))\right|^r+\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t))\right|^r-\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t-\tau_{ij}))\right|^r \\
& +\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^r\left|u_j(t-\tau_{ij})\right|^r\Big\}
\end{aligned}
$$

$$\begin{aligned}
&\le e^{\varepsilon t}\sum_{i=1}^{n}\omega_i\Big\{(\varepsilon-rd_i)\left|u_i(t)\right|^r+(r-1)\sum_{j=1}^{n}\left|a_{ij}\right|^{\frac{r-q_{ij}}{r-1}}\left|\alpha_j^+\right|^{\frac{r-n_{ij}}{r-1}}\left|u_i(t)\right|^r\\
&\quad+\sum_{j=1}^{n}\left|a_{ij}\right|^{q_{ij}}\left|\alpha_j^+\right|^{n_{ij}}\left|u_j(t)\right|^r+(r-1)\sum_{j=1}^{n}\left|b_{ij}\right|^{\frac{r-h_{ij}}{r-1}}\left|\beta_j^+\right|^{\frac{r-j_{ij}}{r-1}}\left|u_i(t)\right|^r\\
&\quad+\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t-\tau_{ij}(t))\right|^r+(r-1)\sum_{j=1}^{n}\left|c_{ij}\right|^{\frac{r-l_{ij}}{r-1}}\left|\gamma_j^+\right|^{\frac{r-p_{ij}}{r-1}}\left|u_i(t)\right|^r\\
&\quad+\sum_{j=1}^{n}\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\left|u_j(t)\right|^r+\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t)\right|^r e^{\varepsilon\rho}-\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\left|u_j(t-\tau_{ij})\right|^r\\
&\quad+\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t))\right|^r e^{\varepsilon\rho}+\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t))\right|^r\\
&\quad-\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|g_j(u_j(t-\tau_{ij}))\right|^r+\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^{r}\left|u_j(t-\tau_{ij})\right|^r\Big\}\\
&\le e^{\varepsilon t}\sum_{i=1}^{n}\omega_i\Big\{(\varepsilon-rd_i)+(r-1)\sum_{j=1}^{n}\left|a_{ij}\right|^{\frac{r-q_{ij}}{r-1}}\left|\alpha_j^+\right|^{\frac{r-n_{ij}}{r-1}}+\sum_{j=1}^{n}\left|a_{ij}\right|^{q_{ij}}\left|\alpha_j^+\right|^{n_{ij}}\\
&\quad+(r-1)\sum_{j=1}^{n}\left|b_{ij}\right|^{\frac{r-h_{ij}}{r-1}}\left|\beta_j^+\right|^{\frac{r-j_{ij}}{r-1}}+(r-1)\sum_{j=1}^{n}\left|c_{ij}\right|^{\frac{r-l_{ij}}{r-1}}\left|\gamma_j^+\right|^{\frac{r-p_{ij}}{r-1}}+\sum_{j=1}^{n}\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\\
&\quad+\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}e^{\varepsilon\rho}+\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^{r}e^{\varepsilon\rho}+\sum_{j=1}^{n}\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^{r}\Big\}\left|u_i(t)\right|^r<0
\end{aligned}$$

2, when r =1, we must let

$$q_{ij}=n_{ij}=h_{ij}=j_{ij}=l_{ij}=p_{ij}=q_{ji}=n_{ji}=h_{ji}=j_{ji}=l_{ji}=p_{ji}=1\ (i,j=1,2,\ldots,n),$$

$$\begin{aligned}
&D^+V(u,t)\\
&\le e^{\varepsilon t}\sum_{i=1}^{n}\omega_i\left[(\varepsilon-rd_i)+\sum_{j=1}^{n}\left|a_{ij}\right|\left|\alpha_j^+\right|+\sum_{j=1}^{n}\left|c_{ij}\right|\left|\gamma_j^+\right|+\sum_{j=1}^{n}\left|c_{ij}\right|\left|\gamma_j^+\right|\left|\beta_j^+\right|+\sum_{j=1}^{n}\left|b_{ij}\right|\left|\beta_j^+\right|e^{\varepsilon\tau_{ij}(t)}\right]\left|u_i(t)\right|<0
\end{aligned}$$

Thus, We can learn that when $r = 1$, the conclusions are valid.

Corollary 1. f_j, g_j, h_j are Lipschitz continuous and $\dot{\tau}_{ij}(t)<0$, if there are constants $r,\omega_i,q_{ij},n_{ij},h_{ij},j_{ij},l_{ij},p_{ij},q_{ji},n_{ji},h_{ji},j_{ji},l_{ji},p_{ji}\in R\ (i=1,2,\ldots,n)$, $\omega_i>0, r\ge 1$ (when r =1, we must let $q_{ij}=n_{ij}=h_{ij}=j_{ij}=l_{ij}=p_{ij}=q_{ji}=n_{ji}=h_{ji}=j_{ji}=l_{ji}=p_{ji}=1$),

$$\begin{aligned}
&-\omega_i rd_i+(r-1)\sum_{j=1}^{n}\omega_i\left|a_{ij}\right|^{\frac{r-q_{ij}}{r-1}}\left|\alpha_j^+\right|^{\frac{r-n_{ij}}{r-1}}+\sum_{j=1}^{n}\omega_i\left|a_{ij}\right|^{q_{ij}}\left|\alpha_j^+\right|^{n_{ij}}+(r-1)\sum_{j=1}^{n}\omega_i\left|b_{ij}\right|^{\frac{r-h_{ij}}{r-1}}\left|\beta_j^+\right|^{\frac{r-j_{ij}}{r-1}}\\
&+(r-1)\sum_{j=1}^{n}\omega_i\left|c_{ij}\right|^{\frac{r-l_{ij}}{r-1}}\left|\gamma_j^+\right|^{\frac{r-p_{ij}}{r-1}}+\sum_{j=1}^{n}\omega_i\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}+\sum_{j=1}^{n}\omega_i\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}+\sum_{j=1}^{n}\omega_i\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^{r}<0
\end{aligned}$$

Then, the equilibrium point of multi-delay and distributed delay cellular neural network x* is global exponential stability.

Proof. If we design the Lyapunov functional as follows

$$V(u,t)=\sum_{i=1}^{n}\omega_i\Bigg\{\left|u_i(t)\right|^r e^{\varepsilon t}+\sum_{j=1}^{n}\left|b_{ij}\right|^{h_{ij}}\left|\beta_j^+\right|^{j_{ij}}\int_{t-\tau_{ij}}^{t}\left|u_j(s)\right|^r e^{\varepsilon(s+\tau_{ij}(t))}\mathrm{d}s$$

$$+\sum_{j=1}^{n}\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\int_{t-\tau_{ij}}^{t}\left|g_j(u_j(\xi))\right|^r e^{\varepsilon(\xi+\tau_{ij}(t))}\mathrm{d}\xi+\sum_{j=1}^{n}\left|c_{ij}\right|^{l_{ij}}\left|\gamma_j^+\right|^{p_{ij}}\left|\beta_j^+\right|^r\int_0^t\left|u_j(s-\tau_{ij})\right|^r e^{\varepsilon s}\mathrm{d}s\Bigg\}$$

This proof is similar to the proof of Theorem 3, we can easily derive the result. Its proof is straightforward and hence omitted.

4. Conclusion

A new sufficient condition is derived to guarantee the global exponential stability of the equilibrium point for cellular neural network with multi-delay and distributed delay. Comparing with traditional methods, this approach is effective.

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