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# Image Rectification Algorithm Based on Immune Monoclonal Strategy

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**Abstract.** To rapidly and accurately search the corresponding points along scan-lines, rectification of stereo image pairs are performed so that all the epipolar lines are parallel to the horizontal scan-lines and the vertical difference between the corresponding epipolar lines is zero. According to the layered rectification algorithm presented by charles loop, we can divide the process of rectification into three steps including projective transformation, affine transformation and shearing transformation. The key to proposed algorithm is the computation of projective matrix, the algorithm uses the affine epipolar geometry constraint to compute projective matrix and determines the optimal value of unknown parameters in projective matrix by immune monoclonal strategy. In the process of solving the matrix, the algorithm does not require the relative matrix be positive definite. The experiments show that the proposed algorithm is an effective image rectification algorithm and it has the obvious advantage in mean vertical difference, distortion and speed of rectification.

**Keywords:** image rectification, affine epipolar geometry, immune monoclonal strategy

## 1. Introduction

In stereovision, stereo matching is the key to solving various problems. Searching the corresponding points in stereo image pairs is along the corresponding epipolar lines of the two images. To improve the speed and precision of search, rectification is used to align the epipolar line with the conjugate epipolar line, making them coincide with the horizontal scan-lines and reducing stereo matching from a 2-D to a 1-D search.

The rectification of stereo image pairs can be performed under the condition of calibrated camera [2], but generally the rectification is under the uncalibrated condition, which has become an important research field of stereovision. Therefore, photogrammetrists all over the world present various algorithms for rectifying epipolar line. Hartley [3] determines the projective matrix through the constraint that difference between the corresponding points is minimum. Francesco [4] presents an algorithm of image rectification without computation of fundamental matrix and only dependent on coordinates of corresponding points, but the initial value computed by nonlinear optimization in the rectification lacks credit. Al-Zahrani [5] defines a reference plane by using arbitrary three groups of corresponding points in the stereo image pairs to determine the projective matrix, where the distortion after the rectification directly depends on the selection of three groups of corresponding points. Hsien-Huang P.Wu [6] determines and optimizes some parameters of projective matrix by minimizing the square sum of distance between image point and corresponding epipolar line, and estimates other parameters by shearing transform. Charles Loop [7] divides the transform matrix into a projective and affine component, and effectively reduces the distortion after image transform during the process of projective transformation, which is a high precise rectification algorithm of stereo image pairs. But the involving matrix must be positive-definite so as to compute projective matrix. Nevertheless these matrices may not be positive-definite because of the noise disturbance on corresponding image points. Therefore we cannot compute projective matrix and thus the rectification process fails.

According to layered rectification algorithm presented by Charles Loop, the paper divides the process of rectification into three steps including projective transformation, affine transformation and shearing transformation, in which the computation of projective matrix is the important and difficult problem. The paper utilizes affine epipolar geometry constraint to compute projective matrix and determines the optimal values of unknown parameters in projective matrix by immune monoclonal strategy, which makes the

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computation of the matrix not require the relative matrix be positive definite and ensures the completion of rectification for arbitrarily given corresponding image points.

## 2. Layered rectification algorithm of stereo image pairs

According to the algorithm presented by Charles Loop, the layered rectification algorithm of stereo image pairs includes three steps, i.e. projective transformation, affine transformation and shearing transformation. Rectifying matrices have the following form:

$$H_l = S_l A_l P_l \ , \ H_r = S_r A_r P_r$$

First, we compute projective transformation matrices $P_l$ and $P_r$ such that all epipolar lines are parallel in stereo image pairs and the epipoles are mapped to infinity. Secondly, we compute affine transformation matrices $A_l$ and $A_r$ such that all epipolar lines are parallel to horizontal scan-lines and the vertical difference between corresponding epipolar lines of image pairs is minimized. Thirdly, we compute shearing transformation $S_l$ and $S_r$ to reduce the horizontal distortion between the rectified images. The algorithm requires the involving matrices be positive definite when computing projective transformation matrix. These matrices are not positive definite if the noise has great influence on the corresponding points between image pairs, which makes the rectification not completed. To avoid the problem, the paper proposes the method that uses the epipolar constraint based on affine model to solve the projective transformation matrix. Furthermore, the precision of solving is raised by using immune monoclonal strategy in the process of computation.

### 2.1. The epipolar constraint based on affine model

According to the algorithm presented by Charles Loop, the projective transformation matrices $P_l$ and $P_r$ are determined by the corresponding epipolar lines $w_l$ and $w_r$ , which have the following form

$$P_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ w_{la} & w_{lb} & 1 \end{pmatrix}, P_r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ w_{ra} & w_{rb} & 1 \end{pmatrix} \tag{1}$$

Given an infinite point $z$ in image $I_l$, the following relation exists between epipolar lines $w_l$ and $w_r$

$$w_l = [e_l]_\times z, w_r = Fz \qquad z = \begin{pmatrix} \lambda & \mu & 0 \end{pmatrix}^T \tag{2}$$

By determining infinite point $z$, $P_l$ and $P_r$ should satisfy the constraint, i.e. mapping the epipoles $e_l$ and $e_r$ to points at infinity.

Let $m_l$ and $m_r$ be the projections of a 3D point $M$ in images $I_l$ and $I_r$ respectively, $F$ is a fundamental matrix of rank 2, then $m_l$ and $m_r$ satisfy the epipolar constraint equation

$$m_r^T F m_l = 0$$

The epipolar lines in each image are parallel after we use $P_l$ and $P_r$ to transform the images, and the epipolar plane consisting of corresponding epipolar lines are parallel. So the image pairs satisfy the epipolar constraint based on affine model, as shown in Figure 1. The corresponding points satisfy epipolar geometry constraint after transforming the original image pairs with $P_l$ and $P_r$, i.e.

$$m_r^T P_r^T F_a P_l m_l = 0 \tag{3}$$

where $F_a$ is the fundamental matrix between the transformed images. The fundamental matrix $F_a$ has the form

$$F_a = \begin{pmatrix} 0 & 0 & e_{ry} \\ 0 & 0 & -e_{rx} \\ e_{ly} & -e_{lx} & e \end{pmatrix} \tag{4}$$

where $e_l$=（$e_{lx}$、$e_{ly}$、1）$^T$, $e_r$=（$e_{rx}$、$e_{ry}$、1）$^T$ and e is the unknown parameter. By substituting Eq. (1), (2), (4) into Eq. (3), we can see that parameters $\lambda$ , $\mu$ and e are needed to be solved.

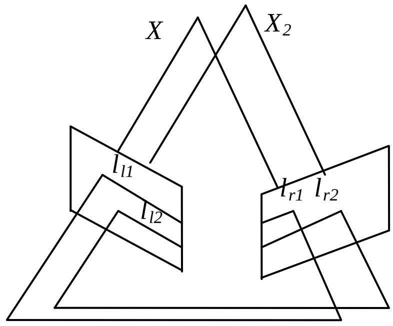


Fig. 1: the epipolar constraint based on affine model

Given the corresponding points of stereo image pairs, parameters $\lambda$ , $\mu$ and e can be solved by minimizing Eq. (5)

$$S = \sum_{i=1}^{N} [m_{r,i}^T P_r^T F_a P_l m_{l,i}]^2 \tag{5}$$

where $N$ is the number of corresponding point pairs, and $m_{l,i}$ and $m_{r,i}$ are corresponding points in images $I_l$ and $I_r$. In the situation of practical application, the rectified images may have larger distortion if we directly use the Eq. (5) as optimization function. Therefore, we add the Eq. (5) with three component constraint to make the optimization function change into

$$\sum_{i=1}^{N} [m_{r,i}^T P_r^T F_a P_l m_{l,i}]^2 + \lambda^2 + \mu^2 + e^2 \tag{6}$$

Whether we use Eq. (5) or Eq. (6) as optimization object function, projective transformation matrices $P_l$ and $P_r$ have multiple solutions, i.e. there exist different $P_l$ and $P_r$ such that the optimization function achieve minimization. But some matrices $P_l$ and $P_r$ lead to larger distortion on the rectified images.

For the optimization function Eq. (5) or Eq. (6), we can arbitrarily set the value of parameters λ, *u* and *e* unless that λ and *u* are not zero simultaneously when selecting the initial values of unknown parameters. Although the method is simple and effective, the precision of rectification on image pairs is lower. To solve the problem, the paper uses the immune monoclonal strategy to optimize the objective function, which not only achieves optimization, but also reduces the distortion on the rectified images.

# 3. Solution of projective matrices based on immune monoclonal strategy

The paper uses immune monoclonal strategy to optimize and solve the objective function Eq. (6) related to projective transformation. In the process of optimization, the monoclonal operator is used to determine the optimal value of unknown parameters in the projective transformation.

## 3.1. Monoclonal operator

The monoclonal operator is an antibody random map induced by affinity, the state transfer of antibody population is denoted as the following stochastic process:

$$C_{MS}: A(k) \xrightarrow{clone} B(k) \xrightarrow{mutation} C(k) \xrightarrow{selection} A(k+1)$$

where $A(k)$ is the k-th generation antibody population at present, $A(k)$ change into $B(k)$ by clone, and $B(k)$ change into $C(k)$ by mutation, finally $C(k)$ change into the next generation antibody population $A(k+1)$ by selection.

According to the definition of algorithm in the artificial immune system, antigen, antibody, the affinity between antigen and antibody correspond to the objective function and all kinds of restrictive conditions, the optimal solution, matching degree between solution and objective function respectively. According to the affinity function between antibody and antigen, a point in the solution space is divided into several same points by using clonal operator, a new antibody population is attained after performing clonal mutation and clonal selection, therefore the monoclonal operator includes clone, clonal mutation and clonal selection.

In the problem of optimizing the objective function Eq. (6), antigen and antibody correspond to optimal solution and candidate solution respectively, and the affinity function between antigen and antibody is the optimized objective function itself. During one generation of evolution, the population of mutation solution nearby candidate solution is produced on the basis of affinity to enlarge the search range, realizing global search and local search simultaneously.

## 3.2. Cauchy distribution

In the process of function optimization, the paper selects the operator of Cauchy mutation such that the algorithm can overcome prematurity effectively, keep the diversity of solution and enhance the convergent speed. Because Cauchy mutation and Cauchy distribution are closely related, we introduce the Cauchy distribution here to show the feasibility of selecting Cauchy mutation.

One-dimensional Cauchy density function centered on the origin is defined as:

$$f_t(x) = \frac{1}{\pi} \frac{t}{t^2 + x^2}, -\infty < x < \infty$$

where $t > 0$ and $t$ is the scale parameter, and the corresponding Cauchy distribution function is defined as:

$$F_t(x) = \frac{1}{2} + \frac{1}{\pi} \arctan(\frac{x}{t})$$

Cauchy density function $f_t(x)$ is similar to Gaussian density function, the difference between them is that: the former is slightly smaller than the latter in vertical direction, furthermore the former infinitely approaches to x axis, and the latter intersects with x axis. The relation between them is showed in the following Figure 2.

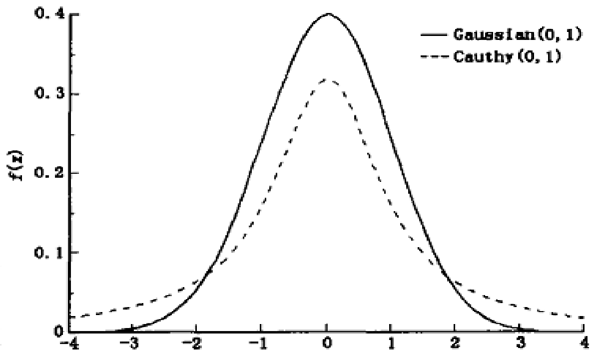


Fig. 2: The comparison between Gaussian density function and Cauchy density function

According to the above figure, we can see that Cauchy distribution easier produces random number far away from origin than Gaussian distribution, so we substitute Cauchy mutation for Gaussian mutation to generate offspring such that the algorithm is prevented falling into local optimum, effectively overcomes prematurity and has the ability of quick convergence.

## 3.3. Implementation steps of algorithm

When solving the projective transformation, the paper uses the immune monoclonal strategy to optimize objective function Eq. (6). The implementation steps of algorithm are as follows:

(1) Initialize parameter. We set the initial value of unknown parameter $\lambda, \mu$ and e as random number produced by random function.

(2) Judge halt condition. We judge whether the algorithm satisfies the halt condition, i.e., the optimal solutions of antibody population in the consecutive two iterations are not improved. If satisfying the halt condition, the algorithm will stop, determine the current value as the optimal solution and turn to step 8 for continuing. Otherwise the algorithm continues to implement step 3.

(3) Clone. We clone the current parent individual of the k-th generation, and the size of clone is 10, then the population becomes new population *B(k)* .

(4) Clonal mutation. The individual in the *B(k)* is added with Cauchy random number produced when t in the Cauchy density function equal to 1, and the population becomes *C(k)*.

(5) Calculate the affinity function. The affinity function in the algorithm is the optimized objective function Eq. (6). According to the result of every iteration, we compute affinity $Aff(C(k))$.

(6) Clonal selection. We select the individual as new parent, whose affinity is minimum.

(7) Set k equal to k+1, the algorithm turns to step 2.

(8) Get the optimal parameter, we can solve $P_l$ and $P_r$ according to the optimal parameter.

When using above algorithm to optimize the objective function, the algorithm is needed to run some times such that we can select the optimal parameter estimation not only to obtain small distortion of rectified images, but to achieve good rectification effect. For solving the projective matrices, the algorithm does not require any matrices be positive definite when estimating infinite point, completing the computation of projective matrices under the condition of arbitrarily given corresponding points in the images.

# 4. Experiments and analysis

This paper considers two aspects to evaluate the distortion of before and after rectification. Firstly, we compute the mean of difference in y coordinate of corresponding points in images to evaluate rectification accuracy of vertical difference between corresponding epipolar lines of images; the formula of computation is as follows:

$$\begin{cases} \overline{\Delta y}_{org} = \dfrac{1}{N}\sum_{i=1}^{N}\left|(m_{l,i})_y - (m_{r,i})_y\right| \\ \overline{\Delta y}_{rec} = \dfrac{1}{N}\sum_{i=1}^{N}\left|(m_{l,i}')_y - (m_{r,i}')_y\right| \end{cases} \tag{7}$$

where $\overline{\Delta y}_{org}, \overline{\Delta y}_{rec}$ are denoted as the mean of difference in y coordinate of corresponding points before and after image rectification; $(\cdot)_y$ indicates y coordinates [6]. Secondly, we compute the mean of difference in x coordinate of corresponding points to evaluate the distortion of horizontal difference after rectification, the formula of computation is as follows

$$\begin{cases} \overline{\Delta x}_{org} = \dfrac{1}{N}\sum_{i=1}^{N}\left|(m_{l,i})_x - (m_{r,i})_x\right| \\ \overline{\Delta x}_{rec} = \dfrac{1}{N}\sum_{i=1}^{N}\left|(m_{l,i}')_x - (m_{r,i}')_x\right| \end{cases} \tag{8}$$

where $\overline{\Delta x}_{org}, \overline{\Delta x}_{rec}$ are denoted as the mean of difference in x coordinate of corresponding points before and after image rectification; $(\cdot)_x$ indicates x coordinates [6].

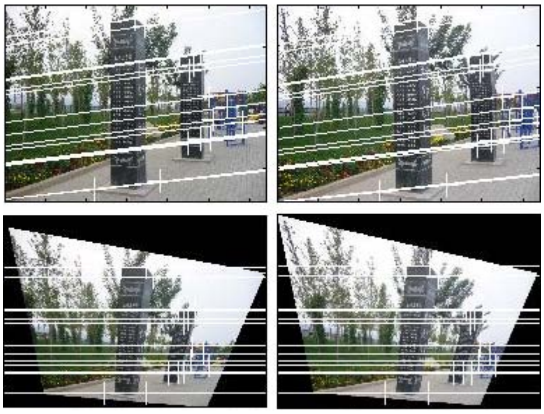

Fig. 3 stone pillar

The paper carries out two groups of experiments to observe the effect of immune monoclonal strategy on the result of image rectification in the problem of solving projective matrices. In the first group of experiments, the proposed algorithm is used to rectify the original image pairs. We use the immune monoclonal strategy to optimize two unknown parameters in the projective matrices in the course of rectification. In the second group of experiments, the algorithm sets initial values of λ, *u* and e as one and determines the projective matrices by Levenberg-Marquardt nonlinear optimization algorithm to complete rectification. In the two groups of experiments, the methods of solving affine transformation and shearing transformation are same.

In experiments, selected stereo image pairs are representative and reflect different imaging situation of vision system. The figures of experimental results are divided into two parts: the upper parts are original image pairs, the lower parts are results of rectification getted by using the proposed method. In these figures, the decussating points express corresponding points, the beelines express epipolar lines.

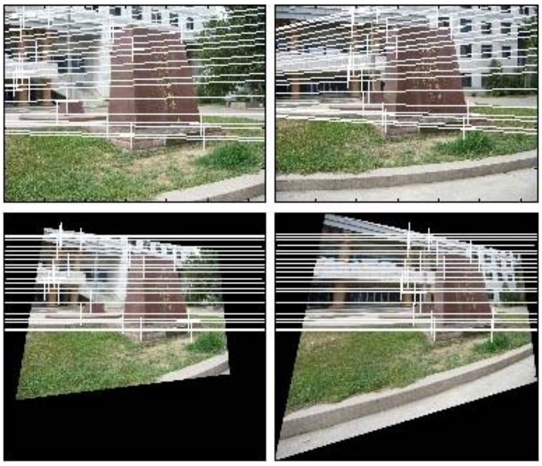

Fig. 4 stele

Fig. 5 board

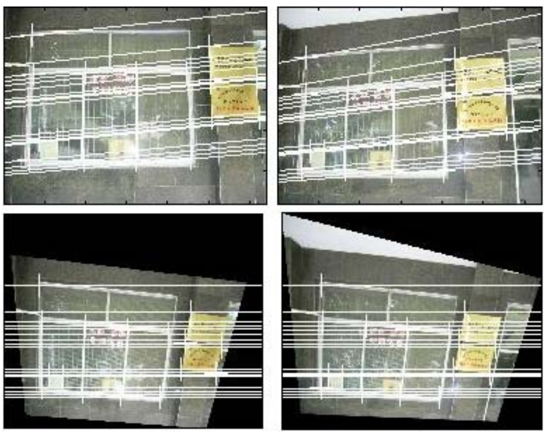

Fig. 6 window

When the proposed algorithm is used for rectification, it is needed to run about 5 times to get the optimal parameter estimation such that the rectified images have small distortion and the effect of rectification is good. The algorithm uses affine epippolar geometry constraints to compute projective matrices, therefore it is capable of completing the computation of projective matrices for any given image pairs.

Because the projective transformation is the first stage of image rectification, it has greater influence on the result of rectification. In the two groups of experiments, the results of rectification are different, which depend on whether use the immune monoclonal strategy to determine projective matrices or not. And the results of rectification are showed in the Table 1. By comparing the results of these two groups of experiments, we draw a conclusion that the mean of difference in y coordinate of corresponding points in the first group of experiments is evidently less than the mean in the second group of experiments. From here we know that it has the absolute advantage that immune monoclonal strategy is used to determine projective matrices in the process of rectification.

Table 1 two results of rectification

| method / name | Original image pairs | | The result of the first group of experiments | | The result of the second group of experiments | |
|---|---|---|---|---|---|---|
| | $\overline{\Delta x}_{org}$ | $\overline{\Delta y}_{org}$ | $\overline{\Delta x}_{rec}$ | $\overline{\Delta y}_{rec}$ | $\overline{\Delta x}_{rec}$ | $\overline{\Delta y}_{rec}$ |
| Fig.3 | 78.438 | 6.9375 | 40.891 | 0.39333 | 78.331 | 0.88948 |
| Fig.4 | 188.55 | 33.85 | 369.19 | 0.59451 | 402.63 | 0.63263 |
| Fig.5 | 63.417 | 152.33 | 196.24 | 0.3806 | 179.72 | 0.48182 |
| Fig.6 | 83.278 | 63.5 | 11.982 | 0.38727 | 24.884 | 0.5422 |

## 5. Conclusions

On the basis of layered rectification algorithm presented by Charles Loop, the paper divides the process of rectification into three steps including projective transformation, affine transformation and shearing transformation, in which the computation of projective matrices is the key of rectification algorithm. The paper uses affine epipolar geometry constraints to compute projective matrices and determines the projective transformation by immune monoclonal strategy, which makes the projective matrices have the global optimal solution for any given corresponding image points. The experiments show that the proposed algorithm in the paper has the obvious advantage in mean vertical difference, distortion and speed of rectification.

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