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Controlling a Novel Chaotic Attractor using Linear Feedback

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Abstract. In this paper, the linear feedback controlling of a novel chaotic system is discussed. The chaotic system is a new attractor which is similar to the Lorenz chaotic attractor, but it is not topological equivalent with the Lorenz chaotic system. This Letter proposes a novel approach for controlling this new attractor by linear feedback functions. The results obtained in this paper show that the trajectories of a new chaotic attractor can be controlled to any periodic target orbits or points. Furthermore, some numerical simulations show that the developed controller design method is effective and feasible. Therefore, the linear feedback controlling of new chaotic system may have good application prospects.

Keywords: Lorenz attractor, Chen attractor, Lü attractor, chaos control.

1. Introduction

The chaotic dynamical system is an interesting research area and has drawn wide attention. The responses of many nonlinear dynamical systems show some random phenomena. Today, this irregular and random-like behavior is termed as chaos and has attracted a great deal of researchers. Much work on analyzing and predicting the behavior of chaotic dynamical systems has been reported in the literature over the last years [1-2]. In order to make these results truly beneficial, many subtle issues on controlling chaos must be carefully investigated.Chaos control, in a broader sense, can be divided into two categories: one is to suppress the chaotic dynamical behavior and the other is to generate or enhance chaos in nonlinear systems. It is well known that most conventional control methods and many special techniques, such as Lyapunov function methods in paper [1], active control method in paper [2], linear state space feedback method in paper [3], inverse optimal control technique in paper [4], output feedback control technique in paper [5], and etc.

In this paper, the feedback control technique is employed to develop a very simple control law for guiding controlled a new chaotic system to any desired target periodic orbits or points. The organization of this paper is as follows. Firstly, some chaotic systems, include Lorenz system, Chen system, Lü system and our new system are described in Section II. Next, the controller design and the results on theoretical analysis of controlling chaos are presented in Section III. In Section IV, some numerical simulations are show the effectiveness and feasibility of the controller design method. Finally, the conclusions of this paper are summarized in Section V.

2. Some chaotic systems description

Since Lorenz found the first chaotic attractor in a smooth three-dimensional autonomous system, considerable research interests have been made in searching for the new chaotic attractors. In 1976, Rössler found a three-dimensional autonomous smooth chaotic system. Later, more and more attractors were found. Some new chaotic systems were recently coined, such as Chen system, Lü system, Liu system, the generalized Lorenz system family, and the hyperbolic type of the generalized Lorenz canonical form [1,2]. For the purpose of system, now we describe the fellowing three chaotic systems and their chaotic attractors, respectively as follows.

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Firstly, the model of Lorenz system is

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - xz - y, \\ \dot{z} = xy - bz. \end{cases} \tag{1}$$

with the paramethers $a = 10$, $b = 8/3$ and $c = 28$.

Next, the model of Chen system is

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = (c - a)x - xz + cy, \\ \dot{z} = xy - bz. \end{cases} \tag{2}$$

with the parameters $a = 35$, $b = 3$ and $c = 28$.

Then the model of Lü system is

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = -xz + cy, \\ \dot{z} = xy - bz. \end{cases} \tag{3}$$

with the parameters $a = 36$, $b = 3$ and $c = 20$.

The new chaotic system is discussed in the fellow, its coined process is similar to the Lü attractor coined. It is a three-dimensional autonomous system, according to the detailed numerical simulation as well as the theoretical analysis, the chaotic attractor obtained from this new system is also the butterfly-shaped attractor. The chaotic system is a new attractor which is similar to the Lorenz chaotic attractor, but it is not topological equivalent with the Lorenz chaotic system [3,4].

The following is our new chaotic system:

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - xz, \\ \dot{z} = xy - bz, \end{cases} \tag{4}$$

which is chaotic when $a = 10$, $b = 8/3$ and $c = 16$ (see Figs. 1-4).

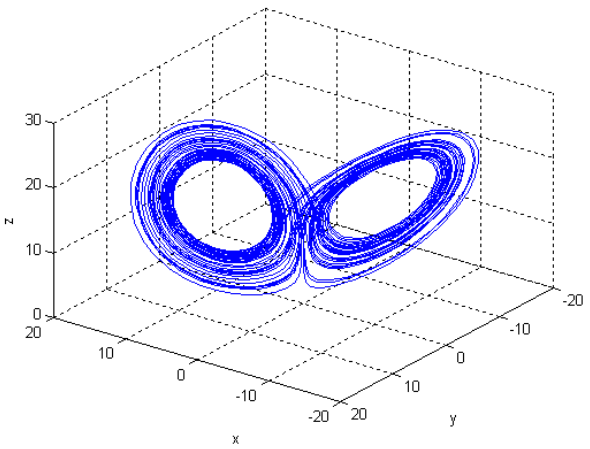


Fig. 1. The new chaotic attractor.

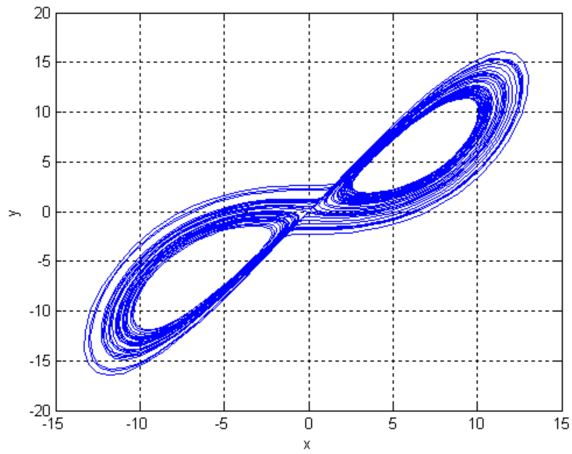


Fig. 2. *x-y* phase plane the new chaotic attractor.

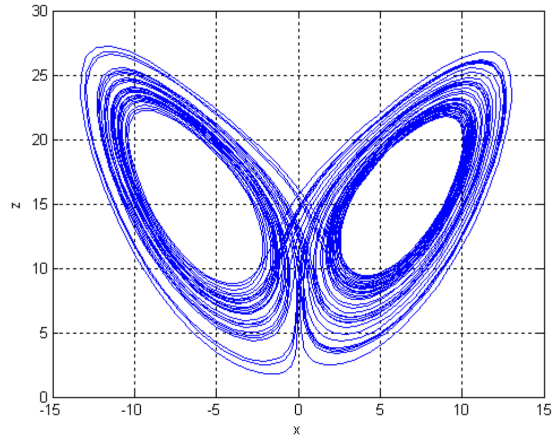


Fig. 3. *x-z* phase plane the new chaotic attractor.

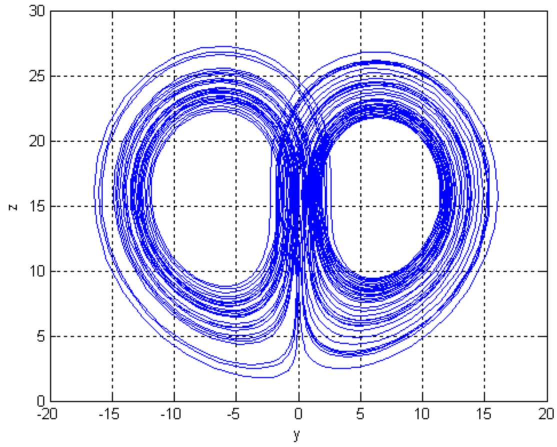


Fig. 4. *y-z* phase plane the new chaotic attractor.

From the control engineering point of view, the new chaotic attractor provides another interesting framework for advanced control techniques since it is more complex than the known Lorenz system and

Chua's system [6]. Moreover, it is more difficult to control the new system than the other than those already known chaotic systems due to the rapid change of the velocity in the z-direction [7]. Therefore, we will introduce a novel control law in the next section.

3. Linear feedback control

In this section, the main results with a new and simple control law will be discussed. The control law can drive the trajectories of the new chaotic system to any target periodic orbits or points.

It is very interesting to control the chaotic system to approach any desired target periodic orbits and points. Here, two control inputs are added to the first two states in (4), and then the controlled system can be stated as follows:

$$\begin{cases} \dot{x} = a(y-x) + u_1, \\ \dot{y} = cx - xz + u_2, \\ \dot{z} = xy - bz, \end{cases} \tag{5}$$

According to the property of chaotic orbits, the new system can reach any point $x = x_0$ near the chaotic attractor, but will not stay at $x = x_0$ without further control. In order to stabilize the new system at $x = x_0$, the control inputs must satisfy the stable condition: $d\dot{x}/dt = d\dot{y}/dt = d\dot{z}/dt = 0$.

Let the control inputs u_1 and u_2 be both linear, in the form:

$$\begin{cases} u_1 = -ay - a|x_0|(x - x_0 - sign(ax_0)), \\ u_2 = -y. \end{cases} \tag{6}$$

And the controlled system (4) becomes

$$\begin{cases} \dot{x} = -ax - a|x_0|(x - x_0 - sign(ax_0)), \\ \dot{y} = cx - xz - y, \\ \dot{z} = xy - bz. \end{cases} \tag{7}$$

Obviously, the system stated in equation (7) has a unique equilibrium point $S_0(x_0, bcx_0/(b+x_0^2), cx_0^2/(b+x_0^2))$.

It can be easily obtained that the Jacobian matrix $J(S_0)$ of system (7) is given by

$$J(S_0) = \begin{pmatrix} -a - a|x_0| & 0 & 0 \\ \dfrac{bc}{b+x_0^2} & -1 & -x_0 \\ \dfrac{bcx_0}{b+x_0^2} & x_0 & -b \end{pmatrix} \tag{8}$$

And the corresponding characteristic equation of $J(x_0)$ is described as $(\lambda + a + a|x_0|)[\lambda^2 + (b+1)\lambda + b + x_0^2] = 0$.

Since the three characteristic roots of the above equation possess negative real part, in case $a > 0$ and $b > \max\{-1, -x_0^2\}$, the unique steady state S_0 is stable. Therefore, with the feedback control inputs (6), the new system will tend to the target point S_0.

In order to guide the controlled system (4) to reach an arbitrary point $x = x_1, y = y_1$, let the linear controller be

$$\begin{cases} u_1 = -ay - a|x_1|(x - x_1 - sign(ax_1)), \\ u_2 = -y - |d|(y - y_1 - sign(d)), \end{cases} \tag{9}$$

where $d = x_1^2 y_1 / b + y_1 - cx_1$. In this case, the controlled system (4) becomes

$$\begin{cases} \dot{x} = -ax - a|x_1|(x - x_1 - sign(ax_1)), \\ \dot{y} = cx - xz - y - |d|(y - y_1 - sign(d)), \\ \dot{z} = xy - bz. \end{cases} \tag{10}$$

Obviously, system (10) has a unique equilibrium point $S_1(x_1, y_1, x_1 y_1 / b)$. It can be easily verified from the Jacobian matrix $J(S_1)$ of system (10) which is given as

$$J(S_1) = \begin{pmatrix} -a - a|x_1| & 0 & 0 \\ c - \dfrac{x_1 y_1}{b} & -1 - |d| & -x_1 \\ y_1 & x_1 & -b \end{pmatrix}. \tag{11}$$

And the corresponding characteristic equation of $J(S_1)$ is

$$(\lambda + a + a|x_1|)[\lambda^2 + (1 + b + |d|)\lambda + b + b|d| + x_1^2] = 0.$$

It has three characteristic roots possess negative real part, and the unique steady state $S_1(x_1, y_1, x_1 y_1 / b)$ is stable. Therefore, with the feedback control inputs (9), the new system will approach the target point $S_1(x_1, y_1, x_1 y_1 / b)$.

Now, in order to guide the trajectories of the new controlled system to approach any target periodic orbits, we assume that x_1 and y_1 are periodic functions. Let

$$x_1 = r\cos\omega t, y_1 = r\sin\omega t \tag{12}$$

and we use the linear time-varying controller

$$\begin{cases} u_1 = -ay - ar|\cos\omega t|(x - r\cos\omega t - sign(ar\cos\omega t)), \\ u_2 = -y - |\overline{d}|(y - r\sin\omega t - sign(\overline{d})), \end{cases} \tag{13}$$

where $\overline{d} = r^3 \sin 2\omega t \cos\omega t / 2b + r\sin\omega t - cr\cos\omega t$.

Similarly, under the feedback control inputs (13), we can prove that new system will tend to the target periodic orbit:

$$x_1 = r\cos\omega t, y_1 = r\sin\omega t, z_1 = \frac{r^2 \sin 2\omega t}{2b}. \tag{14}$$

It is worth to point out that the above feedback controller (13) can be realized easily in practical application. In fact, we can design an outer oscillator, which can produce periodic sine signals. And then using the periodic signal and systematic output signal as control input signal, the new system can be stabilized to the periodic orbit (14) [8].

4. Numerical simulations

In order to verify the control applicability of the proposed control laws (6), (9) and (13), we suppose that system (4) is in its chaotic state currently, with parameters $a = 10$, $b = 8/3$ and $c = 16$, and the initial conditions $x(0) = 1.0$, $y(0) = -1.0$ and $z(0) = 10$. The fourth order Runge-Kutta method is used to solve the systems with different equations, such as (7) and (10), and time step size 0.001 in all numerical simulations.

With the control law (6), for different initial values x_0, the state response curves are shown in Fig. 5 (a) and (b). The numerical results indicate the system (4) can reach the target point $(x_0, bcx_0/(b+x_0^2), cx_0^2/(b+x_0^2))$ within 1.5 seconds for every x_0.

With the control law (9) applied, for different initial values x_1, y_1, the state response curves are shown in Fig. 6 (a) and (b). The numerical results tell us that the system (4) can reach the target point $(x_1, y_1, x_1 y_1 / b)$ within 2 seconds for all x_1, y_1.

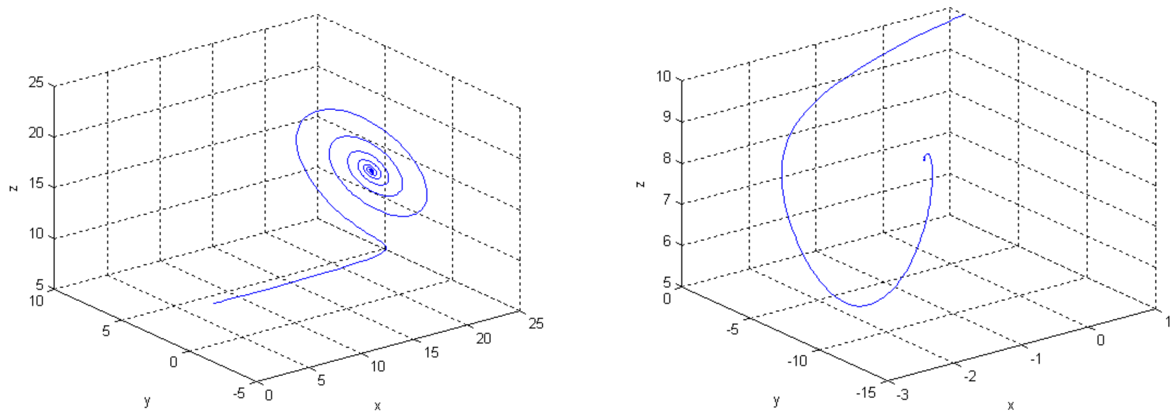
With the control law (13) applied, for different radiuses r and angles ω, the state response curves are shown in Fig. 7 (a) and (b). Moreover, the numerical results indicate us as follows:

(1) For different radiuses r and angles ω, the system can be stabilized at periodic orbits within $\dfrac{2\pi}{\omega}$ seconds. The period is $T = 2\pi/\omega$, and there is no distinct correlation between the period and the orbits.

(2) When $0.3 < r < 1$, the controlled system can be stabilized at the periodic orbit, but the shape and period of the orbit are different from the theoretical results. When $r \to 0$, the system can not be stabilized at the periodic orbit. The possible reason lies in that the feedback signal becomes relatively weak.

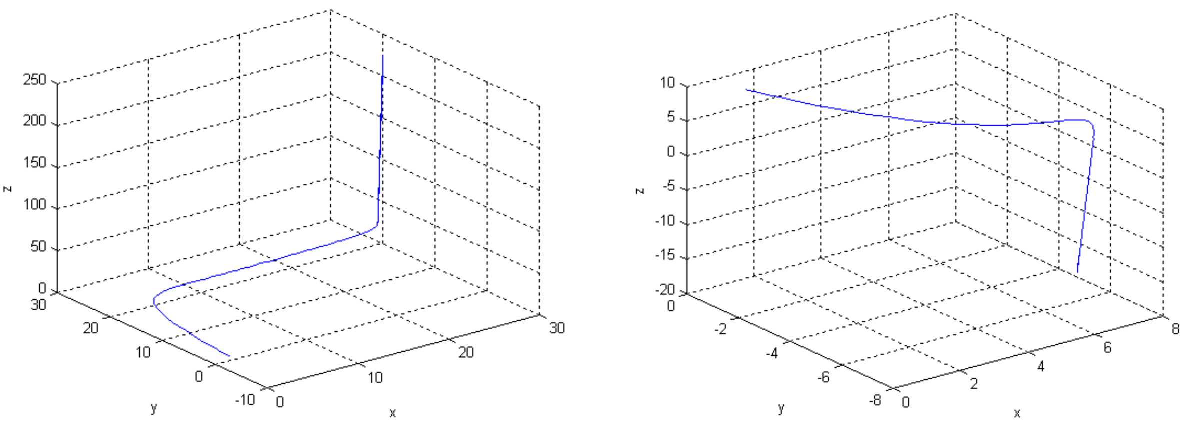
(3) When $r > 1$ and $0.05 < \omega < 120$, the *x*-*y* plane projections of the orbits evolve from similar roundness to ellipse.

All these results conform to our theoretical analysis. The possible difference may be caused by the relatively strong feedback signal [9, 10].



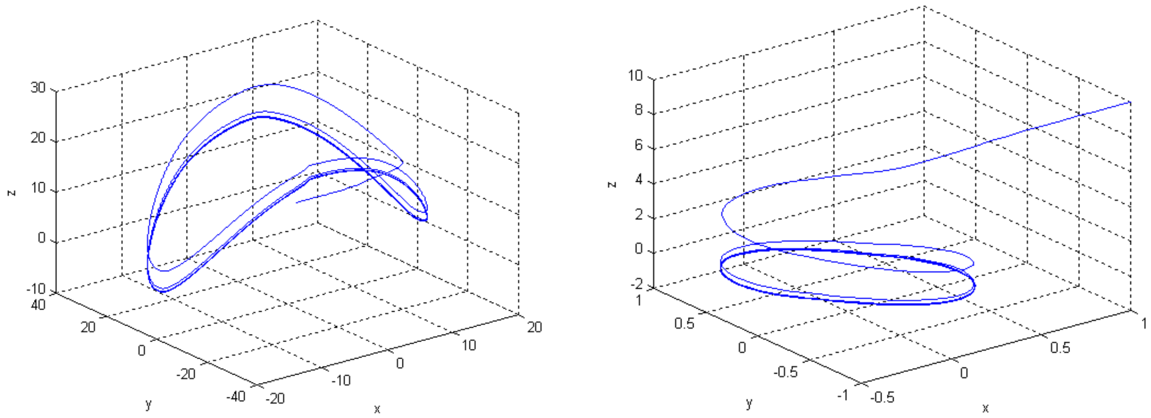
(a) (b)

Fig. 5. The controller guides the controlled system to approach the target points.
(a) $x_0 = 20, S_0(20,2.977667,19.851117)$. (b) $x_0 = -2, S_0(-2,-17.142857,11.428571)$.



(a) (b)

Fig. 6. The controller guides the controlled system to approach the target points.
(a) $x_1 = 30, y_1 = 20, S_1(30,20,200)$. (b) $x_1 = 7, y_1 = -6, S_1(7,-6,-14)$.



(a) (b)

Fig. 7. The controller guides the controlled system to approach the target periodic orbits. (a) $r = 20, \omega = 10$.
(b) $r = 0.5, \omega = 8$.

5. Conclusions and Remarks

We have presented a new effective linear control law for guiding the trajectories of the new chaotic system to approach any target periodic orbits or points. Compared with the other existing chaos control methods, this controller can drive the controlled system to approach any targets within the least time. The method is based on the new chaotic system and the other known system. So, controlling the new chaotic attractor would be more significant than the other existing chaos control methods. Meanwhile, the results of controlling chaos can be deduced beforehand. It should be pointed out that the higher nonlinearity and complexity of the new chaotic attractor justify the practical applications of the proposed method to some other complex dynamical systems as well. And we can change the initial conditions and parameters to fit the target periodic orbits or points in practical applications. So, the linear feedback control of new chaotic system may have good application prospects.

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