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# Adaptive Projective Synchronization in Weighted Dynamical Complex Networks with Time-Varying Coupling Delay

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**Abstract.** Recently, various papers investigated the geometry features, synchronization and control of complex network provided with certain topology. While, sometimes the exact topology of a network is unknown or uncertain. Using Lyapunov theory, we propose an adaptive feedback controlling method to identify the exact topology of a rather general weighted complex dynamical network model. By receiving the network nodes evolution, the topology of such kind of network with identical or different nodes, or even with switching topology can be monitored. Experiments show that the methods presented in this paper are of high accuracy with good performance.

**Keywords:** Complex networks, Time-varying coupling delay, Adaptive control

## 1. Introduction

Today, the complex network present exist in every corner of the world, from the communication networks to social networks, from cellular networks to metabolic networks, from the Internet to the World Wide Web [1-11]. The study of complex networks is under way. In many of the existing literature shows that synchronization and control of a complex dynamic network with some topology related[12-20], and in the real world, sometimes a complex and dynamic network topology is the true unknown or uncertain[21]. Based on the above arguments, to determine the complex network topology becomes a key problem in many disciplines, such as DNA replication, modification, repair and RNA (ribonucleic acid) transcription. It is of significance that by monitoring dynamic behavior of proteins during the process of recognition through NMR (nuclear magnetic resonance) technology [8], Therefore, to determine interactive network nodes that may exist in different topological structure of great significance. By adaptive feedback controlling method, the real network is served as a drive network, and we construct another response network receiving the evolution of each node, then the exact topology of the real network can be identified. Along with it, the evolution of every node is traced. Using Lyapunov stability theory [22-23], mathematical analysis of the mechanism is developed rigorously. Our controlling approach can be applied to a large amount of rather general weighted complex dynamical networks not only with identical nodes, but also with different nodes. Besides, even when the topology of the complex dynamical network changes, it can be monitored as well. All these will contribute to improving efficiency and accuracy of network analysis.

The left paper is organized as follows. Section 2 describes the topology identification method for a general weighted complex dynamical network with identical nodes. Identifying topology mechanism for such kind of network consisting of different nodes are detailed in Section 3. Section 4 gives three computational examples include network with identical nodes, network with different nodes and switching network with different nodes to illustrate effectiveness of the proposed approach. In section 5. we introduce adaptive controlling method, In section 6, we give an example and numerical simulation, The main ideas and conclusions are summarized up in Section 7.

## 2. Model description and preliminaries

In this paper, a complex dynamical network with time-varying coupling delay consisting of N identical nodes with linear couplings is considered, which is characterized by

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$$\dot{x}_i(t) = Bx_i(t) + f(t, x_i(t)) + \sum_{\substack{j=1 \\ j\neq i}}^{N} a_{ij} H(t) x_j(t - \tau(t)), \ i = 1, 2, \ldots, N \quad (1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots x_{iN}(t))^T \in R^N$ is the state vector of the $i$ th node, $H(t) = (h_{ij})_{N\times N} \in R^{N\times N}$ is inner-coupling matrix, $B = (b_{ij})_{N\times N}$ is a constant matrix, $A(t) = (a_{ij}(t))_{N\times N}$ is the unknown or uncertain weight configuration matrix, $f: R\times R^N \to R^N$ is a smooth nonlinear function, $\tau(t) \geq 0$ is the time-varying coupling delay. If there is a connection from node $i$ to node $j$ $(j \neq i)$ , then the coupling $a_{ij}(t) = c_{ij} \neq 0$ ,otherwise, $a_{ij}(t) = 0 \quad (j = i)$ and the diagonal elements of matrix $A(t)$ are defined as $\sum_{j=1}^{N} a_{ij} = 0 \Rightarrow a_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} a_{ij} \quad i = 1, 2, \ldots, N$ .

**Assumption 1.** Time delay $\tau(t)$ is a differential function with $\xi \in [2\dot{\tau}(t) - 1, 1]$.

**Assumption 2.** Suppose there exists a constant *L*, such that

$$\| f(t, x(t)) - f(t, y(t)) \| \leq L \| x(t) - y(t) \|$$

holds for any time-varying vectors $x(t)$ , $y(t)$ , and norm $\|\cdot\|$ of a vector *X* is defined as $\|X\| = (X^T X)^{1/2}$ .

**Lemma**. $\forall\ x(t) = [x_1(t), x_2(t), \cdots x_n(t)]^T \in R^n$ , $y(t) = [y_1(t), y_2(t), \cdots y_n(t)]^T \in R^n$ ,

There exist a positive definite matrix $p \in R^{n\times n}$ , the following matrix inequality holds:

$$2x^T y \leq x^T P x + y^T P^{-1} y .$$

## 3. Adaptive controlling method

In this section, we make drive-response complex dynamical networks with time-varying coupling delay achieve adaptive projective synchronization by using adaptive controlling method. We refer to model (1)as the drive complex network, and consider a response network described as following:

$$\dot{y}_i(t) = By_i(t) + f(t, y_i(t)) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \hat{a}_{ij} H(t) y_j(t - \tau(t)) + u_i, \quad i = 1, 2, \ldots, N \quad (2)$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \ldots y_{iN}(t))^T \in R^N$ is the response state vector of the $i$ th node, $u_i\ i = 1, 2, \ldots N$ are nonlinear controllers to be designed, and $\hat{A} = (\hat{a}_{ij})_{N\times N}$ is estimation of the weight matrix $A(t)$ .

Let $e_i(t) = x_i(t) - \lambda y_i(t), (\lambda \neq 0)$ , $\lambda$ is a scaling factor and $c_{ij} = \hat{a}_{ij} - a_{ij}$ , with the aid of Equations. (1) and (2), the following error dynamical network can be obtained:

$$\dot{e}_i(t) = Be_i(t) + f(t, x_i(t)) - \lambda f(t, y_i(t)) - \lambda \sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij} H(t) y_j(t - \tau(t)) + \sum_{\substack{j=1 \\ j\neq i}}^{N} a_{ij} H(t) e_j(t - \tau(t)) - \lambda u_i \quad (3)$$

Banach space in the definition of norm convergence with n-dimensional domain the absolute value of real convergence and, in accordance with the definition of synchronization we tend to know:

$$\lim_{t\to\infty} |e_i(t)| = \lim_{t\to\infty} |a_{ij} - \hat{a}_{ij}| = 0$$

**Theorem 1.** *Suppose Assumption 1 holds. Using the following adaptive controllers and updated laws:*

$$u_i = \frac{1}{\lambda}[d_i e_i(t) + \lambda f(t, y_i(t)) - f(t, \lambda y_i(t))], (\lambda \neq 0), i = 1, 2, \ldots, N \quad (4)$$

$$\dot{c}_{ij} = \lambda \delta_{ij} H(t) y_j(t - \tau(t)) e_i^T(t), (\lambda \neq 0), i, j = 1, 2, \ldots, N \quad (5)$$

$$\dot{d}_i = k_i (1 - \frac{d_i^*}{d_i}) e_i^T(t) e_i(t), d_i \neq 0, i = 1, 2, \ldots, N \quad (6)$$

*where* $d = (d_1, d_2 \cdots d_N)^T \in R^N$ *is the adaptive feedback gain vector to be designed,* $\delta_{ij} > 0,\ k_i > 0\ (i = 1, 2, \cdots N)$ *are arbitrary constants, then the response network (2) can synchronize with* *the drive network (1), and the weight configuration matrix* $A(t)$ *of network (1) can be identified by* $\hat{A}(t)$ *,i.e.,* $\lim_{t\to\infty}|e_i(t)| = \lim_{t\to\infty}|a_{ij} - \hat{a}_{ij}| = 0 \quad i = 1,2,\ldots,N$.

**Proof.** Choose the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}}c_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{k_i}d_i^2 + \frac{1}{1-\xi}\int_{t-\tau(t)}^{t}\sum_{i=1}^{N} e_i^T(t)e_i(t)dt \quad (7)$$

where $d_i^*$ is a positive constant to be determined. Calculating the derivative of（7）along the trajectories of (3), and with adaptive controllers(4) and updated laws (5) and (6).Thus, we obtain：

$$\dot{V}(t) = \sum_{i=1}^{N} e_i^T(t)[Be_i(t) + f(t,x_i(t)) - \lambda f(t,y_i(t)) - \lambda\sum_{j=1}^{N} c_{ij}H(t)y_j(t-\tau(t)) + \sum_{j=1}^{N} a_{ij}H(t)e_j(t-\tau(t)) - \lambda u_i] +$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}}c_{ij}\dot{c}_{ij} + \sum_{i=1}^{N}\frac{1}{k_i}d_i\dot{d}_i + \frac{1}{1-\xi}\sum_{i=1}^{N} e_i^T(t)e_i(t) - \frac{1-\dot{\tau}(t)}{1-\xi}\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t))$$
$$= \sum_{i=1}^{N} e_i^T(t)Be_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)a_{ij}H(t)e_j(t-\tau(t)) + \sum_{i=1}^{N} e_i^T(t)[f(t,x_i(t)) - f(t,\lambda y_i(t))]$$
$$+ \frac{1}{1-\xi}\sum_{i=1}^{N} e_i^T(t)e_i(t) - \frac{1-\dot{\tau}(t)}{1-\xi}\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t)) - \sum_{i=1}^{N} d_i^* e_i^T(t)e_i(t)$$
$$\le \sum_{i=1}^{N} e_i^T(t)Be_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)a_{ij}H(t)e_j(t-\tau(t)) + L\sum_{i=1}^{N} e_i^T(t)e_i(t)$$
$$+ \frac{1}{1-\xi}\sum_{i=1}^{N} e_i^T(t)e_i(t) - \frac{1-\dot{\tau}(t)}{1-\xi}\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t)) - \sum_{i=1}^{N} d_i^* e_i^T(t)e_i(t)$$
$$\le e^T(t)Be(t) + Le^T(t)e(t) + e^T(t)Pe(t-\tau(t)) - e^T(t)D^*e(t) + \frac{1}{1-\xi}e^T(t)e(t) - \frac{1-\dot{\tau}(t)}{1-\xi}e^T(t-\tau(t))e(t-\tau(t))$$
$$\le e^T(t)Be(t) + Le^T(t)e(t) + \frac{1}{2}e^T(t)PP^Te(t) - e^T(t)D^*e(t) + \frac{1}{1-\xi}e^T(t)e(t)$$
$$-\frac{1-\dot{\tau}(t)}{1-\xi}e^T(t-\tau(t))e(t-\tau(t)) + \frac{1}{2}e^T(t-\tau(t))e(t-\tau(t))$$

Let： $e(t) = (e_1^T(t), e_2^T(t), \cdots e_N^T(t))^T \in R^{N\times N}$ , $P = (A(t)\otimes H(t))$ , where $\otimes$ represents the Kronecker product. From Assumption 1, we get $\frac{1-\dot{\tau}(t)}{1-\xi} \ge \frac{1}{2}$ , thus we have:

$$\dot{V}(t) \le e^T(t)(B + LI + \frac{1}{1-\xi}I - D^* + \frac{1}{2}PP^T)e(t)$$

where $I$ is the identity maximal( $I = diag(\overbrace{1,1,\cdots 1}^{N})$ ), $D^* = diag(d_1^*, d_2^*, \cdots d_N^*)$ .

The constants $d_i^*$ ( $i = 1,2,\ldots N$ ) can be properly chosen to make $\dot{V}(t) \le 0$ .Therefore, based on the Lyapunov stability theory, the errors vector $\lim_{t\to\infty}|e(t)| = 0$ and $\lim_{t\to\infty}\|\hat{A}(t) - A(t)\| = 0$ .This implies the unknown weighs $a_{ij}$ can be successfully using adaptive controllers (4) and update laws (5) and (6). Further, we can further the value of $L$ to determine the value of constant $D^*$, and we define the norm:

$$L_1 = \min_{i,j}\{\|Df(t,x_i(t))\|, \|Df(t,y_i(t))\|, \max_{\substack{1\le j\le n\\ j\in N^*}}\sum_{i=1}^{N}|d_i^*| - \frac{N}{1-\xi} - \max_{\substack{1\le j\le n\\ j\in N^*}}\sum_{i=1}^{N}|b_{ij}| - \frac{1}{2}[\max_{\substack{1\le j\le n\\ j\in N^*}}\sum_{i=1}^{N}|p_{ij}^*|]\} \quad (8)$$

Respectively, $\|Df(t,x_i(t))\|$ and $\|Df(t,y_i(t))\|$ are in the point $x_i(t)$ and $y_i(t)$ the Department of Jacobin Matrix. When $L \le L_1$ , we easily get: $\dot{V}(t) \le 0$ . According to Lyapunov stability theory we get $\lim_{t\to\infty}\|e_i(t)\| = \lim_{t\to\infty}\|x_i(t) - \lambda y_i(t)\| = 0$

## 4. A weighted complex network with different node dynamics

In this subsection, we consider a weighted complex dynamical network consisting of different node dynamics which is described by

$$\begin{cases} \dot{x}_i(t) = B_1 x_i(t) + g(t, x_i(t)) + \sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} H(t) x_j(t-\tau(t)) \quad 1 \le i \le N^* \\ \dot{x}_i(t) = B_2 x_i(t) + h(t, x_i(t)) + \sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} H(t) x_j(t-\tau(t)) \quad N^*+1 \le i \le N \end{cases} \tag{9}$$

$g, h: R^N \times R^+ \to R^N$ are different smooth linear vector functions. Similarly, a useful hypothesis is given as follows：

**Assumption 3.** (A3)Suppose that there exist positive constants $\beta$ and $\gamma$, satisfying

$$\|g(y)-g(z)\| \le \beta \|y-z\| \quad \|h(y)-h(z)\| \le \gamma \|y-z\|$$

where $y$, $z$ are time-varying vectors.

## 5. Adaptive controlling method

For the sake of identifying topology of network model (9) and tracing network nodes evolution, anther generally controlled complex dynamical network is introduced here:

$$\begin{cases} \dot{y}_i(t) = B_1 y_i(t) + g(t, y_i(t)) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \hat{a}_{ij} H(t) y_j(t-\tau(t)) + u_i \quad 1 \le i \le N^* \\ \dot{y}_i(t) = B_2 y_i(t) + h(t, y_i(t)) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \hat{a}_{ij} H(t) y_j(t-\tau(t)) + u_i \quad N^*+1 \le i \le N \end{cases} \tag{10}$$

Then, we have the error system:

$$\begin{cases} \dot{e}_i(t) = B_1 e_i(t) + g(t, x_i(t)) - \lambda g(t, y_i(t)) - \lambda \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} H(t) y_j(t-\tau(t)) \\ \qquad + \sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} H(t) e_j(t-\tau(t)) - \lambda u_i \quad 1 \le i \le N^* \\ \dot{e}_i(t) = B_2 e_i(t) + h(t, x_i(t)) - \lambda h(t, y_i(t)) - \lambda \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij} H(t) y_j(t-\tau(t)) \\ \qquad + \sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} H(t) e_j(t-\tau(t)) - \lambda u_i \quad N^*+1 \le i \le N \end{cases} \tag{11}$$

where $e_i(t) = x_i(t) - \lambda y_i(t) \ (\lambda \neq 0)$ $c_{ij} = \hat{a}_{ij} - a_{ij}$ Similarly, the following adaptive controlling mechanism can be deduced.

**Theorem 2.** *Suppose that* A3 *holds. The weight configuration matrix* $A(t)$ *of general linearly coupled complex dynamical network* (9) *can be identified by the estimation* $\hat{A}(t)$ *using the following response network:*

$$u_i = \frac{1}{\lambda}[d_i e_i(t) + \lambda g(t, y_i(t)) - g(t, \lambda y_i(t))], (\lambda \neq 0), 1 \le i \le N^*$$

$$u_i = \frac{1}{\lambda}[d_i e_i(t) + \lambda h(t, y_i(t)) - h(t, \lambda y_i(t))], (\lambda \neq 0), N^*+1 \le i \le n$$

$$\dot{c}_{ij} = \lambda \delta_{ij} H(t) y_j(t-\tau(t)) e_i^T(t), (\lambda \neq 0), i, j = 1, 2, \ldots, N$$

$$\dot{d}_i = k_i (1 - \frac{d}{d_i}) e_i^T(t) e_i(t), d_i \neq 0, i = 1, 2, \ldots, N$$

**Proof.** Provided with the condition A3, we get $\|g(t, x_i(t)) - \lambda g(t, y_i(t))\| \le \beta \|e_i(t)\|$ for $i = 1, 2, \ldots, N^*$ and $\|h(t, x_i(t)) - \lambda h(t, y_i(t))\| \le \gamma \|e_i(t)\|$ for $i = N^*+1, \ldots, N$. Choose Lyapunov function as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t) e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}} c_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N} \frac{1}{k_i} d_i^2 + \frac{1}{1-\xi} \int_{t-\tau(t)}^{t} \sum_{i=1}^{N} e_i^T(t) e_i(t) dt$$

where $\xi \in [2\tau'(t) - 1, 1]$, $d$ is sufficiently large positive constant to be determined. We then have:

$$\dot{V}(t)=\sum_{i=1}^{N^*}e_i^T(t)\dot{e}_i(t)+\sum_{i=N^*+1}^{N}e_i^T(t)\dot{e}_i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}}c_{ij}\dot{c}_{ij}+\sum_{i=1}^{N}\frac{1}{k_i}d_i\dot{d}_i+\frac{1}{1-\xi}\sum_{i=1}^{N}e_i^T(t)e_i(t)-\frac{1-\dot{\tau}(t)}{1-\xi}\sum_{i=1}^{N}e_i^T(t-\tau(t))e_i(t-\tau(t))$$

$$=\sum_{i=1}^{N^*}e_i^T(t)[B_1e_i(t)+g(t,x_i(t))-\lambda g(t,y_i(t))-\lambda\sum_{\substack{j=1\\j\neq i}}^{N}c_{ij}H(t)y_j(t-\tau(t))+\sum_{\substack{j=1\\j\neq i}}^{N}a_{ij}H(t)e_j(t-\tau(t))-\lambda u_i]$$

$$+\sum_{i=N^*+1}^{N}e_i^T(t)[B_2e_i(t)+h(t,x_i(t))-\lambda h(t,y_i(t))-\lambda\sum_{\substack{j=1\\j\neq i}}^{N}c_{ij}H(t)y_j(t-\tau(t))+\sum_{\substack{j=1\\j\neq i}}^{N}a_{ij}H(t)e_j(t-\tau(t))-\lambda u_i]$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}}c_{ij}\dot{c}_{ij}+\sum_{i=1}^{N}\frac{1}{k_i}d_i\dot{d}_i+\frac{1}{1-\xi}N\sum_{i=1}^{N}e_i^T(t)e_i(t)-\frac{1-\dot{\tau}(t)}{1-\xi}\sum_{i=1}^{N}e_i^T(t-\tau(t))e_i(t-\tau(t))$$

$$\le\sum_{i=1}^{N^*}\beta\|e_i(t)\|^2+\sum_{i=N^*+1}^{N}\gamma\|e_i(t)\|^2+\sum_{i=1}^{N^*}\|B_1\|\|e_i(t)\|^2+\sum_{i=N^*+1}^{N}\|B_2\|\|e_i(t)\|^2+\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}e_i^T(t)H(t)e_i(t-\tau(t))$$

$$-\sum_{i=1}^{N}d\|e_i(t)\|^2+\frac{1}{1-\xi}\sum_{i=1}^{N}\|e_i(t)\|^2-\frac{1-\dot{\tau}(t)}{1-\xi}\sum_{i=1}^{N}\|e_i(t-\tau(t))\|^2$$

$$=e^T(t)Qe(t)$$

where $\|B_1\|=\max_j\sum_{i=1}^{N^*}|a_{ij}|$ , $\|B_2\|=\max_j\sum_{i=N^*+1}^{N}|a_{ij}|$ , $e(t)=(e_1^T(t),e_2^T(t),\cdots e_n^T(t))^T\in R^{n\times n}$

$$Q=\begin{pmatrix}\left(\beta+\frac{1}{1-\xi}+\|B_1\|-d\right)I_{nN^*} & 0\\ 0 & \left(\gamma+\frac{1}{1-\xi}+\|B_2\|-d\right)I_{n(N-N^*)}\end{pmatrix}+A(t)\otimes H(t)$$

Clearly, the matrix $Q$ is negative definite when the positive constant $d$ is large enough. Similar to the proof method of Theorem 1, we obtain $\lim_{t\to\infty}|e_i(t)|=0$ for $i=1,2,\cdots N$ and $\lim_{t\to\infty}|c_{ij}|=0$ for $i,j=1,2,\cdots N$ . That is, the weight configuration matrix $A(t)$ can be identified by the matrix $\hat{A}(t)$. Thus the proof is completed.

**Remark 1.** In this theorem, the weighted complex dynamical network is built up of two types of different nodes. For networks with more types of ones, similar work can be generalized easily. From this theorem, it is shown that using similar adaptive feedback controlling approach, the exact topology of model (9) can be identified, and the evolution of every node can be traced at the same time. On account of the widespread circumstances in which considerable weighted complex dynamical networks with different nodes exist, this mechanism is of great significance in practice.

## 6. Numerical simulation

Let us consider the following Lü chaotic system

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}a(x_2-x_1)\\ cx_2-x_1x_3\\ -bx_3+x_1x_2\end{pmatrix}=B\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+f(t,x(t))$$

where $a=36,b=3,c=20,B=\begin{pmatrix}-a & a & 0\\ 0 & c & 0\\ 0 & 0 & -b\end{pmatrix}, f(t,x(t))=\begin{pmatrix}0\\ -x_1x_3\\ x_1x_2\end{pmatrix}, A(t)=(a_{ij})_{5\times5}=\begin{pmatrix}-6 & 3 & 2 & 0 & 1\\ 3 & -4 & 1 & 0 & 0\\ 2 & 1 & -3 & 0 & 0\\ 1 & 0 & 0 & -5 & 4\\ 0 & 0 & 0 & 4 & -4\end{pmatrix}$

Now, we consider a weighted linearly coupled complex dynamical network (1) with coupling delay consisting of 5 identical Lü chaotic systems. Taking the weight configuration coupling matrix

$$\dot{x}_i(t)=Bx_i(t)+f(t,x_i(t))+\sum_{\substack{j=1\\j\neq i}}^{N}a_{ij}H(t)x_j(t-\tau(t))\quad i=1,2,\dots,5$$

We assume that $H(t)=I_3,\tau(t)=0.1,\lambda=-0.5,k_i=1,\delta_{ij}=1,\hat{a}_{ij}(0)=3,$

$x_i(0)=(0.1+0.1i,0.2+0.1i,0.4+0.1i)^T$ , $y_i(0)=(1.3+0.1i,0.4+0.1i,0.6+0.1i)^T$ Therefore, according to Theorem 1, by using the following response network, the controller and updated laws given by

$$\dot{y}(t) = By_i(t) + f(t, y_i(t)) + \sum_{\substack{j=1 \\ j\neq i}}^{n} \hat{a}_{ij} H(t) y_j(t-\tau(t)) + u_i, \quad i = 1, 2, \ldots, 5$$

$$u_i = \frac{1}{\lambda}[d_i e_i(t) + \lambda f(t, y_i(t)) - f(t, \lambda y_i(t))], \quad i = 1, 2, \ldots, 5$$

$$\dot{c}_{ij} = \lambda \delta_{ij} H(t) y_j(t-\tau(t)) e_i^T(t), \quad i = 1, 2, \ldots, 5$$

$$d_i' = k_i(1 - \frac{d_i^*}{d_i}) e_i^T(t) e_i(t), d_i \neq 0, \quad i = 1, 2, \ldots, 5$$

Then, some elements of matrix $\hat{A}(t)$ and the synchronization errors $\|e_i(t)\|$ $(i = 1, 2, \ldots, 5)$ are shown in Figs. 1. The numerical results show that adaptive scheme for the drive-response complex network is effective in Theorem 1.

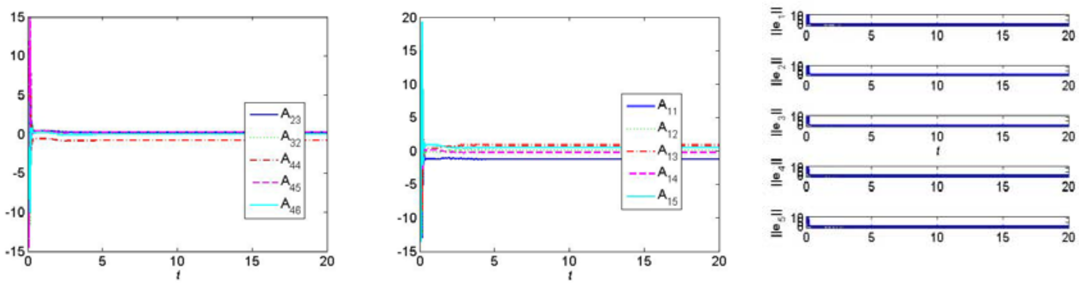


Fig. 1. Estimation of the weight matrix $A(t)$ with time $t$.

From the process to prove theorems in this paper, we can learn that different nodes of the numerical simulation of complex network nodes in a network with the same approximation, in which we have omitted the former numerical simulation.

## 7. Conclusion

In this Letter, the nonlinear controllers and adaptive updated laws have been proposed to study the PS between two complex networks with time-varying coupling delay. With the Lyapunov stability theory and the adaptive control method, two PS theorems have been proposed, and the weight matrix $A(t)$ can be also identified. Numerical results demonstrate that the proposed approach is effective and feasible.

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