A Hilbert Transform N-Dimensional Noisy Phase Unwrapping Algorithm

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Abstract. Phase unwrapping arises as a key step in several imaging technologies, from which we emphasize remote sensing applications like terrain analysis and generation, topographic mapping, interferometric synthetic aperture radar (ISAR). Due to various decorrelation factors, there is much noise in phase images. Phase noise degrades the visual quality of images and interferograms, increases the errors of results derived from the ISAR interferogram, and obstruct phase unwrapping. Hilbert transform (HT) as a phase unwrapping techniques was recently proposed, and its behavior in presence of noise is unknown. In this paper, one-dimensional (1-D) HT phase unwrapping algorithm is proposed to detect and eliminate phase discontinuities in N-dimensional (N-D) noisy phase arrays. The behavior of this method in presence of different noise levels is studied and compared with the other approach of phase jump detection. The phase noise filtration problem and its effect on proposed algorithm behavior are considered. Tests of method presented illustrate the validity and effectiveness of proposed algorithm for difficult problems with noisy and discontinuous original phases.

Keywords: Satellite image processing; phase unwrapping; N-dimensional processing; Hilbert transform; modulo 2π operation; phase noise filtration.

1. Introduction

Remote sensing derives immense applications from this field like terrain analysis and generation, topographic mapping. It is an ever expanding and dynamic area with applications impacting our everyday life. Remotely sensed imagery includes satellite images, images collected through radar systems and other remote sensing techniques that require coherent processing. Satellite image processing is one of the key research areas in the area of remote sensing which requires coherent processing.

Coherent processing requires an accurate estimate of the phase [1-2]. Unfortunately, one is only able to measure a wrapped version of the phase called measured phase not the true phase. The measured phases generated by use of an arctan2-function, are all mapped into the same interval, while any absolute phase offset (an integer multiple of 2π) is lost. The calculation of the unambiguous phase from the measured phase is called phase unwrapping.

Many measurement techniques across a variety of engineering, scientific and medical disciplines deliver quantitative information in the form of true phase or phase images. For example, synthetic aperture radar (SAR) maps terrain and deformation of the Earth's surface through phase images. This means that measured nonlinear phase does not provide useful information. It must be unwrapped before further use through some method to estimate true phase, which is the quantity related to the physical property of interest.

There are many sources would introduce noise to phase images. For example in SAR interferogram the sources are SAR hardware system, SAR image processing, and SAR decorrelating factors, such as spatial baseline, temporal baseline, Doppler-centroid shift and so on. Phase noise is mainly caused by radar thermal noise, speckle noise due to coherent SAR processing, decorrelation, sampling, processing artifacts, interpolation noise, defocusing, registration noise, etc. [3-5]. The measured phases generated from the complex interferogram will be affected by noise in interferogram and this phase noise is characterized by an

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additive noise model rather than the multiplicative one found in SAR amplitude and intensity images [6-7].

Several algorithms for phase unwrapping in the presence of noise have been proposed [8–18]. However, none of the existing phase unwrapping algorithms give satisfactory results when noisy and/or dense fringes occur, with exception a few of them having limitations such as user input, median filtering, computing time, image size, and absence of phase vortices [18].

The filtering of interferogram has great influence on phase. Several common filtering methods are usually used such as non-adaptive filtering methods [19-21] (including mean filtering, median filtering widely used as LPF, two-dimensional Gaussian filter so on), adaptive filtering method [16], multi-looking filtering method [22], and vector filtering method. Linear filtering provides an optimal solution when the input image is corrupted by a Gaussian noise and the mean-square criterion is used to improve the image [23].

In all methods, the unwrapping of measured phase starts from finding phase jump or phase discontinuity between phase values of two adjacent entries that exceeds some constant. The most natural way of detecting phase discontinuity is to take the first or second order derivatives and look for maxima or zero crossings in the output [24]. Then unwrapping is achieved by adding an integer k multiple of 2π to each successive element after zero crossing, and updating k at each phase jump. Finding zero crossing problems is similar to edge detection problem. Traditional edge detection operators like Robert, Sobel and Laplacian [25] detect edges by taking first or second order derivatives and look for maxima. Piggio et al [26] reported that numerical differentiation is an ill-posed problem because its solution depends continuously on the data. Hilbert transform [27-29], provides a means of separating signals based on phase selectivity and uses phase shifts between the pertinent signals to achieve the desired separation. In addition, HT in edge detection has significant advantage in the case of noisy images [30]. The main reason for this advantage is that noise smoothing is performed on the same pixels using only 1-D filtering, thus preserving the edge information in the orthogonal direction. The noise suppression and continuity of the edges obtained by the HT are better than that obtained by the 2-D Canny's method and gives also significant computational advantage compared to the 2-D Canny's method, it takes only about 1=10th the time required 2-D by Canny's method [31].

In this paper, we extend the same logic of edge detection to propose an algorithm for detecting and correcting a noisy phase discontinuity by using 1-D discrete HT as a phase-jump detection filter. Orthogonal noise filtering and smoothing steps are performed before and/or after phase reconstruction. Examples of algorithm applications in 1-D and 2-D cases are provided. The performance of algorithm for different additive Gaussian noise levels was inspected and found the dependence of reconstruction error power on input signal to noise power ratio (SNR). A comparison of method performance in presence of noise with the main approach of phase jump detection is studied and provided.

2. Problem formulation

In most practical cases, the measured signals or satellite images including SAR interferograms are given experimentally as a sequence of discrete values affected by noise. It cannot be sure that phase jumps in the phase images calculated are natural or inserted due to tangent inverse function and noise. Phase noise forms an obstacle to interpret interferogram and satellite images. If the phase noise is too strong, some fringes will be completely lost which will result in errors in interferogram interpretation. The noise in ISAR interferogram will make the visual effect worse, the fringes would be ambiguity, disconnected, and even disappear in speckles. The noise not only worsen the visual appearance, but also hinder phase unwrapping which is absolute necessary to convert the phase to height or deformation, and introduce error in quantitative analysis.

Fig. 1a shows an example of an unambiguous or true phase $\phi_T(n)$, the corresponding measured noisy phase $\phi(n)$ calculated by arctan2– function is shown in Fig. 1b. Jumps of 2π in the measured phase are clearly visible.
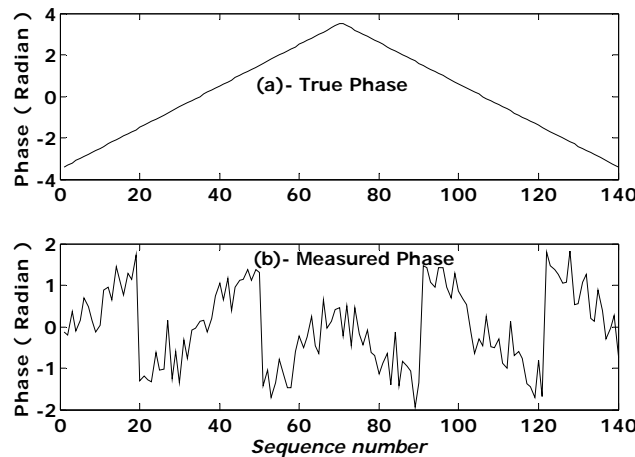
Fig 1: Example of true and measured noisy phase, (a) true phase, (b) measured phase.

3. The Hilbert transform N-D algorithm

In order to obtain more accurate results, a noise filtering steps must be performed before and/or after phase unwrapping. High additive noise levels makes phase unwrapping nearly impossible without filtering, because filtering decreases problems in phase unwrapping by minimizing the residue number. Before true phase calculation, the measured phase $\phi(n)$ filtering step is performed with one or more of the commonly used filtering methods such as mean filtering, median filtering, adaptive filtering, multi-look filtering, two-dimensional (2-D) Gaussian filter or vector filtering.

Calculating true phase starts from finding Hilbert transform of filtered measured phase $\hat{\phi}(n)$ which is used as a phase jump detector. The discrete Fourier transform of 1-D Hilbert signal $\dot{Z}(n)$ of function $\phi(n)$ is given by:

$$\dot{Z}(r) = \dot{H}(r)\dot{\phi}(r) \tag{1}$$

Where $\dot{\phi}(r)$ - the Fourier transform of $\phi(n)$

$$\dot{H}(r) = \begin{cases} 2, & r \in \langle 1, (M/2)-1 \rangle \\ 1, & r = 0, M/2 \\ 0, & r \in \langle (M/2)+1, M-1 \rangle \end{cases} \tag{2}$$

M - number of points in the 1-D array $\dot{H}(r)$

By taking inverse Fourier transform of $\dot{Z}(r)$ we find the needed Hilbert signal in the form:

$$\dot{Z}(n) = \phi(n) + j\hat{\phi}(n) \tag{3}$$

The Hilbert transform $\hat{\phi}(n)$ is found as:

$$\hat{\phi}(n) = \text{Im}[\dot{Z}(n)] \tag{4}$$

Now the true phase in any row or column can be represented through measured phase and some variable k(n):

$$\phi_T(n) = \phi(n) + 2\pi \times k(n) \tag{5}$$

where k(n) is a variable with values depending on sequence number and direction of phase jumps.

Calculating $k(n)$ is achieved by finding maxima and minima of $\hat{\phi}(n)$ and applying the following recurrent equation.

$$k(n) = \begin{cases} k(n) = 0 & \text{if } n = 1 \\ k(n-1)+1 & \text{if } \hat{\phi}(n) \text{ has a maxima} \\ k(n-1)-1 & \text{if } \hat{\phi}(n) \text{ has a minima} \\ k(n-1) & \text{if } \hat{\phi}(n) \text{ has no maxima or minima} \end{cases} \tag{6}$$

After calculation, the reconstructed unwrapped phase may be smoothed with a smoothing median or low pass filter. Caution must be taken when filtering measured phase $\phi(k)$ before reconstruction, because filtration may smoothes phase jumps which may appose phase jumps detection.

The flow chart of proposed algorithm is shown in Fig. 2. This algorithm can work for 1-D, 2-D,…, N-D phase unwrapping. For 1-D array the algorithm is used one time for the array to determine and correct jumps, for 2-D arrays case the algorithm is used for each row or each column, to unwrap a plane. For 3-D arrays case the algorithm is used to determine and correct jumps in each row or each column in all plane.
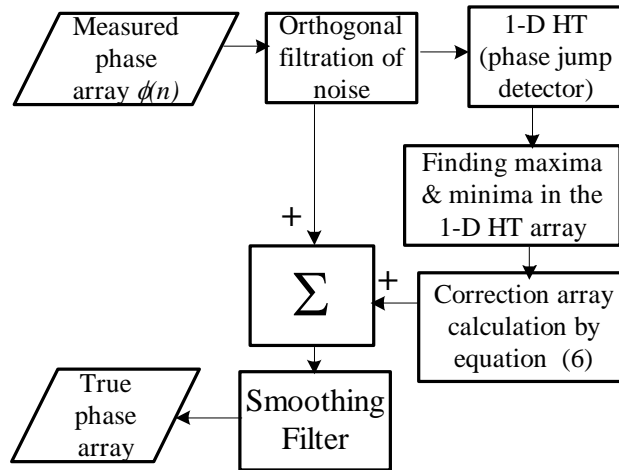

Fig. 2: Block diagram of proposed algorithm

4. Examples

We shall consider a few examples for 1-D and 2-D cases. As a first example we implement proposed algorithm for 1-D phase provided in Fig. 1. The results of running algorithm for this example at 20dB input SNR phase are shown in Fig 3. Fig. 3a depicts the true phase while Fig. 3b depicts the measured noisy phase calculated as arctangent of tangent function of true phase. Fig. 3c shows the reconstructed phase without filtration and finally Fig. 3d plots the reconstructed phase with median filtration and smoothing after reconstruction.

An example of implementation HT algorithm for 2-D case is a complicated mountainous shape phase characteristic of complex function given by:

$$\dot{Z}(s_1, s_2) = \frac{3.414 s_1^3 s_2^3 + 2.613 s_1 s_2^2 + 5 s_1^4 + 4 s_2^4 + 1}{s_1^4 + 2.613 s_1^3 s_2^3 + 3.414 s_1^2 s_2^3 + 12.613 s_1 s_2^2 + 2} \tag{7}$$

where: $s_1 = j\omega_x, \; s_2 = j\omega_y$

The deterministic actual phase of function (7) is shown in Fig. 4. Fig. 5 shows the results of implementing 1-D algorithm to correct measured phase of function (7) for different levels of input additive noise. Fig. 5a shows measured noisy phase and Fig. 5b shows reconstructed phase with no filtration. Fig. 5c shows reconstructed phase with filtration before reconstruction.
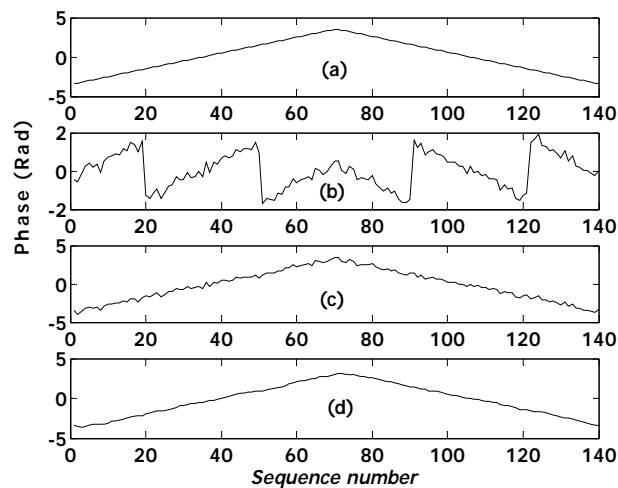
Fig. 3: Implementation of algorithm to 1-D phase unwrapping. (a) Actual true phase. (b) Measured noisy phase. (c) Reconstructed phase (d) Filtered after reconstruction phase.

5. Discussion

In deterministic case, HT algorithm reconstructs phase without any disturbance. Actual and reconstructed phase characteristics are identical and there are no restrictions in its implementation [32-33]. In practice it is very important to study behavior of proposed algorithm in presence of adaptive noise since phase noise is characterized by an additive noise model [6] and this is the case in most practical problems. For this reason we generated random sequences with different power using MATLAB function RAND and added them to known deterministic functions like upper examples. Then we applied proposed algorithm to reconstruct true phase from measured phase corrupted by noise. We calculated standard deviation of reconstruction error and output signal to reconstruction error power ratio as a function of input SNR. Plots for standard deviation are shown in Fig. 6 and plots for output signal to reconstruction error power ratios are shown in Fig. 7. From these plots it can be seen that reconstruction error is very high at low input SNR (below 15dB) and this error decreases with increasing input SNR.

In order to reduce reconstruction error and effects of noise on the algorithm performance, we applied median filtration before reconstruction, after reconstruction, and before-after reconstruction. We used median filtration with proposed algorithm because it is one of the most commonly used nonlinear filters in image processing which are commonly used for smoothing of images and the removal of the noise from a corrupted image. They sharp signal changes and are very effective in removing impulse noise [34]. Filtration is done by filtering each row when the algorithm uses columns for phase reconstruction and to each column when the algorithm uses rows for phase reconstruction. So filtration and phase reconstruction was orthogonal operations in this means. Plots for results with filtration are shown in the same Fig. 6 and Fig. 7. From plots with filtration it can be seen that filtration reduces reconstruction error levels in different manner. Filtration before reconstruction gives better results than filtration after reconstruction. The SNR improvement due to filtration ranges from 2dB when input SNR is about 13dB to 50 dB when input SNR is 45dB. SNR improvement due to pre-reconstruction filtration is plotted in Fig.8.
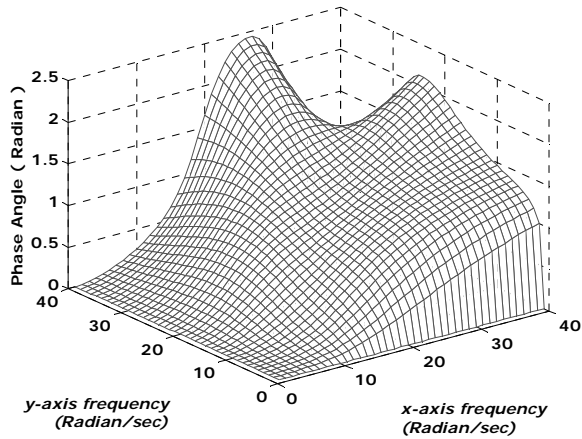
Fig 4: Deterministic phase of function (7).

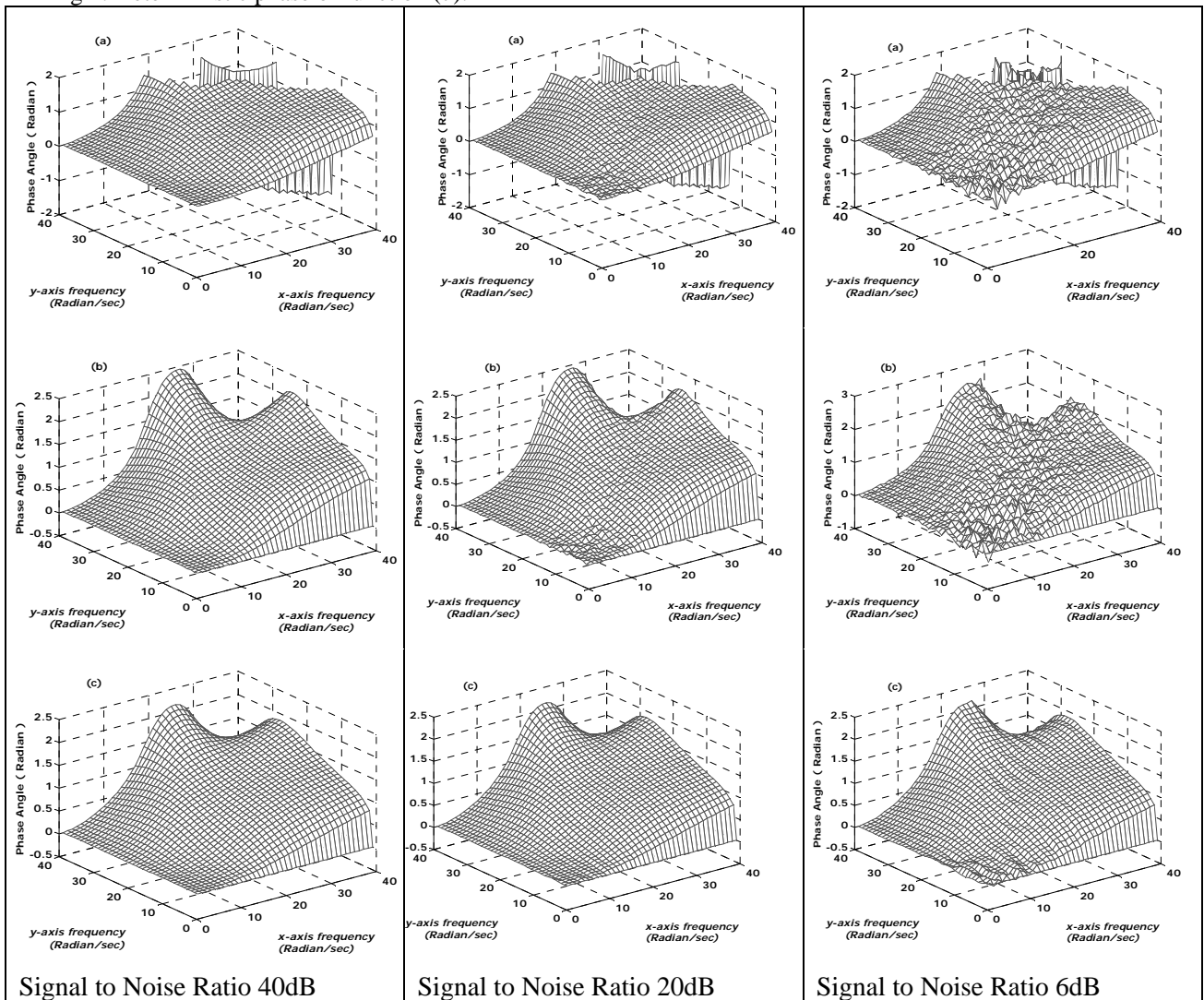| Signal to Noise Ratio 40dB | Signal to Noise Ratio 20dB | Signal to Noise Ratio 6dB |

Fig. 5: Phase characteristic of equation (7) with different levels of additive noise. (a) Measured noisy phase. (b) Reconstructed phase with no filtration. (c) Reconstructed phase with filtration before reconstruction.

Filtration after reconstruction does not have any effect on reconstruction error and so it is not recommended. Also for low level noise signals (input SNR greater than 50dB) and high level noise signals (input SNR less than 10dB) filtration is not recommended.

A comparison between Hilbert transform and differentiation approaches for detecting phase jumps in presence of additive Gaussian noise is made. In Fig. 9 and Fig.10 we provided the relations computed for signal to reconstruction-error power ratio in dB as a function of input SNR in dB for the cases of filtration before reconstruction and no filtration. From plots, we see that HT phase jump detector has much better

performance than differentiation phase jump detector for any input SNR with filtration or without. Filtration improves performance of differentiation however it never reaches that of HT.
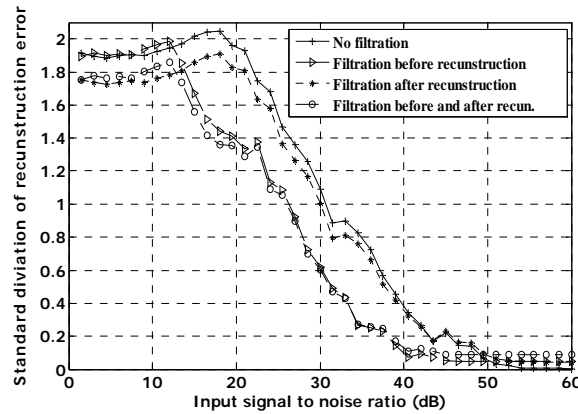

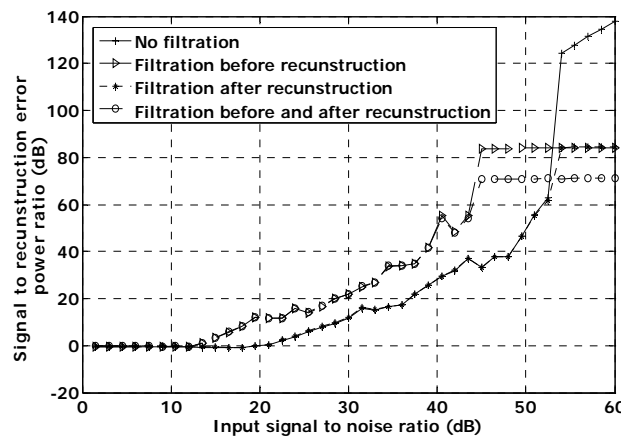Fig. 6: Reconstruction error standard deviation.

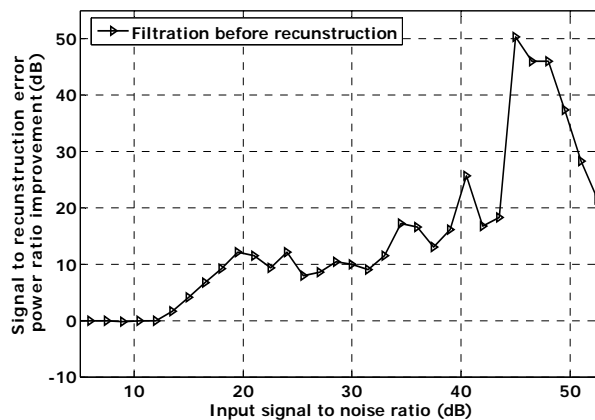Fig. 7: Signal to reconstruction error power ratio

Fig. 8: Signal to reconstruction error power ratio improvement due to pre-reconstruction filtration

In proposed procedure instead of 1-D HT one can propose using 2-D HT. The procedure and processing now becomes 2-D with all other steps of algorithm not changed. We applied such 2-D algorithm for the phase function of equations (7); the results of running this algorithm were not encouraging. The reconstructed phase was strongly damaged and no type of filtration could retrieve phase to an acceptable form. This is because 2-D HT does not have spikes as 1-D HT, the branch cut path is not similar to that of 1-D and reconstructed phase is not the actual. Hence only 1-D HT can be used as a phase jump detector and not other dimensionalities. Also the computational time needed for 2-D is much more than that needed for 1-D algorithm. For $N \times N$ 2-D array: 2-D HT requires $2N$ calculations of 1-D FFT plus N^2 complex

multiplications when multiplying the complex 2-D Hilbert array with the spectrum of the 2-D phase array $\dot{H}(r_1, r_2)\dot{\phi}(r_1, r_2)$ plus $2N$ calculations of 1-D IFFT. 1-D HT algorithm requires N calculations of 1-D FFT plus $N/2$ complex multiplications when multiplying $\dot{H}(r_1)\dot{\phi}(r_1)$ plus N calculations of 1-D IFFT
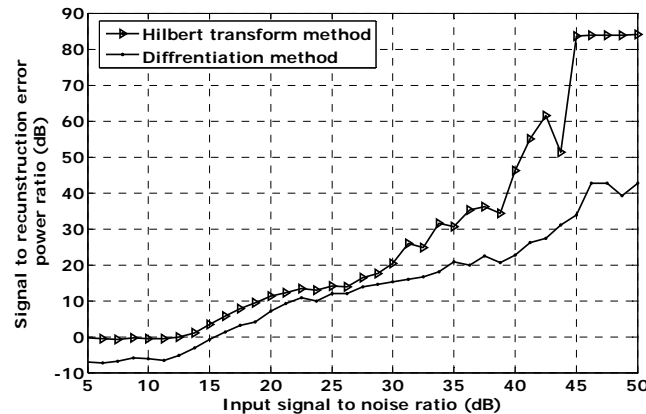

Fig. 9: Performance of the two phase jump detection approaches in presence of adaptive Gaussian white noise with filtration before reconstruction.
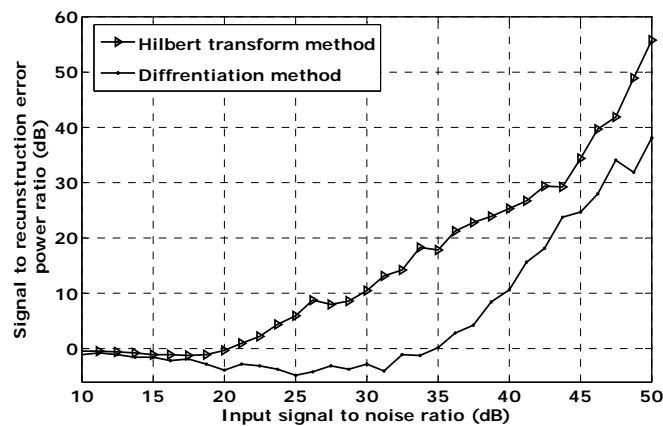

Fig. 10: Performance of the two phase jump detection approaches in presence of adaptive Gaussian white noise with no filtration.

6. Conclusions

In this paper, we extend the same logic of edge detection to propose an algorithm for N-D noisy phase unwrapping by using discrete 1-D HT as a phase jump detector, and studied the effect of different additive noise levels on the algorithm performance. We find that only 1-D HT can be used as a phase jump detector and not other dimensionalities. Orthogonal filtration before phase unwrapping is a very important step and it is recommended for input SNR ranging between (10dB-50dB) since it improves the output SNR of phase unwrapping results. A comparison between Hilbert transform and differentiation approach for detecting phase jumps in presence of noise is made and it was found that performance of HT is much better than differentiation with or without filtration for any level of noise. The algorithm works well and gives satisfactory results even for low signal to noise ratios. It is accurate, easy to implement, has no basic limitations and provides the significant computational advantage due to 1-D processing. Tests performed demonstrate the validity of this approach.

7. References

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