ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 4, No. 1, 2009, *pp*. 033-040

Modified Projective Synchronization of Different Hyperchaotic Systems

HongLan Zhu [1, +], XueBing Zhang [2]

[1] Huaiyin Institute of Technology,Huaian, 223003, PR China,

[2] Huaian College of Information Technology, Huaian, 223003, PR China.

(*Received May 30, 2008, accepted September 29, 2008*)

Abstract. This paper presents modified projective synchronization of two different hyperchaotic systems using active and adaptive control method. The proposed technique is applied to achieve chaos modified projective synchronization for hyperchaotic LÜ system and hyperchaotic Rössler system. Numerical simulations results are presented to demonstrate the effectiveness of the method.

Keywords: Modified projective synchronization; Active control; Adaptive control; Hyperchaotic

1. Introduction

Since the idea of synchronizing chaotic systems was introduced by Pecora and Carroll [1] in 1990, chaos synchronization has received increasing attention due to its potential applications in secure communication, ecological systems, system identification, etc.

The concept of synchronization has been extended to the scope, such as complete synchronization[2], generalized synchronization (GS)[3-6], phase synchronization[7-8], lag synchronization[9-10], and even anti-synchronization[11-13].The generalized synchronization implies the establishment of functional relations between drive and response systems. More recently, Li[14] consider a new GS method, called modified projective synchronization(MPS), where the responses of the synchronized dynamical states synchronize up to a constant scaling matrix.

Our attention in the present paper is to study modified projective synchronization of different hyperchaotic systems .This paper is organized as follows: Section 2 describes the hyperchaotic LÜ system and hyperchaotic Rössler system.

In Section 3 and Section 4 the modified projective synchronization of the hyperchaotic LÜ system and hyperchaotic Rössler system is achieved via active and adaptive control. In Section 5 numerical results demonstrate the effectiveness of the proposed control technique is presented. Finally, Section 6 contains a summarized conclusion of the results.

2. Systems description

LÜ system as a typical transaction system, found by LÜ and Chen, which connects the Lorenz and Chen attractors and represents the transition from one to the other .Recently, Chen Ai-min others proposed hyperchaotic LÜ system[15].The hyperchaotic LÜ system is described by:

$$\begin{cases} \dot{x}_1 = -a(x_1 - x_2) + x_4 \\ \dot{x}_2 = cx_2 - x_1x_3 \\ \dot{x}_3 = x_1x_2 - bx_3 \\ \dot{x}_4 = x_1x_3 + dx_4 \end{cases} \tag{1}$$

The system is based on LÜ system ,the first equation with a controller, the second with three equation non-controller .When $a = 36, b = 3, c = 20$ and d take other different values ,System performance of the

---

[+] Corresponding author. *E-mail address*: zhuhonglan@gmail.com.

Published by World Academic Press, World Academic Union

different dynamics :when $-1.03 \le d \le -0.46$, system is a periodic orbit ;when $-0.46 < d \le -0.35$,system is chaotic attractor ;and when $-0.35 < d \le 1.30$,there are two index greater than zero system is hyperchaotic attractor .The hyperchaotic attractor is shown in Fig.1.

The hyperchaotic Rössler system is studied by Rössler OE[16].The system is described by:

$$\begin{cases} \dot{y}_1 = -y_2 - y_3 \\ \dot{y}_2 = y_1 + a_1 y_2 + y_4 \\ \dot{y}_3 = b_1 + y_1 y_3 \\ \dot{y}_4 = -c_1 y_3 + d_1 y_4 \end{cases} \tag{2}$$

Where y_1, y_2, y_3, y_4 are state variables, and a_1, b_1, c_1, d_1 are real constants. When $a_1 = 0.25, b_1 = 3,\ c_1 = 0.5, d_1 = 0.05$ the system is hyperchaotic .The hyperchaotic attractor is shown in Fig.2.

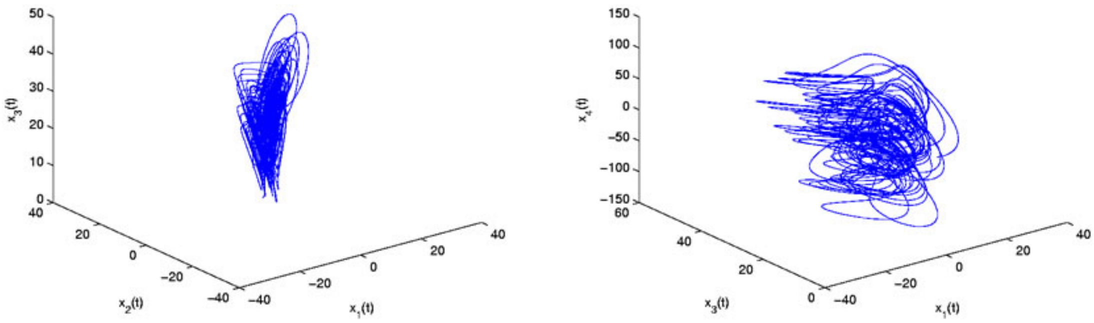

Fig.1:Hyperchaotic attractors for the LÜ system. (a) Hyperchaotic attractor in (x_1, x_2, x_3) space; (b) hyperchaoticattractor in (x_1, x_3, x_4) space.

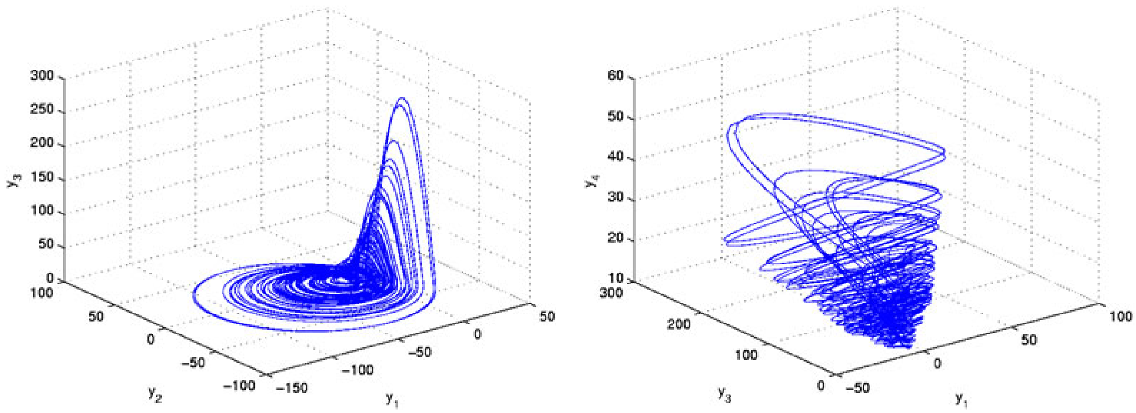

Fig.2: Hyperchaotic attractors for the Rössler system. (a) Hyperchaoti attractor in (y_1, y_2, y_3) space; (b) hyperchaotic attractor in (y_1, y_3, y_4) space.

3. Modified projective synchronization of two hyperchaotic systems via active control

To observe the modified projective synchronization behavior in hyperchaotic LÜ and hyperchaotic Rössler systems, we assume that LÜ system drives the Rössler system. Therefore, we define the master as the hyperchaotc LÜ system (1) and slave systems as follows.

$$\begin{cases} \dot{y}_1 = -y_2 - y_3 + u_1(t) \\ \dot{y}_2 = y_1 + a_1 y_2 + y_4 + u_2(t) \\ \dot{y}_3 = b_1 + y_1 y_3 + u_3(t) \\ \dot{y}_4 = -c_1 y_3 + d_1 y_4 + u_4(t) \end{cases} \tag{3}$$

where u_1, u_2, u_3, u_4 are four control functions to be designed; in order to determine the control functions to realize modified projective synchronization between systems (1) and (3).Let us define the states of the MPS errors for the slave system (3) that is to be controlled and the controlling system (3) as:

$$\begin{aligned} e_1 &= y_1 - \alpha_1 x_1 \\ e_2 &= y_2 - \alpha_2 x_2 \\ e_3 &= y_3 - \alpha_3 x_3 \\ e_4 &= y_4 - \alpha_4 x_4 \end{aligned} \tag{4}$$

Where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are constant .Then two chaotic systems can be synchronized in the sense of MPS, i.e., $\lim_{t\to\infty}\|e_1\| = 0, \lim_{t\to\infty}\|e_2\| = 0, \lim_{t\to\infty}\|e_3\| = 0, \lim_{t\to\infty}\|e_4\| = 0$.

From Eq. (4), we have the following error dynamics:

$$\begin{cases} \dot{e}_1 = -e_2 - e_3 - \alpha_2 x_2 - \alpha_3 x_3 - \alpha_1 a(x_2 - x_1) - \alpha_1 x_4 + u_1 \\ \dot{e}_2 = e_1 + a_1 e_2 + e_4 + \alpha_1 x_1 + a_1 \alpha_2 x_2 + \alpha_4 x_4 - \alpha_2 c x_2 + \alpha_2 x_1 x_3 + u_2 \\ \dot{e}_3 = b_1 + y_1 e_3 + y_1 \alpha_3 x_3 - \alpha_3 x_1 x_2 + \alpha_3 b x_3 + u_3 \\ \dot{e}_4 = -c_1 e_3 - c_1 \alpha_3 x_3 + d_1 e_4 + d_1 \alpha_4 x_4 - \alpha_4 x_1 x_3 - \alpha_4 d x_4 + u_4 \end{cases} \tag{5}$$

Then, by defining the active control inputs u_1, u_2, u_3, u_4 as follows:

$$\begin{cases} u_1 = \alpha_2 x_2 + \alpha_3 x_3 + \alpha_1 a(x_2 - x_1) + \alpha_1 x_4 + v_1 \\ u_2 = -\alpha_1 x_1 - a_1 \alpha_2 x_2 - \alpha_4 x_4 + \alpha_2 c x_2 - \alpha_2 x_1 x_3 + v_2 \\ u_3 = -y_1 \alpha_3 x_3 + \alpha_3 x_1 x_2 - \alpha_3 b x_3 - b_1 + v_3 \\ u_4 = +c_1 \alpha_3 x_3 - (d_1 - d)\alpha_4 x_4 + \alpha_4 x_1 x_3 + v_4 \end{cases} \tag{6}$$

this leads to:

$$\begin{cases} \dot{e}_1 = -e_2 - e_3 + v_1 \\ \dot{e}_2 = e_1 + a_1 e_2 + e_4 + v_2 \\ \dot{e}_3 = y_1 e_3 + v_3 \\ \dot{e}_4 = -c_1 e_3 + d_1 e_4 + v_4 \end{cases} \tag{7}$$

The error system (7) to be controlled is a linear system with control input v_1, v_2, v_3 and v_4 as the function of the error states e_1, e_2, e_3 and e_4 As stated ,as long as $\lim_{t\to\infty}\|e(t)\| = 0$, modified projective synchronization between the driver and response systems is realized, that is, the hyperchaotic LÜ system and hyperchaotic Rössler system are Modified projective synchronization under a active control. According to the original method of active control, v_1, v_2, v_3 and v_4 can be rewritten as

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{8}$$

Where A is a 4×4 constant matrix. In order to make the closed loop system (7) stable, the proper choice of the elements of the matrix A is such that the system (7) must have all eigenvalues with negative real parts. Let

$$A=\begin{pmatrix} -\lambda_1 & 0 & 0 & 0 \\ -1 & -\lambda_2-a_1 & 0 & 0 \\ 0 & 0 & -\lambda_3-y_1 & 0 \\ 0 & 0 & c_1 & -\lambda_4-d_1 \end{pmatrix} \tag{9}$$

with $\lambda_1>0,\lambda_2>0,\lambda_3>0,\lambda_4>0$. So the eigenvalues of the closed loop system (7) are: $-\lambda_1<0,-\lambda_2<0,-\lambda_3<0,-\lambda_4<0$ .Thus modified projective synchronization between hyperchaotic LÜ system and hyperchaotic Rössler system is achieved.

4. Modified projective synchronization of two hyperchaotic systems via adaptive control

The purpose of this section is to introduce a adaptive control law for resolving the modified projective synchronization between the hyperchaotic LÜ system and hyperchaotic Rössler system with complete uncertain system parameters.

In order to observe the modified projective synchronization behavior in the hyperchaotic LÜ system and hyperchaotic Rössler system ,we assume that hyperchaotic LÜ system drives the hyperchaotic Rössler system. Therefore, we define the master system and slave systems as follows

$$\begin{cases} \dot{x}_1=-a(x_1-x_2)+x_4 \\ \dot{x}_2=cx_2-x_1x_3 \\ \dot{x}_3=x_1x_2-bx_3 \\ \dot{x}_4=x_1x_3+dx_4 \end{cases} \tag{10}$$

And

$$\begin{cases} \dot{y}_1=-y_2-y_3+u_1 \\ \dot{y}_2=y_1+a_1y_2+y_4+u_2 \\ \dot{y}_3=b_1+y_1y_3+u_3 \\ \dot{y}_4=-c_1y_3+d_1y_4+u_4 \end{cases} \tag{11}$$

where u_1,u_2,u_3,u_4 are four control functions to be designed; in order to determine the control functions to realize modified projective synchronization between systems (10) and (11).Let us define the states of the MPS errors for the slave system (11) that is to be controlled and the controlling system (3) as:

$$\begin{aligned} e_1&=y_1-\alpha_1x_1 \\ e_2&=y_2-\alpha_2x_2 \\ e_3&=y_3-\alpha_3x_3 \\ e_4&=y_4-\alpha_4x_4 \end{aligned} \tag{12}$$

Where $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ are constant .Then two chaotic systems can be synchronized in the sense of MPS, i.e., $\lim\limits_{t\to\infty}\|e_1\|=0,\lim\limits_{t\to\infty}\|e_2\|=0,\lim\limits_{t\to\infty}\|e_3\|=0,\lim\limits_{t\to\infty}\|e_4\|=0$.

From Eq. (12), we have the following error dynamics:

$$\begin{cases} \dot{e}_1 = -e_2 - e_3 - \alpha_2 x_2 - \alpha_3 x_3 - \alpha_1 a(x_2 - x_1) - \alpha_1 x_4 + u_1 \\ \dot{e}_2 = e_1 + a_1 e_2 + e_4 + \alpha_1 x_1 + a_1 \alpha_2 x_2 + \alpha_4 x_4 - \alpha_2 c x_2 + \alpha_2 x_1 x_3 + u_2 \\ \dot{e}_3 = b_1 + y_1 y_3 - \alpha_3 x_1 x_2 + \alpha_3 b x_3 + u_3 \\ \dot{e}_4 = -c_1 e_3 - c_1 \alpha_3 x_3 + d_1 e_4 + d_1 \alpha_4 x_4 - \alpha_4 x_1 x_3 - \alpha_4 d x_4 + u_4 \end{cases} \tag{13}$$

For systems (10) and (11) without controls u_1, u_2, u_3, u_4, then the trajectories of two systems will quickly separate from each other and become irrelevant. However, when controls are applied, the two systems will approach modified projective synchronization for any initial conditions through control functions. With this mind, we propose the following Theorem 1.

Theorem 1 Let us now define the adaptive control functions u_1, u_2, u_3, u_4 as

$$\begin{cases} u_1 = \alpha_2 x_2 + \alpha_3 x_3 + \alpha_1 x_4 + \hat{a}\alpha_1 (x_2 - x_1) - k_1 e_1 \\ u_2 = -\alpha_1 x_1 - \alpha_4 x_4 - \alpha_2 x_1 x_3 - \hat{a}_1 e_2 - \hat{a}_1 \alpha_2 x_2 + \alpha_2 \hat{c} x_2 - e_4 - k_2 e_2 \\ u_3 = -\hat{b}_1 - y_1 y_3 + \alpha_3 x_1 x_2 - \hat{b}\alpha_3 x_3 - k_3 e_3 \\ u_4 = \hat{c}_1 e_3 + \hat{c}_1 \alpha_3 x_3 - \hat{d}_1 e_4 - \hat{d}_1 \alpha_4 x_4 + \alpha_4 x_1 x_3 + \hat{d}\alpha_4 x_4 - k_4 e_4 \end{cases} \tag{14}$$

Where $k_i (i = 1, 2, 3, 4)$ is the control gains of positive scalars, $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{a}_1, \hat{b}_1, \hat{c}_1, \hat{d}_1$ are estimates of $a, b, c, d, a_1, b_1, c_1, d_1$ respectively and the parameters adaptive laws of $a, b, c, d, a_1, b_1, c_1, d_1$ as below

$$\begin{cases} \dot{\hat{a}} = \alpha_1 e_1 (x_1 - x_2) \\ \dot{\hat{b}} = \alpha_3 x_3 e_3 \\ \dot{\hat{c}} = -\alpha_2 x_2 e_2 \\ \dot{\hat{d}} = -\alpha_4 x_4 e_4 \\ \dot{\hat{a}}_1 = e_2^2 + \alpha_2 x_2 e_2 \\ \dot{\hat{b}}_1 = e_3 \\ \dot{\hat{c}}_1 = -e_3 e_4 - \alpha_3 x_3 e_4 \\ \dot{\hat{d}}_1 = e_4^2 + \alpha_4 x_4 e_4 \end{cases} \tag{15}$$

Then the uncertain mater system and response system is modified projective synchronized.

Proof : According to master system (10) and the controlled slave system (11), we get the error dynamical system (13) can be described by

$$\begin{cases} \dot{e}_1 = \tilde{a}\alpha_1 (x_2 - x_1) - e_2 - k_1 e_1 \\ \dot{e}_2 = -\tilde{a}_1 (e_2 + \alpha_2 x_2) + \tilde{c}\alpha_2 x_2 + e_1 - k_2 e_2 \\ \dot{e}_3 = -\tilde{b}_1 - \tilde{b}\alpha_3 x_3 - k_3 e_3 \\ \dot{e}_4 = \tilde{c}_1 (e_3 + \alpha_3 x_3) - \tilde{d}_1 e_4 - \tilde{d}_1 \alpha_4 x_4 + \tilde{d}\alpha_4 x_4 - k_4 e_4 \end{cases} \tag{16}$$

Where $\tilde{a} = \hat{a} - a, \tilde{b} = \hat{b} - b, \tilde{b} = \hat{c} - c, \tilde{d} = \hat{d} - d, \tilde{a}_1 = \hat{a}_1 - a_1, \tilde{b}_1 = \hat{b}_1 - b_1, \tilde{c}_1 = \hat{c}_1 - c_1, \tilde{d} = \hat{d}_1 - d_1$. Consider a Lyapunov function as

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{a}_1^{\ 2} + \tilde{b}_1^{\ 2} + \tilde{c}_1^{\ 2} + \tilde{d}_1^{\ 2}) \tag{17}$$

The differential of the Lyapunov function along the trajectory of error system (7) is

$$\begin{aligned}\dot{V} &= e_1\dot{e}_1+e_2\dot{e}_2+e_3\dot{e}_3+e_4\dot{e}_4+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{d}\dot{\tilde{d}}+\tilde{a}_1\dot{\tilde{a}}_1+\tilde{b}_1\dot{\tilde{b}}_1+\tilde{c}_1\dot{\tilde{c}}_1+\tilde{d}_1\dot{\tilde{d}}_1\\ &= e_1(\tilde{a}\alpha_1(x_2-x_1)-e_2-k_1e_1)+e_2(-\tilde{a}_1(e_2+\alpha_2x_2)-\tilde{c}\alpha_2x_2+e_1-k_2e_2)\\ &\quad+e_3(-\tilde{b}_1-\tilde{b}\alpha_3x_3-k_3e_3)+e_4(\tilde{c}_1(e_3+\alpha_3x_3)-\tilde{d}_1e_4-\tilde{d}_1\alpha_4x_4+\tilde{d}\alpha_4x_4-k_4e_4)\\ &\quad+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{d}\dot{\tilde{d}}+\tilde{a}_1\dot{\tilde{a}}_1+\tilde{b}_1\dot{\tilde{b}}_1+\tilde{c}_1\dot{\tilde{c}}_1+\tilde{d}_1\dot{\tilde{d}}_1\end{aligned} \tag{18}$$

Substitute (14), (15) into (18), we get

$$\dot{V} = -k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2 = -e^TPe \tag{19}$$

Where $P = diag\{k_1,k_2,k_3,k_4\}$.

Since V is a positive decreasing function and $\dot{V}$ is negative semidefinite, we cannot immediately obtain that the origin of error system (13) is asymptotically stable. In fact, as $\dot{V}\le 0$, then $e_1,e_2,e_3,e_4\in L_\infty$ and $\tilde{a},\tilde{b},\tilde{c},\tilde{d},\tilde{a}_1,\tilde{b}_1,\tilde{c}_1,\tilde{d}_1\in L_\infty$. From the error system (13), we have $\dot{e}_1,\dot{e}_2,\dot{e}_3,\dot{e}_4\in L_\infty$ .Since $\dot{V}=-e^TPe$, then we have $\int_0^t\lambda_{\min}(P)\|e\|^2dt\le\int_0^t e^tPedt\le\int_0^t-\dot{V}dt=V(0)-V(t)\le V(0)$. Where $\lambda_{\min(P)}$ is the minimum eigenvalue of positive-definite matrix P. Thus $e_1,e_2,e_3,e_4\in L_2$. According to the Barbalat's Lemma, we have $e_1\to 0,e_2\to 0,e_3\to 0,e_4\to 0$. Thus in the closed-loop system $y_1\to\alpha_1x_1, y_2\to\alpha_2x_2, y_3\to\alpha_3x_3, y_4\to\alpha_4x_4$, as $t\to\infty$, i.e. $\lim_{t\to\infty}\|e(t)\|=0$. Therefore, the slave system (11) synchronizes the master system (10) in the sense of MPS. This completes the proof.

5. Simulation

In this section, we will show a series of numerical simulations to demonstrate the effectiveness of the proposed control scheme. All simulation procedures are coded and executed using the Matlab software. Fourth order Runge-Kutta integration method is used to solve two systems of differential hyperchoatic systems .In addition, a time step size 0.001 is employed.

5.1. simulations for active control

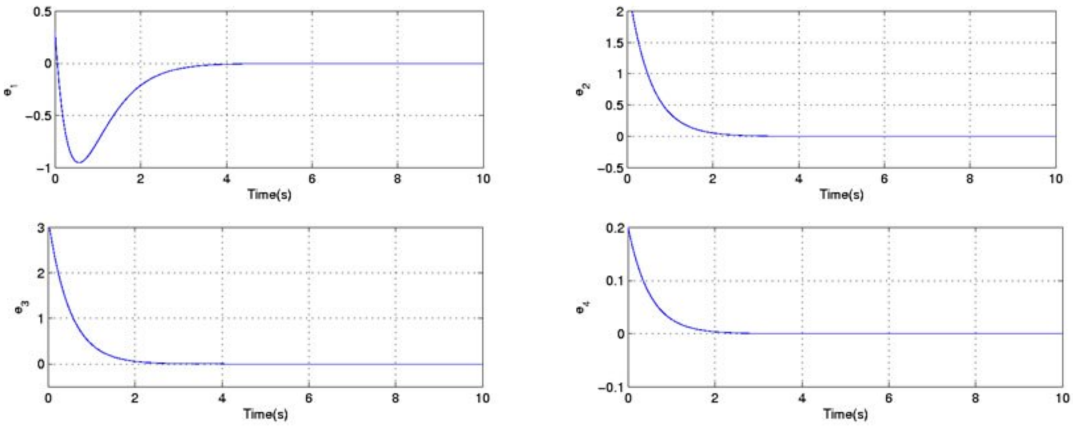

Fig.3: Dynamics of modified projective synchronization errors e_1,e_2,e_3,e_4 between hyperchaotic LÜ system and hyperchaotic Rössler system with time t via active control.

We assume that the control gain $(\lambda_1,\lambda_2,\lambda_3,\lambda_4)=(2,2,2,2)$. We will select the parameters of hyperchaotic LÜ system $a=0.25,b=3,c=20,d=1$ and the parameters of hyperchaotic Rössler system as $a_1=0.25,b_1=3,c_1=0.5,d_1=0.05$ .Therefore, both LÜ and Rössler systems exhibit hyperchaotic behavior. The initial values of the master and slave systems are $(x_1(0),x_2(0),x_3(0),x_4(0))=(0.3,2.5,3.2,0.2)$ and $(y_1(0),y_2(0),y_3(0),y_4(0))=(-20,0,0,15)$. So the initial values ofthe error system is $(e_1(0), e_2(0), e_3(0), e_4(0)) = (-20.3, -0.75, 1.6, 15.4)$. The modified projective synchronization systems (1) and (3) via active control law are shown in Fig.3.

5.2. simulations for adaptive control

We assume that control gains $k_i = 2(i = 1, 2, 3, 4)$. We will select the parameters of hyperchaotic LÜ system $a = 0.25, b = 3, c = 20, d = 1$ and the parameters of hyperchaotic Rössler system $a_1 = 0.25, b_1 = 3, c_1 = 0.5, d_1 = 0.05$ .Therefore, both the hyperchaotic LÜ system and hyperchaotic Rössler system exhibit hyperchaotic behavior. As a test for verification of MPS of the system, let us take $\alpha_1 = 1, \alpha_2 = 0.3, \alpha_3 = -0.5, \alpha_4 = -2$ and the initial values of the master and slave systems are $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.3, 2.5, 3.2, 0.2)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (-20, 0, 0, 15)$ .So the initial values of the error $(e_1(0), e_2(0), e_3(0), e_4(0)) = (-20.3, -0.75, 1.6, 15.4)$ .The modified projective synchronization of systems 10) and (11) are shown in Fig.4.

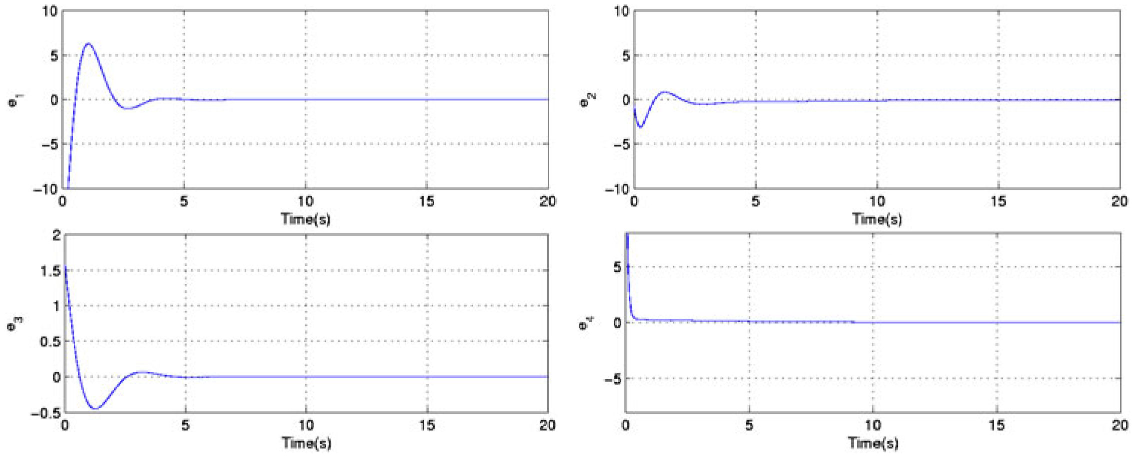

Fig.4: Dynamics of synchronization errors e_1, e_2, e_3, e_4 between the master system and slave system with time t via adaptive control.

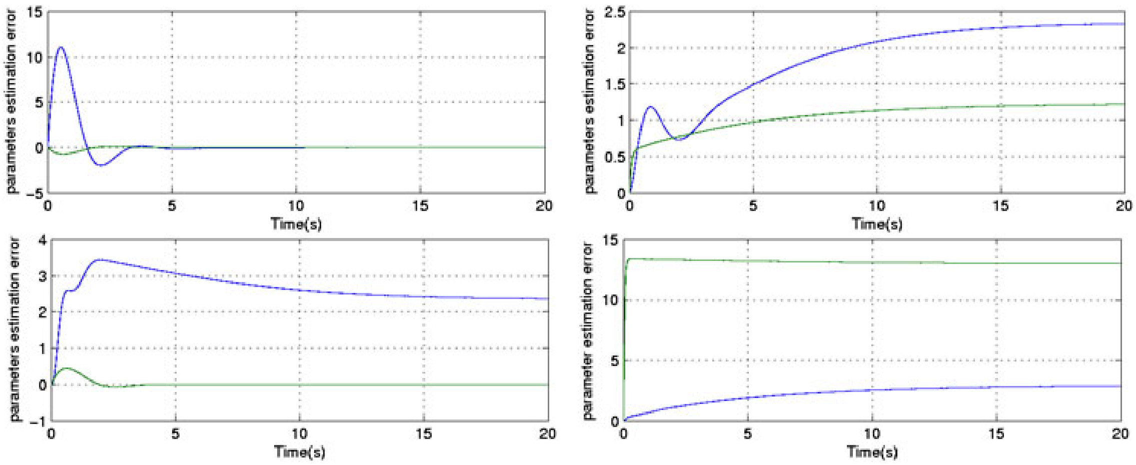

Fig.5: Adaptive parameters estimation errors: $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{a}_1, \hat{b}_1, \hat{c}_1, \hat{d}_1$

In Fig. 3 and Fig.4 we can show that the drive system and response system realize modified projective synchronization quickly. At the same time we can see the estimate values of unknown parameters $a, b, c, d, a_1, b_1, c_1, d_1$ in Fig.5.

6. Conclusion

In this Letter, the modified projective synchronization of two different hyperchaotic systems has been studied. Certain controller and parameters update law are designed to synchronize two different hyperchaotic systems based on the Lyapunov stability theorem, and numerical simulations are also given to show the effectiveness of the proposed method.

7. References

[1] Pecora LM, Carroll TL.. Synchronization in chaotic systems. *Phys. Rev. Lett.*. 1990, **64**: 821-4.

[2] M.T. Yassen. Adaptive control and synchronization of a modified Chua's system. *Appl. Math. Comput*.. 2003, **135**: 113-128.

[3] Y. W. Wang, Z. H. Guan. Generalized synchronization of continuous chaotic system. *Chaos, Solitons and Fractals*. 2006, **27**: 97-101.

[4] K. Murali, M. Lakshmanan. Secure communication using a compound signal from generalized synchronizable chaotic systems. *Physics Letters, A*. 1998, **241**: 303-310.

[5] S. S. Yang, C. K. Duan. Generalized synchronization in chaotic systems. *Chaos, Solitons and Fractals*. 1998, **10**: 1703-1707.

[6] L. Kocarev, U. Parlitz. Gneralized synchronization, predictability, and equivalence of unidirectionally coupled dynamical systems. *Phys. Rev. Lett.*. 1996, **76**: 1816-1819.

[7] G. Santoboni, A. Y. Pogromsky, H. Nijmeijer. An observer for phase synchronization of chaos. *Physics Letters A* . 2001, **291**: 265-273.

[8] G. R. Michael, S. P. Arkady, K. Jürgen. Phase synchronization of chaotic oscillators. *Phys. Rev. Lett.*. 1996, **76**: 1804-1807.

[9] C. Li, X. Liao, K. Wong. Lag synchronization of hyperchaos with application to secure communications. *Chaos, Solitons and Fractals*. 2005, **23**: 183-193.

[10] I. S. Taherion, Y. C. Lai. Observability of lag synchronization of coupled chaotic oscillators. *Phys. Rev. E.*. 1999, **59**: 6247-6250.

[11] G. H. Li, S. P. Zhou. An observer-based anti-synchronization. *Chaos, Solitons and Fractals.* 2006, 29: 495-498.

[12] H. Li. Synchronization and anti-synchronization of Colpitts oscillators using active control. *Chaos, Solitons and Fractals.* 2005, **26**: 87-93.

[13] J. Hu, S. Chen, L. Chen. Adaptive control for anti-synchronization of Chua's chaotic system. *Physics Letters A.* 2005, **339**: 455-460.

[14] Guo-Hui Li. Modified projective synchronization of chaotic system. *Chaos, Solitons and Fractals*. 2007, **32**: 1786-1790.

[15] Chen A. M., Lu J. A. ,Ln J. H., et a1. *Generating hyperehaotic Lit attractor via state feedback control. Physics A.* 2006, **364**: 103-110.

[16] Rössler OE. *An equation for hyperchaos. Phys Lett A*. 1979, **71**: 155-157.