

Uniqueness of positive solutions for a class of p -Laplacion problems

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Abstract: We consider uniqueness of positive radial solutions to the system.

$$\begin{cases} -\Delta_p u = \lambda f(v) & \text{in } \Omega \\ -\Delta_q v = \mu g(u) & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases} \quad (1)$$

Where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$, $\Delta_q v = \operatorname{div}(|\nabla v|^{q-2} \nabla v)$, $p, q > 1$, Ω is the open unit ball in R^N , $N \geq 2$ and $\partial\Omega$ is its boundary.

Keywords: uniqueness, laplacion problems, positive solutions.

1. Introduction

We consider the boundary value problem

$$\begin{cases} -\Delta_p u = \lambda f(v) & \text{in } \Omega \\ -\Delta_q v = \mu g(u) & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases} \quad (1)$$

Where Ω is the open unit ball in R^N , $N \geq 2$, with a smooth boundary $\partial\Omega$, λ, μ are positive parameters and f, g are smooth functions. We shall establish existence and nonexistence of positive radial solutions for (1). Dalmasso [2] investigated the existence and uniqueness of positive radial solutions of the boundary value problem

$$\begin{cases} -\Delta u = \lambda f(v) & \text{in } B \\ -\Delta v = \mu g(u) & \text{in } B \\ u = v = 0 & \text{on } \partial B \end{cases} \quad (2)$$

Where B is the open unit ball in R^N , $N \geq 2$, $f, g : [0, \infty) \rightarrow [0, \infty)$. He obtained some results in the sublinear case when f, g are nondecreasing and there exist positive numbers p, q with $pq < 1$ such that

$$\frac{f(x)}{x^p} \text{ and } \frac{g(x)}{x^q} \text{ are nonincreasing on } R^+. \quad (H)$$

D.D. Hai [3] considered system (2) and extended (H) to hold only for large x . According to a result of Troy [6], positive solutions of the boundary value problem

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$$\begin{cases} -\Delta u = \lambda a(|x|) f(v) & \text{in } \Omega \\ -\Delta v = \mu b(|x|) g(u) & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases} \quad (3)$$

are radially symmetric, where Ω is the open unit ball in R^N ($N \geq 2$), $f, g: R^+ \times R^+ \rightarrow R^+$ and $a, b: R^+ \rightarrow (0, \infty)$ are continuous function, $R^+ = [0, \infty)$. Yulian An [6] investigated uniqueness of positive solutions of the system

$$\begin{cases} \left(r^{(N-1)} u' \right)' = -a(r) r^{(N-1)} f(u(r), v(r)) & r \in (0, 1) \\ \left(r^{(N-1)} v' \right)' = -b(r) r^{(N-1)} g(u(r), v(r)) & r \in (0, 1) \\ u'(0) = v'(0) = u(1) = v(1) = 0. \end{cases} \quad (4)$$

M. Chhetri and P. Girg [5] investigated nonexistence of nonnegative solutions to the BVP:

$$\begin{cases} -\Delta_p u = \lambda f(u) & \text{in } B \\ u = 0 & \text{on } \partial B \end{cases} \quad (5)$$

Where $p > 1, \lambda > 0$ and B is the open unite ball in R^N . We consider the system (1) and make the follwing assumptions:

1.1. (H.1) $f, g: R^+ \rightarrow R^+$ are continuous, nondecreasing and C^1 on $(0, +\infty)$ and

$$\limsup_{x \rightarrow 0^+} x f'(x) < \infty, \quad \limsup_{x \rightarrow 0^+} x g'(x) < \infty$$

1.2. (H.2) There exist $m \in (p-1, p^*)$ such that $\lim_{x \rightarrow \infty} \frac{f(x)}{x^m} > 0$ and $\liminf_{x \rightarrow x^+} \frac{f(x)}{x^m} > 0$

where $p^* = \frac{Np}{N-p}$ for $p < N$ and $p^* = +\infty$ for $p \geq N$ is the critical exponent.

1.3. (H.3) There exist $n \in (q-1, q^*)$ such that $\lim_{x \rightarrow \infty} \frac{g(x)}{x^n} > 0$ and $\liminf_{x \rightarrow 0^+} \frac{g(x)}{x^n} > 0$ where

$q^* = \frac{Nq}{N-q}$ for $q < N$ and $q^* = +\infty$ for $q \geq N$ is the critical exponent and for

$m_1 > m, n_1 > n, \frac{f(x)}{x^{m_1}}$ and $\frac{g(x)}{x^{n_1}}$ are nonincreasing for x large.

Due to [3], studying nonnegative solution of (1) is equivalent to studying positive solution of

$$\begin{cases} -\left(r^{N-1} \phi_p(u') \right)' = \lambda r^{N-1} f(v(r)) & r \in (0, 1) \\ -\left(r^{N-1} \phi_p(v') \right)' = \mu r^{N-1} g(u(r)) & r \in (0, 1) \\ u'(0) = v'(0) = u(1) = v(1) = 0 \end{cases} \quad (6)$$

where $\phi_p(s) = |s|^{p-2}s, \phi_q(s) = |s|^{q-2}s$ for $s \neq 0$ and $\phi_p(0) = \phi_q(0) = 0$. Our main result is:

1.4. Theorem 1. Let (H.1)-(H.3) hold.

Then there exist a positive number η , such that the system (6) has at most one positive solution for $\min(\lambda\mu^{m(q'-1)}, \mu\lambda^{n(p'-1)}) \geq \eta$.

1.5. Remark 2. For reader's convenience we list some of the properties of the function

$\phi_p : \mathbb{R} \rightarrow \mathbb{R}$, defined above, that are relevant in this paper. Namely,

(i) ϕ_p is an odd increasing homeomorphism of $\mathbb{R}$ onto itself;

(ii) the inverse mapping of ϕ_p , denoted by $(\phi_p)_{-1}$, is given by $(\phi_p)_{-1} = \phi_{p'}$

$$\text{where } \frac{1}{p} + \frac{1}{p'} = 1.$$

1.6. Lemma 3. Let h be continuous on $\mathbb{R}^+$ and C^1 on $(0, +\infty)$ such that $\limsup_{x \rightarrow 0^+} xh < \infty$.

Let M, ε, r be positive numbers with $\varepsilon < 1$. Then there exists a positive number C such that $|h(\gamma x) - \gamma^r h(x)| \leq C(1 - \gamma)$ for $\varepsilon \leq \gamma < 1$ and $0 \leq x \leq M$.

Poof. Let $0 \leq x \leq M$. Define $H(\gamma) = h(\gamma x) - \gamma^r h(x)$, $\varepsilon \leq \gamma < 1$. By the mean value theorem, there exists $c \in (\gamma, 1)$ such that

$$\begin{aligned} |H(\gamma)| &= |H(\gamma) - H(1)| = (1 - \gamma) |xh'(cx) - rc^{r-1}h(x)| \\ &\leq C(1 - \gamma), \end{aligned}$$

where

$$C = \frac{\sup\{|yh'(y)| : 0 < y \leq M\}}{\varepsilon} + r \max(\varepsilon^{r-1}, 1) \sup\{|h(y)| : y \leq M\}.$$

Lemma 4. Let (u, v) be a positive solution of the system (6). Then there exist positive constants $M_i, i = 1, 2, 3, 4$ and M independent of u, v such that

$$\begin{aligned} M_1 \left(\lambda \mu^{m(q'-1)} \right)^{\frac{(p'-1)}{1-mn(q'-1)(p'-1)}} (1-r) &\leq u(r) \leq M_2 \left(\lambda \mu^{m(q'-1)} \right)^{\frac{(p'-1)}{1-mn(q'-1)(p'-1)}} (1-r) \\ M_3 \left(\mu \lambda^{n(p'-1)} \right)^{\frac{q'-1}{1-mn(q'-1)(p'-1)}} (1-r) &\leq v(r) \leq M_4 \left(\mu \lambda^{n(p'-1)} \right)^{\frac{q'-1}{1-mn(q'-1)(p'-1)}} (1-r) \end{aligned}$$

for $\min(\lambda\mu^{m(q'-1)}, \mu\lambda^{n(p'-1)}) \geq M$ and $0 < r < 1$.

Proof. Let (u, v) be a positive solution of the system (6). By integrating two equations in (6) we obtain respectively,

$$\phi_p(u'(r)) = -\frac{1}{r^{N-1}} \int_0^r \lambda \tau^{(N-1)} f(v) d\tau ds \quad (7)$$

$$\phi_q(v'(r)) = -\frac{1}{r^{N-1}} \int_0^r \mu \tau^{(N-1)} g(u) d\tau ds \quad (8)$$

It is clear that

$$u(r) = \int_r^1 \left(\frac{1}{s^{(N-1)}} \right)^{p'-1} \left(\int_0^s \lambda \tau^{(N-1)} f(v) d\tau \right)^{p'-1} ds. \quad (9)$$

$$v(r) = \int_r^1 \left(\frac{1}{s^{(N-1)}} \right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(u) d\tau \right)^{q'-1} ds. \quad (10)$$

From now on, we shall denote by $C_i, i = 1, 2, \dots$, positive constants independent of u, v, λ, μ .

$$\begin{aligned} u\left(\frac{1}{2}\right) &\geq \int_{\frac{1}{2}}^1 \left(\frac{1}{s^{N-1}}\right)^{p'-1} \left(\int_0^{\frac{1}{2}} \lambda \tau^{(N-1)} f(v) d\tau\right)^{(p'-1)} ds \\ &\geq \left(\frac{\lambda}{2} f\left(v\left(\frac{1}{2}\right)\right)\right)^{(p'-1)} \left(\int_0^{\frac{1}{2}} \tau^{(N-1)} d\tau\right)^{(p'-1)} \\ &= \left(\frac{\lambda f\left(v\left(\frac{1}{2}\right)\right)}{N 2^{(N+1)}}\right)^{p'-1}. \end{aligned} \quad (11)$$

Similary,

$$v\left(\frac{1}{2}\right) \geq \left(\frac{\mu g\left(u\left(\frac{1}{2}\right)\right)}{N 2^{(N+1)}}\right)^{q'-1}. \quad (12)$$

By (H.2) and (H.3), we have

$$f(v) \geq K_1 v^m \quad (13)$$

$$g(u) \geq K_2 u^n. \quad (14)$$

Both (13), (14) together with (11), (12) give us

$$u\left(\frac{1}{2}\right) \geq \left(\frac{K_1}{N 2^{(N+1)}}\right)^{p'-1} \lambda^{p'-1} \left(v\left(\frac{1}{2}\right)\right)^{m(p'-1)}, \quad (15)$$

$$v\left(\frac{1}{2}\right) \geq \left(\frac{K_2}{N 2^{(N+1)}}\right)^{q'-1} \mu^{q'-1} \left(u\left(\frac{1}{2}\right)\right)^{n(q'-1)}. \quad (16)$$

It is clear that

$$u\left(\frac{1}{2}\right) \geq C_1 \left(\lambda \mu^{m(q'-1)}\right)^{\frac{p'-1}{1-mn(q'-1)(p'-1)}}, \quad (17)$$

$$v\left(\frac{1}{2}\right) \geq C_2 \left(\mu \lambda^{n(p'-1)}\right)^{\frac{q'-1}{1-mn(q'-1)(p'-1)}}. \quad (18)$$

It follows from (13), (14), (17), (18) that for $r \geq \frac{1}{2}$,

$$\begin{aligned}
-u'(r) &= \left(\frac{\lambda}{r^{(N-1)}} \right)^{p'-1} \left(\int_0^r s^{(N-1)} f(v(s)) ds \right)^{(p'-1)} \\
&\geq \lambda^{p'-1} \left(\int_0^{\frac{1}{2}} s^{(N-1)} f(v(s)) ds \right)^{p'-1} \\
&\geq \left(\frac{\lambda f\left(v\left(\frac{1}{2}\right)\right)}{N2^N} \right)^{p'-1} \geq \left(\frac{K_1}{N2^N} \right)^{p'-1} \lambda^{p'-1} \left(v\left(\frac{1}{2}\right) \right)^{m(p'-1)} \\
&= C_3 \left(\lambda^{p'-1} \mu^{m(q'-1)(p'-1)} \right)^{\frac{1}{1-mn(q'-1)(p'-1)}},
\end{aligned}$$

and after integrating ,

$$u(r) \geq C_3 \left(\lambda \mu^{m(q'-1)} \right)^{\frac{p'-1}{1-mn(q'-1)(p'-1)}} (1-r), \quad r \geq \frac{1}{2}. \quad (19)$$

In a similar manner,

$$v(r) \geq C_4 \left(\mu \lambda^{n(p'-1)} \right)^{\frac{q'-1}{1-mn(q'-1)(p'-1)}} (1-r), \quad r \geq \frac{1}{2}. \quad (20)$$

Since u, v are decreasing , this implies that there exist positive constants M_1, M_2 independent of u, v such that the left-side inequalities for u, v in theorem 1 hold. From (9) , (10) , we have

$$|u|_0 \leq \left(\lambda f(|v|_0) \right)^{p'-1}, \quad |v|_0 \leq \left(\mu g(|u|_0) \right)^{q'-1} \quad (21)$$

where $|\cdot|_0$ denotes the sup-norm. By the conditions (H.2) , (H.3) we have from (21) that

$$\begin{aligned}
|u|_0 &\leq \left(\lambda f(|v|_0) \right)^{p'-1} \leq C_5 \lambda^{p'-1} \left(|v|_0 \right)^{m(p'-1)}, \\
|v|_0 &\leq \left(\mu g(|u|_0) \right)^{q'-1} \leq C_6 \mu^{q'-1} \left(|u|_0 \right)^{n(q'-1)}.
\end{aligned}$$

It is clear that

$$|u|_0 \leq C_7 \left(\lambda \mu^{m(q'-1)} \right)^{\frac{(p'-1)}{1-mn(q'-1)(p'-1)}},$$

similarly,

$$|v|_0 \leq C_8 \left(\mu \lambda^{n(p'-1)} \right)^{\frac{(q'-1)}{1-mn(q'-1)(p'-1)}}.$$

Using this in the equation for u' gives

$$-u'(r) \leq \left(\lambda f(|v|_0) \right)^{(p'-1)} \leq C_9 \left(\lambda \mu^{m(q'-1)} \right)^{\frac{(p'-1)}{1-mn(q'-1)(p'-1)}},$$

In a similar manner, we have the upper estimate for $v(r)$. This complete the proof of Lemma 4.

Proof of Theorem 1. Let (u, v) and (u_1, v_1) be positive solution of (6) and let $\min(\lambda \mu^{m(q'-1)}, \mu \lambda^{n(p'-1)})$ be large enough so that Lemma 4 applies . By Lemma 4,

$$\frac{M_1}{M_2} u_1 \leq u \leq \frac{M_2}{M_1} u_1 \quad \text{on } (0,1).$$

Let $\alpha = \sup\{c > 0 : u \geq cu_1 \text{ in } (0,1)\}$. Then clearly $\alpha_0 < \alpha < \infty$ and $u \geq \alpha u_1$ in $(0,1)$, where $\alpha_0 = \frac{M_1}{M_2}$. We claim that $\alpha \geq 1$. Suppose to the contrary that $\alpha < 1$. Since

$$\begin{aligned} (r^{N-1} \phi_p(u'))' &= -\lambda r^{N-1} f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)\right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(u) d\tau\right)^{q'-1} ds \\ (r^{N-1} \phi_p(\alpha u_1'))' &= -\lambda r^{N-1} \alpha f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)\right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(u) d\tau\right)^{q'-1} ds, \end{aligned}$$

it follows that

$$\begin{aligned} \left[r^{N-1} (\phi_p(u') - \phi_p(\alpha u_1')) \right]' &\leq -\lambda r^{N-1} \left[f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)\right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(\alpha u_1) d\tau\right)^{q'-1} ds \right. \\ &\quad \left. - \alpha f\left(\int_r^1 \left(\frac{1}{s^{(N-1)}}\right)\right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} ds \right] \end{aligned} \quad (22)$$

Let $n_1 > n_2 > n, m_1 > m_2 > m$. We claim that

$$\left(\int_0^s \tau^{(N-1)} g(\alpha u_1) d\tau\right)^{q'-1} > \alpha^{n_1(q'-1)} \left(\int_0^s \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1}, \quad s \geq 0. \quad (23)$$

Since $\alpha \geq \alpha_0$ and $\frac{g(x)}{x^{n_2}}$ is nonincreasing for $x \gg 1$,

$$g(\alpha x) \geq \alpha^{n_2} g(x) \text{ for } x \gg 1.$$

Let $\frac{1}{2} < T < 1$. By Lemma 4,

$$u_1(s) \geq M_1(1-T) \left(\lambda \mu^{m(q'-1)} \right)^{\frac{(p^*-1)}{1-mn(q'-1)(p'-1)}} > 1, \quad s \leq T,$$

and therefore

$$\left(\int_0^s \tau^{(N-1)} (g(\alpha u_1) - \alpha^{n_1} g(u_1)) d\tau\right)^{q'-1} \geq (\alpha^{n_2} - \alpha^{n_1})^{(q'-1)} \left(\int_0^s \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} \geq 0, \quad s \leq T.$$

For $s > T$,

$$\begin{aligned} &\left(\int_0^s \tau^{(N-1)} (g(\alpha u_1) - \alpha^{n_1} g(u_1)) d\tau\right)^{q'-1} \\ &= \left(\int_0^T \tau^{(N-1)} (g(\alpha u_1) - \alpha^{n_1} g(u_1)) d\tau\right)^{q'-1} + \left(\int_T^s \tau^{(N-1)} (g(\alpha u_1) - \alpha^{n_1} g(u_1)) d\tau\right)^{q'-1} \\ &\geq (\alpha^{n_2} - \alpha^{n_1})^{q'-1} \left(\int_0^T \tau^{(N-1)} g(u_1) d\tau\right)^{q'-1} - (C(1-T)(1-\alpha))^{q'-1}, \end{aligned}$$

where we have used Lemma 3 with $h = g$. Since

$$\left(\int_0^T \tau^{(N-1)} g(u_1) d\tau \right)^{q'-1} \geq \left(\int_0^{\frac{1}{2}} \tau^{(N-1)} g(u_1) d\tau \right)^{q'-1} \geq \left(\frac{g\left(u_1\left(\frac{1}{2}\right)\right)}{N2^N} \right)^{q'-1}$$

and since there exist a positive number $k > 0$ such that

$$\alpha^{n_2} - \alpha^{n_1} \geq k(1 - \alpha) \text{ for } \alpha_0 \leq \alpha \leq 1,$$

It follow that

$$\left(\int_0^s \tau^{(N-1)} (g(\alpha u_1) - \alpha^{n_1} g(u_1)) d\tau \right)^{q'-1} > 0, s > T$$

if T is sufficiently close to 1. This proves the claim.

Inserting (23) into (22) and integrating gives

$$z^{N-1} (\phi_p(u') - \phi_p(\alpha u'_1))(z) \leq -\lambda \int_0^z B(\alpha, r) dr,$$

where

$$B(\alpha, r) = r^{N-1} \left[f \left(\alpha^{n_1(q'-1)} \int_r^1 \left(\frac{1}{s^{(N-1)}} \right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(u_1) d\tau \right)^{q'-1} ds \right. \right. \\ \left. \left. - \alpha f \left(\int_r^1 \left(\frac{1}{s^{(N-1)}} \right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(u_1) d\tau \right)^{q'-1} ds \right) \right]$$

Using (13), (14) and Lemma 4, we obtain for $r \leq T$,

$$\begin{aligned} \int_r^1 \left(\frac{1}{s^{(N-1)}} \right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(u_1) d\tau \right)^{q'-1} ds &\geq \int_T^1 \left(\frac{1}{s^{(N-1)}} \right)^{q'-1} \left(\int_0^T \mu \tau^{(N-1)} g(u_1) d\tau \right)^{q'-1} \\ &\geq \left(\frac{\mu T^N (1-T)}{N} \right)^{q'-1} (g(u_1(T)))^{q'-1} \\ &\geq \left(\frac{\mu T^N (1-T)}{N} \right)^{q'-1} (K_2)^{q'-1} (u_1(T))^{n(q'-1)} \\ &\geq c_1(T) (\lambda \mu^{m(q'-1)})^{\frac{p'-1}{1-mn(q'-1)(p'-1)}} \gg 1 \end{aligned}$$

where $c_1(T) = \left(\frac{\mu T^N K_2}{N} \right)^{q'-1} M_1^{n(q'-1)} (1-T)^{(n+1)(q'-1)}$. Since $\frac{f(x)}{x^{m_1}}$ is nonincreasing for $x \gg 1$,

$$f(\alpha^{n_1} x) \geq \alpha^{n_1 m_1} f(x) \text{ for } x \gg 1$$

and therefore

$$\begin{aligned} B(\alpha, r) &\geq r^{N-1} \left(\alpha^{n_1 m_1 (q'-1)} - \alpha \right) f \left(\int_r^1 \left(\frac{1}{s^{(N-1)}} \right)^{q'-1} \left(\int_0^s \mu \tau^{(N-1)} g(u_1) d\tau \right)^{q'-1} ds \right) \\ &\geq c_2(T) r^{N-1} (\lambda \mu^{m(q'-1)})^{\frac{m(p'-1)}{1-mn(q'-1)(p'-1)}} (1-\alpha) > 0, r \leq T \end{aligned} \quad (24)$$

where $c_2(T) = K_1 (c_1(T))^m k_0$ and k_0 is a positive constant such that

$$\alpha^{n_1 m_1 (q'-1)} - \alpha \geq k_0 (1 - \alpha) \text{ for } \alpha_0 \leq \alpha \leq 1.$$

This shows that

$$z^{N-1} (\phi_p(u') - \phi_p(\alpha u_1'))(z) < 0, \quad 0 < z \leq T.$$

For $z > T$, we have by lemma 3 and (24) that

$$\begin{aligned} \int_0^z B(\alpha, r) dr &\geq \int_0^{\frac{1}{2}} B(\alpha, r) dr + \int_T^z B(\alpha, r) dr \\ &\geq \frac{c_2 \left(\frac{1}{2}\right)}{N 2^N} (\lambda \mu^{m(q'-1)})^{\frac{m(p'-1)}{1-mn(q'-1)(p'-1)}} (1-\alpha) - (C(1-T)(1-\alpha))^{(q'-1)} > 0 \end{aligned}$$

for large $\lambda \mu^{m(q'-1)}$ and $\mu \lambda^{n(p'-1)}$ and T sufficiently close to 1. Hence

$$(\phi_p(u') - \phi_p(\alpha u_1'))(z) < 0, \quad 0 < z \leq 1,$$

from which it follows that there exists $\tilde{\alpha} > \alpha$ such that $u \geq \tilde{\alpha} u_1$ in $(0, 1)$, a contradiction.

Thus $\alpha \geq 1$ and so $u = u_1$ in $(0, 1)$. Using the formulas for v and v_1 it follows that $v = v_1$ in $(0, 1)$, completing the proof of Theorem 1.

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