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Anti-synchronization in different Hyperchaotic systems

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Abstract. This paper analyzed the anti-synchronization of the four-dimensional autonomous different structural hyperchaotic systems, and achieved the anti-synchronization of the hyperchaotic Lorenz-Chen system, hyperchaotic Chen-Lü system and hyperchaotic Lü-Lorenz system with each other via the active control theory. Numerical simulations are provided for illustration and verification of the proposed method.

Keywords: anti-synchronization; four-dimensional autonomous system, active control.

1. Introduction

Since Pecora and Carrol [1] introduced a method to synchronize two identical chaotic systems with different initial conditions, Synchronization in chaotic dynamic systems has received deal of interest among scientists various research fields including secure communication, chemical reactions, biological systems, information science, plasma technologies, etc. The concept of synchronization has been extended to the scope, such as generalized synchronization [2-3], phase synchronization [4], lag synchronization [5], and anti-synchronization (AS) [6-9], in which the state values of synchronized systems have the same absolute values but opposite signs. Therefore, the sum of two signals can converge to zero when AS appears.

Recently, active control has been applied to synchronize chaotic systems [10-12]. However, the approach of synchronization between two different hyperchaotic systems is seldom reported. In this paper, we focus on the AS of two different hyperchaotic systems. The active control methods will be employed. We derive rigorously a sufficient condition for the anti-synchronization. Numerical experiments on hyperchaotic Lorenz system, Chen system, and Lü system are performed, which demonstrate the effectiveness and feasibility of the proposed control strategy.

2. Systems Analysis

At present, a new hyperchaotic Lorenz system [13] was constructed by Q.Jia. The new hyperchaotic system is described by

$$\begin{cases} \dot{x} = a(y-x)+w \\ \dot{y} = cx - y - xz \\ \dot{z} = xy - bz \\ \dot{w} = -xz + dw \end{cases} \tag{1}$$

The hyperchaotic Chen system [14] and hyperchaotic Lü system [15] are described by the following four different equations

$$\begin{cases} \dot{x} = \alpha(y-x)+w \\ \dot{y} = \beta x - xz + ry \\ \dot{z} = xy - \rho z \\ \dot{w} = yz + \mu w \end{cases} \tag{2}$$

and

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$$\begin{cases}\dot{x}=m(y-x)+w\\ \dot{y}=-xz+ny\\ \dot{z}=xy-pz\\ \dot{w}=xz+qw\end{cases}\tag{3}$$

respectively.

Where a, b, c, d, α, β, γ, μ, ρ, m, n, p and q are constant parameters, when a=10, b=8/3, c=28, d=1.3, α=35, β=7, γ=12, μ=0.5, ρ=3, m=36, n=20, p=3 and q=1.3, the three four-dimensional dynamical systems show hyperchaotic behaviors, respectively. For more detailed analysis of the complex dynamics of the systems, please see relative Ref [13-15].

3. Anti-synchronization between hyperchaotic Lorenz and Chen systems

In this section, based on the active control theory, AS two different hyperchaotic Lorenz and Chen systems is achieved.

Let the drive and response systems are given as follows

$$\begin{cases}\dot{x}_1=a(y_1-x_1)+w_1\\ \dot{y}_1=cx_1-y_1-x_1z_1\\ \dot{z}_1=x_1y_1-bz_1\\ \dot{w}_1=-x_1z_1+dw_1\end{cases}\tag{4}$$

and

$$\begin{cases}\dot{x}_2=\alpha(y_2-x_2)+w_2+u_1\\ \dot{y}_2=\beta x_2-x_2z_2+ry_2+u_2\\ \dot{z}_2=x_2y_2-\rho z_2+u_3\\ \dot{w}_2=y_2z_2+\mu w_2+u_4\end{cases}\tag{5}$$

Where the system parameters are chosen such that both systems (4) and (5) are in hyperchaotic states when the control function ui=0 (i=1, 2, 3, 4) .our goal is to determine the control function from active control method. In order to observe the AS, we should add (4) to (5) instead of subtracting (4) from (5)

$$\begin{cases}\dot{e}_1=\alpha(e_2-e_1)+(a-\alpha)(y_1-x_1)+e_4+u_1\\ \dot{e}_2=\beta e_1+re_2+(c-\beta)x_1-(1+r)y_1-x_2z_2-x_1z_1+u_2\\ \dot{e}_3=-\rho e_3+(\rho-b)z_1+x_2y_2+x_1y_1+u_3\\ \dot{e}_4=\mu e_4+(d-\mu)w_1+y_2z_2-x_1z_1+u_4\end{cases}\tag{6}$$

where

$$e_1=x_2+x_1, e_2=y_2+y_1, e_3=z_2+z_1, e_4=w_2+w_1.$$

According to the clue of active control, the control functions ui (i=1, 2, 3, 4) can be designed

$$\begin{cases}u_1=(\alpha-a)(y_1-x_1)+v_1\\ u_2=(\beta-c)x_1+(1+r)y_1+x_2z_2+x_1z_1+v_2\\ u_3=(b-\rho)z_1-x_2y_2-x_1y_1+v_3\\ u_4=(\mu-d)w_1-y_2z_2+x_1z_1+v_4\end{cases}\tag{7}$$

Hence the error system becomes

$$\begin{cases} \dot{e}_1 = \alpha(e_2 - e_1) + e_4 + v_1 \\ \dot{e}_2 = \beta e_1 + re_2 + v_2 \\ \dot{e}_3 = -\rho e_3 + v_3 \\ \dot{e}_4 = \mu e_4 + v_4 \end{cases} \tag{8}$$

The error system (6) to be controlled is a linear system with control input v1, v2, v3 and v4 as the function of the error states e1, e2, e3 and e4. As long as these feedbacks stabilize the system, e1, e2, e3 and e4 converge to zero as time $t\to\infty$. This implies that two different hyperchaotic systems are anti-synchronized with feedback control. There are many possible choices for the control v1, v2, v3 and v4. We choose

$$\begin{cases} v_1 = -\alpha e_2 - e_4 \\ v_2 = -\beta e_1 - (1+r)e_2 \\ v_3 = 0 \\ v_4 = -(1+\mu)e_4 \end{cases} \tag{9}$$

Then the error dynamical system is

$$\dot{e}_1 = -\alpha e_1, \dot{e}_2 = -e_2, \dot{e}_3 = -\rho e_3, \dot{e}_4 = -e_4 \tag{10}$$

Choose the following Lyapunov function

$$V = \frac{1}{2}(e_1^{\ 2} + e_2^{\ 2} + e_3^{\ 2} + e_4^{\ 2})$$

The time derivation of the Lyapunov function along the trajectory is

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = -[\alpha e_1^{\ 2} + e_2^{\ 2} + \rho e_3^{\ 2} + e_4^{\ 2}]$$

Since the Lyapunov function V is positive definite and its derivative $\dot{V}$ is negative definite in the neighborhood of the zero solution for the system (8). In light of the Lyapunov stability theory, the error dynamical system can converge to the origin asymptotically. This implies that the two different hyperchaotic systems are anti-synchronized.

4. Anti-synchronization between hyperchaotic Chen and Lü systems

In this section, we consider the hyperchaotic Chen system as the drive system and the hyperchaotic Lü system as the response system.

Let the drive and response systems are given as follows

$$\begin{cases} \dot{x}_1 = \alpha(y_1 - x_1) + w_1 \\ \dot{y}_1 = \beta x_1 - x_1 z_1 + ry_1 \\ \dot{z}_1 = x_1 y_1 - \rho z_1 \\ \dot{w}_1 = y_1 z_1 + \mu w_1 \end{cases} \tag{11}$$

and

$$\begin{cases} \dot{x}_2 = m(y_2 - x_2) + w_2 + u_1 \\ \dot{y}_2 = -x_2 z_2 + ny_2 + u_2 \\ \dot{z}_2 = x_2 y_2 - pz_2 + u_3 \\ \dot{w}_2 = x_2 z_2 + qw_2 + u_4 \end{cases} \tag{12}$$

Our goal is to determine the control functions u_i (i=1, 2, 3, 4) to make AS in the two different hyperchaotic systems via using active control. Similarly, adding (11) to (12), we obtain the following error dynamical system

$$\begin{cases}\dot{e}_1 = m(e_2 - e_1) + (\alpha - m)(y_1 - x_1) + e_4 + u_1 \\ \dot{e}_2 = ne_2 + (r - n)y_1 + \beta x_1 - x_2 z_2 - x_1 z_1 + u_2 \\ \dot{e}_3 = -pe_3 + (p - \rho)z_1 + x_2 y_2 + x_1 y_1 + u_3 \\ \dot{e}_4 = qe_4 + (\mu - q)w_1 + x_2 z_2 + y_1 z_1 + u_4\end{cases} \quad (13)$$

Where

$$e_1 = x_2 + x_1, e_2 = y_2 + y_1, e_3 = z_2 + z_1, e_4 = w_2 + w_1.$$

To achieve the asymptotic stability of the zero solution of the error system (13), we take the active control functions ui (i=1, 2, 3, 4) as follows:

$$\begin{cases}u_1 = (m - \alpha)(y_1 - x_1) + v_1 \\ u_2 = -\beta x_1 + (n - r)y_1 + x_2 z_2 + x_1 z_1 + v_2 \\ u_3 = (\rho - p)z_1 - x_2 y_2 - x_1 y_1 + v_3 \\ u_4 = (q - \mu)w_1 - x_2 z_2 - y_1 z_1 + v_4\end{cases} \quad (14)$$

Hence the error system becomes

$$\begin{cases}\dot{e}_1 = m(e_2 - e_1) + e_4 + v_1 \\ \dot{e}_2 = ne_2 + v_2 \\ \dot{e}_3 = -pe_3 + v_3 \\ \dot{e}_4 = qe_4 + v_4\end{cases} \quad (15)$$

According to the original method of active control, v_i (i=1, 2, 3, 4) are chosen as

$$\begin{cases}v_1 = -me_2 - e_4 \\ v_2 = -(1 + n)e_2 \\ v_3 = 0 \\ v_4 = -(1 + q)e_4\end{cases} \quad (16)$$

Then the error dynamical system (12) becomes

$$\dot{e}_1 = -me_1, \dot{e}_2 = -e_2, \dot{e}_3 = -pe_3, \dot{e}_4 = -e_4 \quad (17)$$

Therefore, the closed loop system (13) has the eigenvalues: $-m$, -1, $-p$, and -1. By the Lyapunov stability theory, this choice will lead to the error states e_1, e_2, e_3 and e_4 converge to zero as time $t \to \infty$. That is to say, systems (11 and (12) can achieve global asymptotical anti-synchronization.

5. Anti-synchronization between hyperchaotic Lü and Lorenz systems

In this section, we choose the hyperchaotic Lü system as the drive system and the hyperchaotic Lorenz system as the response system.

Let the drive and response systems are given as follows

$$\begin{cases}\dot{x}_1 = m(y_1 - x_1) + w_1 \\ \dot{y}_1 = -x_1 z_1 + ny_1 \\ \dot{z}_1 = x_1 y_1 - pz_1 \\ \dot{w}_1 = x_1 z_1 + qw_1\end{cases} \quad (18)$$

and

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2) + w_2 + u_1 \\ \dot{y}_2 = cx_2 - y_2 - x_2 z_2 + u_2 \\ \dot{z}_2 = x_2 y_2 - bz_2 + u_3 \\ \dot{w}_2 = -x_2 z_2 + dw_2 + u_4 \end{cases} \tag{19}$$

Similarly, adding the drive system (18) to the response system (19), we obtain the following error dynamical system

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + (m-a)(y_1 - x_1) + e_4 + u_1 \\ \dot{e}_2 = ce_1 - e_2 + (n+1)y_1 - cx_1 - x_2 z_2 - x_1 z_1 + u_2 \\ \dot{e}_3 = -be_3 + (b-p)z_1 + x_2 y_2 + x_1 y_1 + u_3 \\ \dot{e}_4 = de_4 + (q-d)w_1 - x_2 z_2 + x_1 z_1 + u_4 \end{cases} \tag{20}$$

where $e_1 = x_1 + x_2, e_2 = y_2 + y_1, e_3 = z_2 + z_1, e_1 = w_2 + w_1$.

To achieve the asymptotic stability of the zero solution of the error system (20), we choose the active control functions ui (i=1, 2, 3, 4) as follows:

$$\begin{cases} u_1 = (a-m)(y_1 - x_1) + v_1 \\ u_2 = cx_1 - (n+1)y_1 + x_2 z_2 + x_1 z_1 + v_2 \\ u_3 = (p-b)z_1 - x_2 y_2 - x_1 y_1 + v_3 \\ u_4 = (d-q)w_1 + x_2 z_2 - x_1 z_1 + v_4 \end{cases} \tag{21}$$

This leads to

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + v_1 \\ \dot{e}_2 = ce_1 - e_2 + v_2 \\ \dot{e}_3 = -be_3 + v_3 \\ \dot{e}_4 = de_4 + v_4 \end{cases} \tag{22}$$

According to the original method of active control, v_i (i=1, 2, 3, 4) are chosen as

$$\begin{cases} v_1 = -ae_2 - e_4 \\ v_2 = -ce_1 \\ v_3 = 0 \\ v_4 = -(1+d)e_4 \end{cases} \tag{23}$$

Then the error dynamical system (20) becomes

$$\dot{e}_1 = -ae_1, \dot{e}_2 = -e_2, \dot{e}_3 = -be_3, \dot{e}_4 = -e_4 \tag{24}$$

Therefore, the closed loop system (20) has the eigenvalues: -a, -1, -b, and -1. This choice will lead to the error states e_1, e_2, e_3 and e_4 converge to zero as time $t \to \infty$. This guarantees the asymptotic stability of the system (24), which implies that AS between the hyperchaotic Lü system and hyperchaotic Lorenz system is realized.

6. Numerical simulations

In this section, to verify and demonstrate the effectiveness of the proposed method, we consider three numerical examples. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. We choose the parameters of hyperchatic systems as a=10, b=8/3, c=28, d=1.3, α=35, β=7, γ=12, μ=0.5, ρ=3, m=36, n=20, p=3 and q=1.3, so that the three systems have hyperchaotic attractors, respectively.

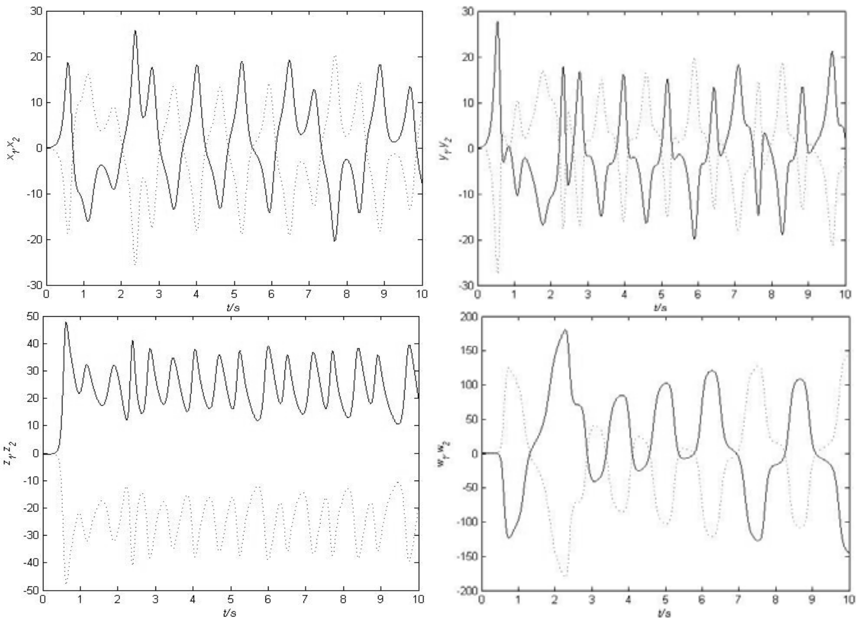

Fig.1 State trajectories drive system states (-), response system states (:)

Example 1. Anti-synchronization **between hyperchaotic Lorenz-Chen systems**

Consider the systems given in (4) and (5). The initial values of the drive system and response system are taken as ($x_1(0)$, $y_1(0)$, $z_1(0)$,$w_1(0)$)=(-0.1,0.2,-0.6,0.4), ($x_2(0)$, $y_2(0)$, $z_2(0)$,$w_2(0)$)=(-1,0.4,-0.2,1), respectively; hence, the initial errors are $e_1(0)$= -1.1, $e_2(0)$= 0.6, $e_3(0)$=-0.8 and $e_4(0)$=1.4. The simulation results are illustrated in Fig.1. Fig.1 displays the time evolutions of the drive system and the response system.

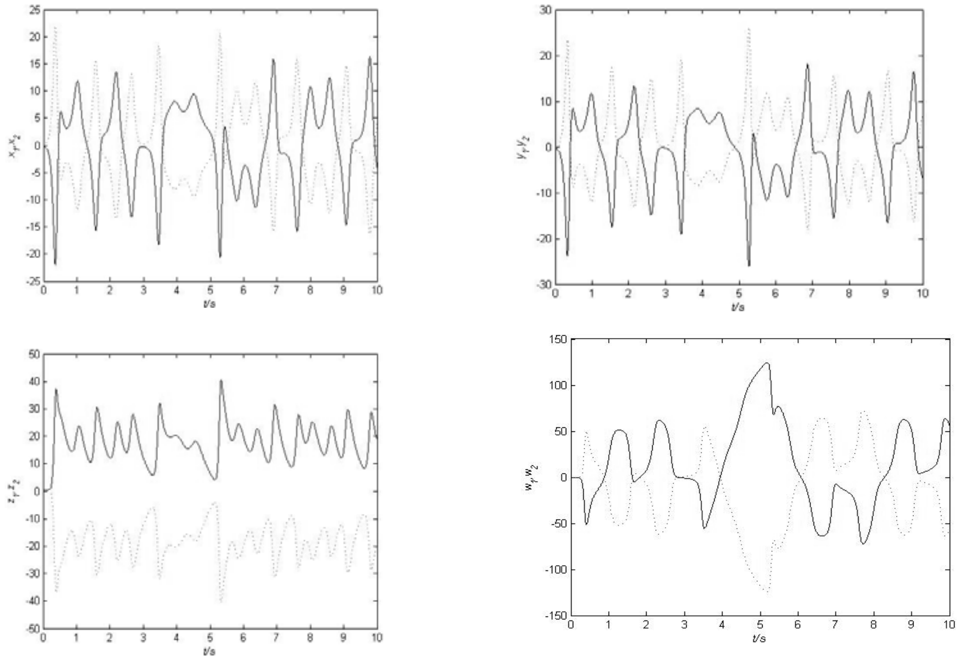

Fig.2 State trajectories drive system states (-), response system states (:)

Example 2. Anti-synchronization between hyperchaotic Chen-Lü systems

The initial values of the drive system (11) and response system (12) are taken as (x1(0), y1(0), z1(0),w1(0))=(0.1,-0.2,0.6,-0.4), (x2(0), y2(0), z2(0),w2(0))=(1,-0.4,0.2,-1), respectively. Thus, the initial errors are e1(0)=1.1, e2(0)=-0.6, e3(0)=0.8 and e4(0)=-1.4. The results of the two different hyperchaotic systems with the active control are shown in Fig.2. Fig.2 displays the time evolutions of the drive system and the response system.

Example 3. Anti-synchronization between hyperchaotic Lü-Lorenz systems

The initial values of the drive system and response system are taken as (x1(0), y1(0), z1(0),w1(0))=(-

1,2,-6, 4), (x2(0), y2(0), z2(0),w2(0))=(-1, 4, -2, 1), respectively. Thus, the initial errors are e1(0)=-2, e2(0)=6, e3(0)=-8 and e4(0)=5. The results of the two different hyperchaotic systems with the active control are shown in Fig.3. Fig.3 displays the time evolutions of the drive system and the response system.

7. Conclusion

In this paper, we study the problem of anti-synchronization for two different hyperchaotic systems. Using the active control method, we have realized the anti-synchronization for hyperchaotic Lorenz-Chen system, hyperchaotic Chen-Lü system and hyperchaotic Lü-Lorenz system. Numerical stimulations are used to verify effectiveness of the proposed control methods. It is clear that one can use active control theory to achieve anti-synchronization in other chaotic systems.

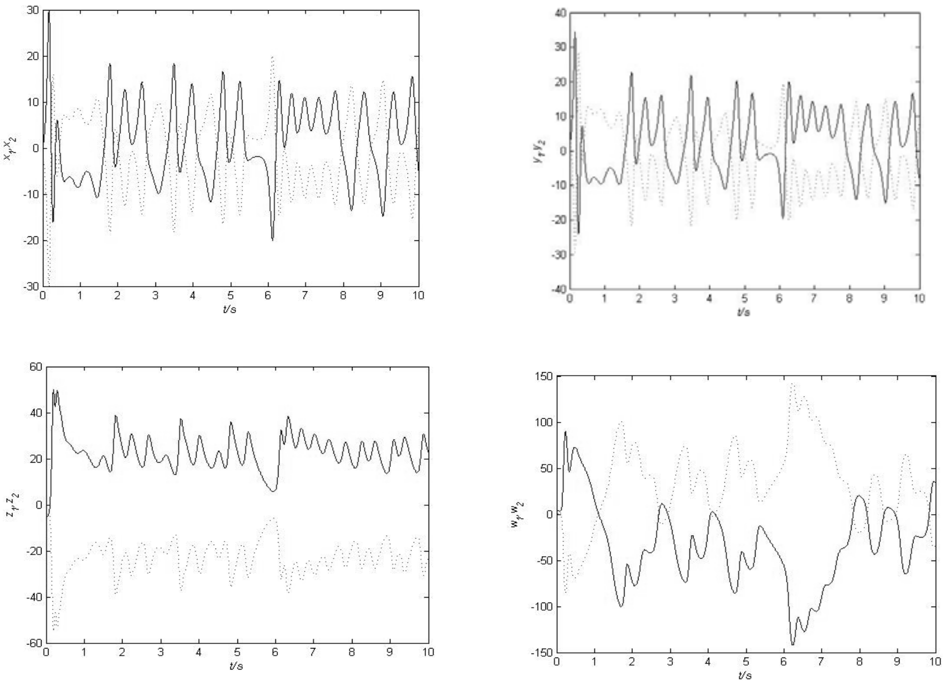

Fig.3 State trajectories drive system states (-), response system states (:)

8. Acknowledgements

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