A Fuzzy Optimization Method for Multi-criteria Decision-making Problem Based on the Inclusion Degrees of Intuitionistic Fuzzy Sets

Yan Luo[1] and Changrui Yu[1,2+]

[1] Institute of System Engineering, Shanghai Jiao Tong University, Shanghai 200052, China

[2] School of Information Management and Engineering, Shanghai University of Finance and Economics, Shanghai 200433, China

Abstract. This paper presents a fuzzy optimization method based on the inclusion degrees of intuitionistic fuzzy sets to solve multi-criteria decision making problems under fuzzy environments. First, the inclusion degree of intuitionistic fuzzy sets is defined and a series of specific formulas of inclusion degree are presented by means of the normal implication operators. Some formulas of inclusion degree of intuitionistic fuzzy sets are generalized by defining the cardinal number of intuitionistic fuzzy sets. Then, we give multi-criteria fuzzy decision-making method based on inclusion degree of intuitionistic fuzzy sets. Finally, we illustrate the effectiveness of the method proposed in this paper by an example.

Keywords: Multi-criteria Decision-making, Fuzzy Optimization, Inclusion Degrees, IFS.

1. Introduction

Fuzzy sets were introduced by Zadeh in 1965 [1]. In the following several decades, fuzzy set theory has been used for handling fuzzy multi-criteria decision-making problems [2-5]. The main characteristic of fuzzy sets is that: the membership function assigns to each element *u* in a universe of discourse a membership degree ranging between 0 and 1 and the non-membership degree equals one minus the membership degree, i.e., this membership degree combines the evidence for *u* and the evidence against *u*. The single number tells us nothing about the lack of knowledge. In real applications, however, the information of an object corresponding to a fuzzy concept may be incomplete, i.e., the sum of the membership degree and the non-membership degree of an element in a universe corresponding to a fuzzy concept may be less than one. In fuzzy set theory, there is no means to incorporate the lack of knowledge with the membership degrees. A possible solution is to use intuitionistic fuzzy sets (IFSs for short), introduced by Atanassov in [6]. IFS as an extension of Zadeh fuzzy set was also applied to the decision-making problems [7-12].

In the present paper, the inclusion degree of intuitionistic fuzzy sets is applied to multi-criteria decision making in fuzzy environment. In Section 2 we define the inclusion degree of intuitionistic fuzzy sets and present a series of specific formulas of inclusion degree by means of the normal implication operators. Then some formulas of inclusion degree of fuzzy sets are generalized to intuitionistic fuzzy sets by defining the cardinal number of intuitionistic fuzzy sets. In Section 3 we give a fuzzy method for multi-criteria decision-making method based on inclusion degree of intuitionistic fuzzy sets. Finally, we illustrate the effectiveness of the method proposed in this paper by an example.

2. Inclusion degrees of IFSs

The concept of IFSs is an extension of Zadeh's fuzzy sets. It give us the possibility to model unknown information by using an additional degree. In [6] [13,14], IFSs are defined as follows:

Definition 1. An intuitionistic fuzzy set *A* on a universe *U* is defined as an object of the following form:

$$A = \{(u, \mu_A(u), v_A(u)) \mid u \in U\},$$

where the functions $u_A : U \to [0,1]$ and $v_A : U \to [0,1]$ define the degree of membership and the degree of non-membership of the element $u \in U$ in A, respectively, and for every $u \in U$:

$$0 \le \mu_A(u) + v_A(u) \le 1,$$

Obviously, each ordinary fuzzy set may be written as $\{(u, \mu_A(u), 1 - \mu_A(u)) \mid u \in U\}$. For simplicity, the class of intuitionistic fuzzy sets on a universe U will be denoted by *IFS(U)*.

2.1. Inclusion degrees based on implicational operators

Definition 2 If the mapping $I : IFS(U) \times IFS(U) \to [0,1]$ satisfies

(i) $A \subseteq B \Rightarrow I(A, B) = 1$;

(ii) $I(U, \phi) = 0$;

(iii) $A \subseteq B \subseteq C \Rightarrow I(C, A) \le \min(I(B, A), I(C, B))$

then we call $I(A,B)$ the inclusion degree of *A* in *B*, and call *I* a inclusion degree function on *IFS(U)*.

Definition 3 If mapping $R : [0,1]^2 \to [0,1]$ satisfies

(i) $R(1,0) = 0$

(ii) $R(0,0) = R(0,1) = R(1,1) = 1$

then *R* is called fuzzy implicational operator (briefly, implication).

Definition 4 If mapping $T : [0,1]^2 \to [0,1]$ satisfies

(i) $T(a,b) = T(b,a)$

(ii) $a \le c, b \le d \Rightarrow T(a,b) = T(c,d)$

(iii) $T(T(a,b), c) = T(a, T(b,c))$

(iv) $T(a,1) = a(a \in [0,1])$

then it is called triangular norm (briefly, *t*-norm). If T satisfies (i)-(iii) and (iv)' $T(0,a) = a(a \in [0,1])$, then we call it triangular conorm (briefly, *t*-conorm).

Theorem 1 Let $A, B \in IFS(U)$ and *R* be an implication. If *R* satisfies

(i) $\forall a, b \in [0,1]$, and $a \le b \Rightarrow R(a,b) = 1$

(ii) $R(a,b)$ is non-decreasing with respect to *b* non-increasing with respect to *a*,

then the following I_1, I_2 are the inclusion degree functions of IFSs:

$$I_1(A, B) = \inf_{u \in U} [\lambda R(\mu_A(u), \mu_B(u)) + (1 - \lambda) R(v_B(u), v_A(u))], \lambda \in [0,1],$$

$$I_2(A, B) = \inf_{u \in U} T(R(\mu_A(u), \mu_B(u)), R(v_B(u), v_A(u)))$$

where $T : [0,1]^2 \to [0,1]$ is a *t*-norm

Proof. We only prove I_2. The proof of I_1 is similar.

(i) $A \subseteq B \Rightarrow \mu_A(u) \le \mu_B(u), v_A(u) \ge v_B(u), \forall u \in U$

$\Rightarrow R(\mu_A(u), \mu_B(u)) = 1, R(v_B(u), v_A(u)) = 1, \forall u \in U$

$\Rightarrow I_2(A, B) = \inf_{u \in U} T(1,1) = 1$

(ii) $I_2(U, \phi) = \inf_{u \in U} T(R(1,0), R(1,0)) = \inf_{u \in U} T(0,0) = 0$

(iii) $A \subseteq B \subseteq C \Rightarrow \mu_A(u) \le \mu_B(u) \le \mu_C(u), v_A(u) \ge v_B(u) \ge v_C(u)$

Since $R(a, \bullet)$ is non-decreasing and $R(\bullet, b)$ is non-increasing, we have

$$R(\mu_C(u), \mu_A(u)) \le R(\mu_C(u), \mu_B(u)), R(v_A(u), v_C(u)) \le R(v_B(u), v_C(u))$$
$$\Rightarrow T(R(\mu_C(u), \mu_A(u)), v_A(u), v_C(u)) \le T(R(\mu_C(u), \mu_B(u)), R(v_B(u), v_C(u)))$$
$$\Rightarrow \inf_{u \in U} T(R(\mu_C(u), \mu_A(u)), v_A(u), v_C(u)) \le \inf_{u \in U} T(R(\mu_C(u), \mu_B(u)), R(v_B(u), v_C(u)))$$
$$\Rightarrow I_2(C, A) \le I_2(C, B)$$

In a similar, we can get $I_2(C, A) \le I_2(B, A)$.

Theorem 2 Assume that U is a finite universe and R is an implication. If R satisfies

(i) $\forall a, b \in [0,1]$, and $a \le b \Rightarrow R(a,b) = 1$

(ii) $R(a,b)$ is non-decreasing with respect to b non-increasing with respect to a,

then the following I_3, I_4 are the inclusion degree functions of IFSs:

$$I_3(A, B) = \frac{1}{|U|} \sum_{u \in U} [\lambda R(\mu_A(u), \mu_B(u)) + (1-\lambda) R(v_B(u), v_A(u))], \lambda \in [0,1],$$

$$I_4(A, B) = \frac{1}{|U|} \sum_{u \in U} T(R(\mu_A(u), \mu_B(u)), R(v_B(u), v_A(u)))$$

where $|U|$ denotes the cardinality of U and $T : [0,1]^2 \to [0,1]$ is a *t*-norm.

The proof of theorem 2 is similar to the proof of theorem 1.

The implications R satisfy the conditions of above theorems, $\forall a, b \in [0,1]$:

(i) Lukasiewicz implication: $R_L(a,b) = \min(1 - a + b, 1)$

(ii) Goguen implication: $R_\pi(a,b) = \begin{cases} 1, & if \quad a = 0, \\ \min\left(\dfrac{b}{a}, 1\right), & if \quad a > 0. \end{cases}$

(iii) Gödel implication: $R_G(a,b) = \begin{cases} 1, & if \quad a \le b \\ b, & if \quad a > b. \end{cases}$

(iv) Gaines-Recher implication: $R_{GR}(a,b) = \begin{cases} 1, & if \quad a \le b \\ 0, & if \quad a > b. \end{cases}$

(v) R₀-implication: $R_\pi(a,b) = \begin{cases} 1, & if \quad a \le b \\ \max(1-a, b), & if \quad a > b \end{cases}$

2.2. Inclusion degrees based on the cardinalities of sets

Definition 5 Let U be a finite set, $A \in IFS(U)$. The cardinality of A is defined as

$$|A| = \sum_{u \in U} \frac{1 + \mu_A(u) - v_A(u)}{2},$$

It is easy to prove that the following inclusion functions of fuzzy sets I_5 - I_{10} [15] still hold to IFSs.

Theorem 3 Let U be a finite set, $A \in IFS(U)$. Then the following I_5 - I_{10} are inclusion degree functions of IFSs:

$I_5(A,B) = \begin{cases} 1, & A = \phi \\ \dfrac{\|A \cap B\|}{\|A\|}, & A \ne \phi \end{cases};$	$I_6(A,B) = \begin{cases} 1, & A = B = \phi \\ \dfrac{\|B\|}{\|A \cup B\|}, & otherwise \end{cases};$
$I_7(A,B) = \begin{cases} 1, & B = U \\ \dfrac{\|A^C \cap B^C\|}{\|B^C\|}, & B \ne U \end{cases};$	$I_8(A,B) = \begin{cases} 1, & A = B = U \\ \dfrac{\|A^C\|}{\|A^C \cup B^C\|}, & otherwise \end{cases};$

$$I_9(A,B) = \frac{\left|A^C \cup B\right|}{\left|A^C \cup A \cup B \cup B^C\right|};$$

$$I_{10}(A,B) = \begin{cases} 1, & A = \phi \quad or \quad B = U \\ \dfrac{\left|A^C \cap A \cap B \cap B^C\right|}{\left|A \cup B^C\right|}, & otherwise \end{cases}.$$

2.3. Generation of inclusion degrees

Theorem 4 Let *I* be an inclusion degree function on *IFS(U)* and mapping $h : [0,1]^2 \rightarrow [0,1]$ satisfies

(i) $h(0,0) = 0, h(1,1) = 1$;

(ii) $h(a,b)$ is non-decreasing with respect to a and b

then $GI(A,B) = h\big(I(A,B), I(B^C, A^C)\big)(\forall A, B \in IFS(U))$ is inclusion degree of *A* in *B*, and *GI* is an inclusion degree on *IFS(U)*.

proof. (i) $A \subseteq B \Rightarrow I(A,B) = 1, I(B^C, A^C) = 1 \Rightarrow GI(A,B) = h(1,1) = 1$

(ii) $GI(U, \phi) = h\big(I(U, \phi), I(\phi^C, U^C)\big) = h(0,0) = 0$

(iii) $A \subseteq B \subseteq C \Rightarrow C^C \subseteq B^C \subseteq A^C$

$\Rightarrow I(C,A) \le I(C,B), I(A^C, C^C) \le I(B^C, C^C)$

$\Rightarrow h\big(I(C,A), I(A^C, C^C)\big) \le h\big(I(C,B), I(B^C, C^C)\big)$

$\Rightarrow GI(C,A) \le GI(C,B)$

In the similar way, we can get $GI(C,A) \le GI(B,A)$

Theorem 5 Let I_1, I_2 be inclusion degree functions on *IFS(U)*, and $h : [0,1]^2 \rightarrow [0,1]$ satisfies

(i) $h(0,0) = 0, h(1,1) = 1$;

(ii) $h(a,b)$ is non-decreasing with respect to *a* and *b*,

then $GI(A,B) = h\big(I_1(A,B), I_2(A,B)\big)(\forall A, B \in IFS(U))$ is inclusion degree of *A* in *B*, and *GI* is an inclusion degree on *IFS(U)*.

The proof of theorem 5 is similar to the proof of theorem 4.

3. Multi-criteria fuzzy decision-making based on inclusion degrees

Definition 6 (Multi-criteria fuzzy decision-making problem [7]). Let *M* be a set of alternatives and let *C* be a set of criteria, where

$$M = \{M_1, M_2, \cdots, M_m\}, \quad C = \{C_1, C_2, \cdots, C_n\},$$

Assume that the characteristics of the alternative M_i are presented by the IFS shown as follows:

$$M_i = \{(C_1, \mu_{i1}, v_{i1}), (C_2, \mu_{i2}, v_{i2}), \cdots, (C_n, \mu_{in}, v_{in})\},$$

where μ_{ij} indicates the degree to which the alternative M_i satisfies criterion C_j, v_{ij} indicates the degree to which the alternative M_i does not satisfy criterion $C_j (\mu_{ij}, v_{ij}) \in L^*(j = 1,2,\cdots,n; i = 1,2,\cdots,m)$.

Assume that there is a decision-maker who wants to choose an alternative which satisfies the criteria $C_j, C_k, \cdots,$ and C_p or which satisfies the criterion C_s. This decision-maker's requirement is represented by the following expression:

$$C_j \, and \, C_k \, and \cdots and \, C_p \, or \, C_s,$$

It is noted that we say an alternative satisfies a criterion if it meets some desirable level of an evaluation criterion. The satisfaction is gradual and is characterized by a dual information: a degree of satisfaction and a degree of non-satisfaction.

We now use the inclusion degrees of IFSs to solve the mufti-criteria fuzzy decision-making problem (definition in [7]).The basic idea is similar to the TOPSIS (Technique for Order Preference by Similarity to

Ideal Solution) [16-20]. Firstly, the ideal solution and the anti-ideal solution are constructed, where the ideal solution and the anti-ideal solution are respectively the best and the worst solution supposed but not existing in the set of alternatives. Then we compare the inclusion degrees of the ideal solution in alternative M_i and the inclusion degrees of alternative M_i in the anti-ideal solution. The alternative containing the ideal solution maximally as well as being contained by the negatively ideal solution minimally is the best choice. For this purpose, we introduce the following definition.

Definition 7 Let $M_i = (i = 1, \cdots, m)$ be the set of alternatives and $C_j = (j = 1, \cdots, n)$ be the set of criteria,

(i) The ideal solution and the anti-ideal solution satisfying the criteria $C_j, C_k, \cdots C_p$ are defined as follows:

$$
\begin{aligned}
G_1 &= \left\{ \left(C_j, \bigvee_{i=1}^m \mu_{ij}, \bigwedge_{i=1}^m v_{ij} \right), \left(C_k, \bigvee_{i=1}^m \mu_{ik}, \bigwedge_{i=1}^m v_{ik} \right), \cdots, \left(C_p, \bigvee_{i=1}^m \mu_{ip}, \bigwedge_{i=1}^m v_{ip} \right) \right\}, \\
&= \left\{ \left(C_j, \mu_{gj}, v_{gj} \right), \left(C_k, \mu_{gk}, v_{gk} \right), \cdots, \left(C_p, \mu_{gp}, v_{gp} \right) \right\}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
B_1 &= \left\{ \left(C_j, \bigwedge_{i=1}^m \mu_{ij}, \bigvee_{i=1}^m v_{ij} \right), \left(C_k, \bigwedge_{i=1}^m \mu_{ik}, \bigvee_{i=1}^m v_{ik} \right), \cdots, \left(C_p, \bigwedge_{i=1}^m \mu_{ip}, \bigvee_{i=1}^m v_{ip} \right) \right\} \\
&= \left\{ \left(C_j, \mu_{bj}, v_{bj} \right), \left(C_k, \mu_{bk}, v_{bk} \right), \cdots, \left(C_p, \mu_{bp}, v_{bp} \right) \right\}
\end{aligned}
\tag{2}
$$

Definition 8 The inclusion degree $D(M_i)$ of the ideal solution in alternative M_i and the inclusion degree $d(M_i)$ of alternative M_i in the anti-ideal solution are respectively defined as follows:

$$
D(M_i) = \max\left(I(G_1, M_{i1}), I(G_2, M_{i2}) \right),
\tag{5}
$$

$$
d(M_i) = \min\left(I(M_{i1}, B_1), I(M_{i2}, B_2) \right)
\tag{6}
$$

where I denotes the inclusion degree function,

$$
M_{i1} = \left\{ \left(C_j, \mu_{ij}, v_{ij} \right), \left(C_k, \mu_{ik}, v_{ik} \right), \cdots, \left(C_p, \mu_{ip}, v_{ip} \right) \right\}, M_{i2} = \left\{ \left(C_s, \mu_{bs}, v_{bs} \right) \right\}, i = 1, \cdots, m
$$

Definition 9 The ranking index of alternative $M_i = (i = 1, \cdots, m)$ is defined as follows:

$$
p_i = \frac{D(M_i)}{d(M_i) + D(M_i)},
\tag{7}
$$

The procedure of solving multi-criteria fuzzy decision-making problem (**Definition 6**) is as follows:

(i) calculate the ideal solution G_1 and the anti-ideal solution B_1 satisfying the criteria $C_j, C_k, \cdots C_p$, calculate the ideal solution G_2 and the anti-ideal solution B_2 satisfying the criteria C_s;

(ii) calculate the inclusion degree $I(G_j, M_{ij})$ of G_j in M_{ij} and the inclusion degree $I(M_{ij}, B_j)$ of M_{ij} in $B_j, j = 1,2; i = 1, \cdots, m$;

(iii) calculate the inclusion degree $D(M_i)$ of the ideal solution in alternative M_i and the inclusion degree $d(M_i)$ of M_i in the anti-ideal solution;

(iv) calculate the ranking index p_i of alternative $M_i = (i = 1, \cdots, m)$

(v) if there exists $i_0 \in \{1, 2, \cdots, m\}$ such that $p_{i_0} = \max(p_1, p_2, \cdots, p_m)$, then alternative M_{i_0} is the best choice.

The reason for introducing the ideal solution and the anti-ideal solution simultaneously in above method is that when two alternatives contain the ideal solution by the same inclusion degree, we introduce the negatively ideal solution for differentiating which alternative is superior. Then the alternative contained by the anti-ideal solution with the less inclusion degree is the better choice.

4. An example

Let M_1, M_2, M_3, M_4, M_5 be five alternative, and let C_1, C_2, C_3 be three criteria. Assume that the characteristics of the alternatives are represented by the IFSs shown as follows:

$$M_1 = \{(C_1,(0.2,0.2)),(C_2,(0.3,0.1)),(C_3,(0.2,0))\}$$
$$M_2 = \{(C_1,(0.3,0.3)),(C_2,(0.2,0.2)),(C_3,(0.3,0.1))\}$$
$$M_3 = \{(C_1,(0.4,0.4)),(C_2,(0.5,0.4)),(C_3,(0.3,0.2))\}$$
$$M_4 = \{(C_1,(0.5,0.3)),(C_2,(0.4,0.4)),(C_3,(0.5,0.3))\}$$
$$M_5 = \{(C_1,(0.4,0.4)),(C_2,(0.6,0.3)),(C_3,(0.6,0.4))\}$$

and assume that the decision-maker wants to choose an alternative which satisfies the criteria C_1, C_2 or which satisfies the criterion C_3.

Firstly, we construct the ideal solution and the negatively ideal solution satisfying the criteria C_1, C_2 and satisfying criterion C_3.

$$G_1 = \{(C_1,0.5,0.2),(C_2,0.6,0.1)\}, G_2 = \{(C_3,0.6,0)\}$$
$$B_1 = \{(C_1,0.2,0.4),(C_2,0.2,0.4)\}, B_2 = \{(C_3,0.2,0.4)\}$$

Take I_3 in theorem 2 as the inclusion degree function I and take $\lambda = \dfrac{1}{2}$, i.e.,

$$I(A,B) = \frac{1}{|U|}\sum_{u \in U}\frac{R(\mu_A(u_i),\mu_B(u_i)) + R(v_B(u_i),v_A(u_i))}{2}$$

where we choose Lukasiewicz implication R_L as implication R, i.e., $R_L(a,b) = \min(1 - a + b,1), \forall a,b \in [0,1]$. So we obtain the inclusion degree $I(G_j,M_{ij})$ of G_j in M_{ij} and the inclusion degree $I(M_{ij},B_j)$ of M_{ij} in $B_j (j = 1,2; i = 1,2,3,4,5)$ listed in table 1 and 2

	M_{11}	M_{21}	M_{31}	M_{41}	M_{51}
$I(G_1,M_{i1})$	0.85	0.8	0.825	0.85	0.875
$I(M_{i1},B_1)$	0.8	0.9	0.875	0.85	0.825

Table 1: Inclusion degrees of G_1 in M_{i1} and M_{i1} in B_1

	M_{12}	M_{22}	M_{32}	M_{42}	M_{52}
$I(G_2,M_{i2})$	0.8	0.8	0.75	0.8	0.8
$I(M_{i2},B_2)$	0.8	0.8	0.85	0.8	0.8

Table 2. Inclusion degrees of G_2 in M_{i2} and M_{i2} in B_2

Using formulae (5) and (6) we get the inclusion degree $D(M_i)$ of the ideal solution in alternative M_i and the inclusion degree $d(M_i)$ of M_i in the negatively ideal solution. We fist them in table 3.

	M_1	M_2	M_3	M_4	M_5
$D(M_i)$	0.85	0.8	0.825	0.85	0.875
$d(M_i)$	0.8	0.8	0.85	0.8	0.8

Table 3: Inclusion degrees of G_2 in M_{i2} and M_{i2} in B_2

From (7), we get the ranking index of alternatives as follows:

$$p_1 = 0.515, p_2 = 0.5, p_3 = 0.4925, p_4 = 0.515, p_5 = 0.5224$$

Therefore, alternative M_5 is the best choice.

5. Conclusion

This paper presents a fuzzy optimization method based on the inclusion degrees of intuitionistic fuzzy sets to solve multi-criteria decision making problems under fuzzy environments. Inclusion degree is a quantity describing that a set is contained by another set and is quantitative description of containment relation. It holds the uncertainty of the relation. The inclusion degree theory and IFS theory are the important tools in studying the uncertain knowledge. The proposed method in the paper has been demonstrated by an example, illustrating the power of the approach to solve multi-criteria fuzzy decision making problems. This research work not only develops and enriches the fundamental theory of IFSs but also provides a new idea

for the applications of IFS theory.

6. Acknowledgement

7. References

[1]   Zadeh L. A. Fuzzy sets. Information and Control 1965; 8(3): 338－356.

[2]   Bortolan G., Degani R. A review of some methods for ranking fuzzy subsets. Fuzzy Sets and Systems 1985; 15(1) (1985) pp: 1-19.

[3]   Liou T. S., Wang M. J. Ranking fuzzy numbers with integral value. Fuzzy Sets and Systems 1992; 50(3): 247－255.

[4]   Tran L. and Duckstein L. Comparison of fuzzy numbers using a fuzzy distance measure. Fuzzy Sets and Systems 2002; 130(3): 331－341.

[5]   Wang X. Z. and Kerre E.E. Reasonable properties for the ordering of fuzzy quantities (I) and (II). Fuzzy Sets and Systems 2001; 118(3): 387－405.

[6]   Atanassov K. Intuitionistic fuzzy sets, VII ITKR's Session, Sofia, June 1983 (Deposed in Central Science-Technical Library of Bulg. Academy of Science, 1697/84) (in Bulgarian).

[7]   Chen S. M. and Tan J. M. Handling multi-criteria fuzzy decision-making problems based on vague set theory. Fuzzy Sets and Systems 1994; 67(2): 163–172.

[8]   Hong D.H.and Choi C.H. Multi-criteria fuzzy decision-making problems based on vague set theory. Fuzzy Sets and Systems 2000; 114: 103–113.

[9]   Szmidt E. and Kacprzyk J. Group decision making under intuitionistic fuzzy preference relations. Proc. of the Seventh International Conference on IPMU 1998; 172–178.

[10]   Atanassov K., Pasi G. and Yager R. Intuitionistic fuzzy interpretations of multi-measurement tool multi-criteria decision making. Proc. of the Sixth International Conference on Intuitionistic Fuzzy Sets, Varna, 13–14 September 2002, Lecture Notes on Intuitionistic Fuzzy Sets 2002; 8(3): 66–74.

[11]   Atanassov K., Pasi G., Yager R.and Atanassova V. Intuitionistic fuzzy group interpretations of multi-person multi-criteria decision making. Proc. of the Third Conference of the European Society for Fuzzy Logic and Technology EUSFLAT'2003. Zittau, 10–12 September 2003; 177–182.

[12]   Pasi G., Yager Y.and Atanassov K. Intuitionistic fuzzy graph interpretations of multi-person multi-criteria decision making: Generalized net approach. Proc. of 2004 second International IEEE Conference Intelligent Systems 2004; 2: 434–439.

[13]   Atanassov K. Intuitionistic fuzzy sets. Fuzzy Sets and Systems 1986; 20: 87－96.

[14]   Atanassov K. Intuitionistic fuzzy sets. Physica-Verlag, Heidelberg, New York, 1999.

[15]   Fan J., Xie W.and Pei J. Subsethood measure: new definitions. Fuzzy Sets and Systems 1999; 106: 201-209.

[16]   Chen C. T. Extensions of the TOPSIS for group decision-making under fuzzy environment. Fuzzy Sets and Systems 2000; 114(1): 1-9.

[17]   Chu T. C.and Lin Y. C. A fuzzy TOPSIS method for robot selection. The International Journal of Advanced Manufacturing Technology 2003; 21: 284-290.

[18]   Wang Y. M.and Elhag T. Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment. Expert Systems with Applications 2006; 31(2): 309－319.

[19]   Abo-Sinna M. A.and Amer A. H. Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems. Applied Mathematics and Computation 2005; 162(1): 243－256.

[20]   Liang G. S. Fuzzy MCDM based on ideal and anti-ideal concepts. European Journal of Operational Research 1999; 112(3): 682－691.