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Adaptive Backstepping Control of the Uncertain Liu Chaotic System

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Abstract: This paper investigates the method for controlling the uncertain Liu chaotic system with known parameters k and h via adaptive backstepping control. With the method, parameters identification and control can be achieved simultaneously and quickly with only one controller within finite steps based on Lyapunov stabilization theorem. Numerical stimulations are provided to show the effectiveness and feasibility of this method.

Keywords: uncertain Liu chaotic system, adaptive backstepping, chaotic control

1. Introduction

In recent years, there has been considerable interest in the control and application of chaos in nonlinear dynamical systems. For past years, many different techniques have been proposed to control chaos, such as OGY method, differential geometric approach, linear state space feedback, adaptive control, fuzzy control and backtepping design [1-8].

However, for some uncertain systems, many of the aforementioned methods will fail. An important problem is how to achieve nonlinear control of uncertain complex dynamics systems. This problem involves both the identification of the unknown parameters and the approach of chaos control. In this paper, we investigate the method for controlling the uncertain Liu chaotic system with known parameters k and h. Since the Liu chaotic system cannot be controlled directly using the backstepping method for its singularity problem, we transform it into the so-called general strict-feedback form [7]. Then an adaptive controller is presented to identify the unknown parameters and control Liu chaotic system to a bounded point simultaneously. Especially, the designed update laws of the unknown parameters can remarkably improve the efficiency of the identification of the unknown parameters and quickly control the Liu system to a bounded point. Numerical simulations are given for illustration and verification.

2. System description

In 1963, Lorenz found the first canonical chaotic attractor [9]. In 1999, Chen found another similar but not topological equivalent chaotic attractor [10]. In 2002, Lü and Chen found the critical attractor between the Lorenz and Chen attractor [11]. In the same year, Lü et al. unified above three chaotic systems into a chaotic system which is called unified chaotic system [12]. In 2004, Liu et al. found a new chaotic system, bearing the name of Liu system [13].

The nonlinear differential equations that describe the Liu system are

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = bx - kxz, \\ \dot{z} = -cz + hx^2, \end{cases} \tag{1}$$

where a, b, c, k and h are system parameters. The system (1) have a chaotic attractor, as shown in Fig.1, when a=10, b=40, c=2.5, k=1, h=4.

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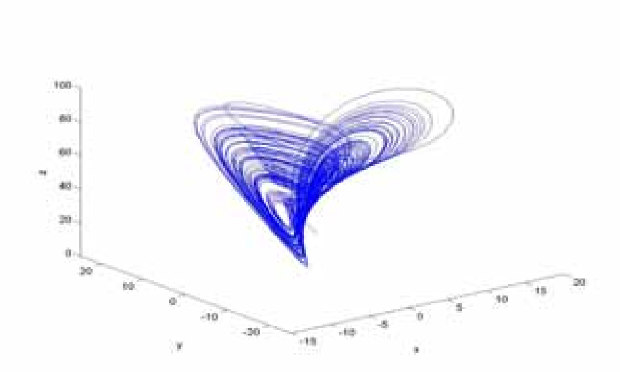


Fig.1.Liu chaotic attractor

3. The general strict-feedback normal form

Since the Liu chaotic system cannot be controlled directly using the backstepping method for its singularity problem, we transform it into the so-called general strict-feedback form.

We assume that the parameters a, b and c of the system (1) are unknown. The other parameters k and h are known constant parameters. Besides, we add one control input u to the third equation. Thus the controlled system of system (1) becomes

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = bx - kxz, \\ \dot{z} = -cz + hx^2 + u. \end{cases} \tag{2}$$

defining

$$x = x_1, y = x_2, z = hx_3 \ , \tag{3}$$

then

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = bx_1 - k_1 x_1 x_3, \\ \dot{x}_3 = u_1 - cx_3 + x_1{}^2, \end{cases} \tag{4}$$

where $k_1 = kh \neq 0, u_1 = \dfrac{u}{h}$.

The so-called general strict-feedback normal form is described by

$$\begin{cases} \dot{x}_i = b_i g_i(\overline{x}_i) x_{i+1} + \theta^T F_i(\overline{x}_i) + f_i(\overline{x}_i), \\ \quad \cdots\cdots \\ \dot{x}_n = b_n g_n(\overline{x}_n) u + \theta^T F_n(\overline{x}_n) + f_n(\overline{x}_n), \\ y = x_1, \end{cases} , i = 1, 2, \cdots, n-1. \tag{5}$$

where $\overline{x}_i = (x_1, \cdots, x_i)^T \in R^i, i = 1, 2, \cdots n,$ $u \in R$ and $y \in R$ are the states, input and output respectively; $b = (b_1, b_2 \cdots b_n)^T \in R^n$ and $\theta = (\theta_1, \theta_2 \cdots \theta_n)^T \in R^n$ are the vectors of the unknown constant parameters of interest; $g_i(\overline{x}_i), F_i(\overline{x}_i), f_i(\overline{x}_i), i = 1, 2, \cdots n$ are known smooth nonlinear functions.

Compared with the general strict-feedback normal form (5), for the controlled Liu system (4), we have

$$\left. \begin{aligned} & g_1(x_1) = 1, F_1(x_1) = (x_1, 0, 0)^T, \ f_1(x_1) = 0, \\ & g_2(x_1, x_2) = x_1, F_2(x_1, x_2) = (x_1, 0, 0)^T, \ f_2(x_1, x_2) = 0, \\ & g_3(x_1, x_2, x_3) = 1, F_3(x_1, x_2, x_3) = (0, 0, x_3)^T, \ f_3(x_1, x_2, x_3) = x_1{}^2, \\ & \theta = (\theta_1, \theta_2 \cdots \theta_n)^T = (a, b, -c)^T, b = (b_1, b_2 \cdots b_n)^T = (a, -k_1, 1)^T. \end{aligned} \right\} 5 \tag{6}$$

since the system parameters *a*, *b* and *c* are unknown, the parameters θ and *b* are also unknown.

The backsteping design procedure cannot be directly applied to the general strict-feedback form (5) when there are some $g_i(\overline{x}_i)=0, i=1,2,\cdots n$ [7]. For system (4), we have $g_2(x_1,x_2)=x_1$, which can take the value of zero when $x_1=0$. In this paper, the adaptive backstepping design procedure can overcome the singularity problem caused by $g_2(x_1,x_2)=x_1=0$. We also design an adaptive feedback controller for system (4), which can control the Liu system to a bounded point.

4. Adaptive backstepping control

The backstepping design procedure take x_{i+1} in each subsystem of system (5) as a virtual controlled variable [14]. Then the stabilization of the state x_i can be achieved asymptotically via an appropriate virtual feedback $x_{i+1}=\alpha_i, i=1,2,\cdots,n-1$. Generally, the solution of the system (5) does not satisfy $x_{i+1}=\alpha_i$. However, the stabilization of the system can be achieved asymptotically by the control of error variables which describe the asymptotic characteristic between x_{i+1} and α_i. For system (4), there are three steps in the backstepping design procedure. At step *i*, the intermediate control function α_i should be developed using an appropriate Lyapunov function V_i.

Defining three error variables

$$\begin{cases} z_1 = x_1, \\ z_2 = x_2 - \alpha_1, \\ z_3 = x_3 - \alpha_2, \end{cases} \tag{7}$$

where α_1 and α_2 are functions to be defined.

Step 1: The derivative of z_1 is express as

$$\dot{z}_1 = \dot{x}_1 = a(x_2 - x_1) = az_2 - az_1 + a\alpha_1 . \tag{8}$$

Using α_1 as a control to stabilize the z_1-subsystem defined by Eq.(8), we choose the following Lyapunov function:

$$V_1 = \frac{1}{2} z_1^{\ 2} . \tag{9}$$

Calculating the derivative of V_1 along system (4), we have

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1(az_2 - az_1 + a\alpha_1) = az_1 z_2 + a\alpha_1 z_1 - az_1^{\ 2} . \tag{10}$$

Here we suppose that *a* is positive. We can choose

$$\alpha_1 = c_0 x_1 = c_0 z_1 , \tag{11}$$

where $0< c_0 <1$. Letting $c_1 = a(1-c_0) > 0$, we have

$$\dot{V}_1 = az_1 z_2 - c_1 z_1^{\ 2} . \tag{12}$$

We will cancel the first term in the next step. According Eq.(11), the Eq.(8) can be written in the form

$$\dot{z}_1 = az_2 - a(1-c_0) z_1 = az_2 - c_1 z_1 . \tag{13}$$

Step 2: In this step, we deal with the singularity problem caused by the term $-k_1 x_1 x_3$ in the second equation of system (4).

Defining $\overline{a}$, $\overline{b}$ and $\overline{c}$ are the estimates of *a*, *b* and *c*, and introduce the parameters errors

$$\hat{a} = \overline{a} - a, \hat{b} = \overline{b} - b, \hat{c} = \overline{c} - c. \tag{14}$$

Then the derivative of z_2 is

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = bx_1 - k_1 x_1 x_3 - c_0 \dot{z}_1 = -k_1 z_1 z_3 - k_1 \alpha_2 z_1 - ac_0 z_2 - (\bar{b} - b) z_1$$
$$-c_0(1-c_0)(\bar{a}-a)z_1 + \bar{b}z_1 + \bar{a}c_0(1-c_0)z_1 . \tag{15}$$

We obtain the (z_1, z_2) -subsystem:

$$\begin{cases} \dot{z}_1 = az_2 - a(1-c_0)z_1, \\ \dot{z}_2 = -k_1 z_1 z_3 - k_1 \alpha_2 z_1 - ac_0 z_2 - (\bar{b} - b) z_1 \\ \qquad - c_0(1-c_0)(\bar{a}-a)z_1 + \bar{b}z_1 + \bar{a}c_0(1-c_0)z_1. \end{cases} \tag{16}$$

Using α_2 as a control to stabilize the (z_1, z_2) -subsystem (16), we choose the following Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2} z_2{}^2 + \frac{1}{2}(\bar{a}-a)^2 + \frac{1}{2}(\bar{b}-b)^2 . \tag{17}$$

Its time derivative is given by

$$\dot{V}_2 = \dot{V}_1 + z_2 \dot{z}_2 + (\bar{a}-a)\dot{\bar{a}} + (\bar{b}-b)\dot{\bar{b}} = -c_1 z_1{}^2 + z_2\{z_1[\bar{b} + \bar{a}c_0(1-c_0) - k_1\alpha_2] - ac_0 z_2\}$$
$$+(\bar{a}-a)(\dot{\bar{a}} - c_0(1-c_0)z_1 z_2) + (\bar{b}-b)(\dot{\bar{b}} - z_1 z_2) - k_1 z_1 z_2 z_3 . \tag{18}$$

Choosing

$$\begin{cases} \dot{\bar{a}} = c_0(1-c_0)z_1 z_2 - m(\bar{a}-a), \\ \dot{\bar{b}} = z_1 z_2 - n(\bar{b}-b), \end{cases} \tag{19}$$

where $m > 0$ and $n > 0$.

According to Eq.(11) and Eq.(15), the Eq.(18) can be written as

$$\dot{V}_2 = -c_1 z_1{}^2 + z_2\{z_1[\bar{b} + \bar{a}c_0(1-c_0) - k_1\alpha_2] - ac_0 z_2\} - m(\bar{a}-a)^2 - n(\bar{b}-b)^2 - k_1 z_1 z_2 z_3 \tag{20}$$

The possibility of $z_1 = x_1 = 0$ makes α_2 incapable of canceling the term $ac_0 z_2$ in Eq.(20). However, we can choose an appropriate c_0 such that $-ac_0 < 0$. Since a is positive, it is sufficient to choose $0 < c_0 < 1$.

Letting $c_2 = ac_0$, then we have

$$c_2 = ac_0 > 0 . \tag{21}$$

According to $k_1 = kh \neq 0$, we choose

$$\alpha_2 = \frac{1}{k_1}[\bar{b} + \bar{a}c_0(1-c_0)] . \tag{22}$$

Then we have

$$\dot{V}_2 = -c_1 z_1{}^2 - c_2 z_2{}^2 - m(\bar{a}-a)^2 - n(\bar{b}-b)^2 - k_1 z_1 z_2 z_3 . \tag{23}$$

We will cancel the fifth term $k_1 z_1 z_2 z_3$ in the next step. By using Eq.(21) and Eq.(22), the Eq.(15) can be written in the form

$$\dot{z}_2 = -k_1 z_1 z_3 - c_2 z_2 - (\bar{b}-b)z_1 - c_0(1-c_0)(\bar{a}-a)z_1 . \tag{24}$$

Step 3: The derivative of z_3 is

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = u_1 - cx_3 + x_1{}^2 - \frac{\partial \alpha_2}{\partial \bar{a}}\dot{\bar{a}} - \frac{\partial \alpha_2}{\partial \bar{b}}\dot{\bar{b}}$$
$$= u_1 + (\bar{c}-c)z_3 + c\alpha_2 + z_1{}^2 - \bar{c}z_3 - \frac{\partial \alpha_2}{\partial \bar{a}}\dot{\bar{a}} - \frac{\partial \alpha_2}{\partial \bar{b}}\dot{\bar{b}} . \tag{25}$$

Then we get the following system in the (z_1, z_2, z_3) -coordinates:

$$\begin{cases} \dot{z}_1 = az_2 - a(1-c_0)z_1, \\ \dot{z}_2 = -k_1 z_1 z_3 - c_2 z_2 - (\overline{b} - b)z_1 - c_0(1-c_0)(\overline{a} - a)z_1, \\ \dot{z}_3 = u_1 + (\overline{c} - c)z_3 + c\alpha_2 + z_1{}^2 - \overline{c}z_3 - \dfrac{\partial \alpha_2}{\partial \overline{a}}\dot{\overline{a}} - \dfrac{\partial \alpha_2}{\partial \overline{b}}\dot{\overline{b}}. \end{cases} \tag{26}$$

We will choose an appropriate input u to stabilize the system (26) in the following.

Considering the following Lyapunov function

$$V_3 = V_2 + \frac{1}{2} z_3{}^2 + \frac{1}{2}(\bar{c} - c)^2, \tag{27}$$

we can get the derivative of V_3

$$\dot{V}_3 = \dot{V}_2 + z_3 \dot{z}_3 + (\overline{c} - c)\dot{\overline{c}} = -c_1 z_1{}^2 - c_2 z_2{}^2 - m(\overline{a} - a)^2 - n(\overline{b} - b)^2 + (\overline{c} - c)(\dot{\overline{c}} + z_3{}^2)$$

$$+ z_3[-k_1 z_1 z_2 + u_1 - \overline{c}z_3 - c\alpha_2 + z_1{}^2 - \frac{\partial \alpha_2}{\partial \overline{a}}\dot{\overline{a}} - \frac{\partial \alpha_2}{\partial \overline{b}}\dot{\overline{b}}]. \tag{28}$$

We choose the following update law

$$\dot{\overline{c}} = -z_3{}^2 - r(\overline{c} - c), \tag{29}$$

where $r > 0$.

Let

$$u_1 = -c_3 z_3 + k_1 z_1 z_2 + \overline{c}z_3 + c\alpha_2 - z_1{}^2 + \frac{\partial \alpha_2}{\partial \overline{a}}\dot{\overline{a}} + \frac{\partial \alpha_2}{\partial \overline{b}}\dot{\overline{b}}, \tag{30}$$

where $c_3 > 0$. So we have

$$\dot{V}_3 = -c_1 z_1{}^2 - c_2 z_2{}^2 - c_3 z_3{}^2 - m(\overline{a} - a)^2 - n(\overline{b} - b)^2 - r(\overline{c} - c)^2, \tag{31}$$

i.e. $\dot{V}_3$ is negative definite.

According to Eq.(30), the Eq.(25) can be written in the form

$$\dot{z}_3 = -c_3 z_3 + k_1 z_1 z_2 + (\overline{c} - c)z_3. \tag{32}$$

According to Eq.(14), Eq.(19), Eq.(20), Eq.(29) and Eq.(32), we get the following $(z_1, z_2, z_3, \hat{a}, \hat{b}, \hat{c})$-system:

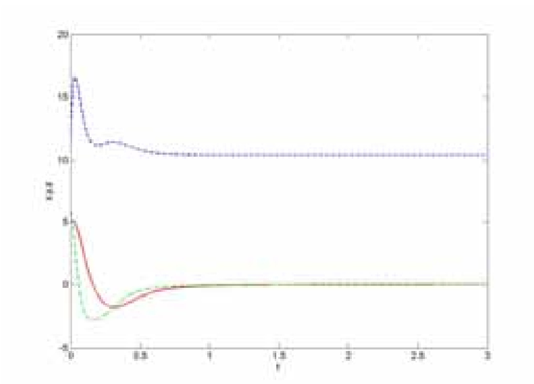
$$\begin{cases} \dot{z}_1 = az_2 - a(1-c_0)z_1, \\ \dot{z}_2 = -k_1 z_1 z_3 - c_2 z_2 - \hat{b}z_1 - c_0(1-c_0)\hat{a}z_1, \\ \dot{z}_3 = -c_3 z_3 + k_1 z_1 z_2 + \hat{c}z_3, \\ \dot{\hat{a}} = c_0(1-c_0)z_1 z_2 - m\hat{a}, \\ \dot{\hat{b}} = z_1 z_2 - n\hat{b}, \\ \dot{\hat{c}} = -z_3{}^2 - r\hat{c}. \end{cases} \tag{33}$$

Since $\dot{V}_3$ is negative definite, we prove that system (33) is globally asymptotically stabilized at the origin point. In view of $z_1 = x_1, \alpha_1 = c_0 x_1 = c_0 z_1, z_2 = x_2 - \alpha_1$, $x = x_1$ and $y = x_2$, we know that the states x and y go to zero asymptotically. From $\alpha_2 = \frac{1}{k_1}[\overline{b} + \overline{a}c_0(1-c_0)]$, $z_3 = x_3 - \alpha_2$ $z = hx_3$ and $k_1 = kh \neq 0$, we have $z \to \frac{1}{k}[b + ac_0(1-c_0)]$, as $t \to +\infty$, i.e. z is bounded. At the same time, from Eq.(30) and $u_1 = \frac{u}{h}$, we can conclude that the control u is also bounded.

This adaptive backstepping controlling method presents a systematic procedure for selecting a proper controller in chaotic control. It needs only one controller, so it is easy to implement. With the controller and updating laws designed above, the control and the parameters identification of the system (2) can be achieved asymptotically.

5. Numerical simulations

Numerical simulations show the effectiveness of the above methods. We assume that $[a,b,c,h,k]=[10,40,2.5,1,4]$, initial conditions $[x_0,y_0,z_0]=[5,6,11.3]$, $c_0=0.2$, $c_3=10$, $[\overline{a},\overline{b},\overline{c}]=[20,20,20]$, $[m,n,r]=[100,100,100]$.



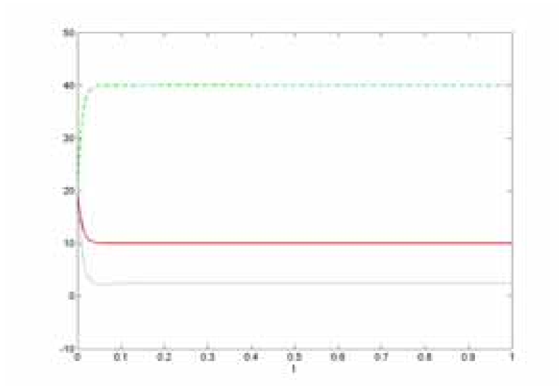
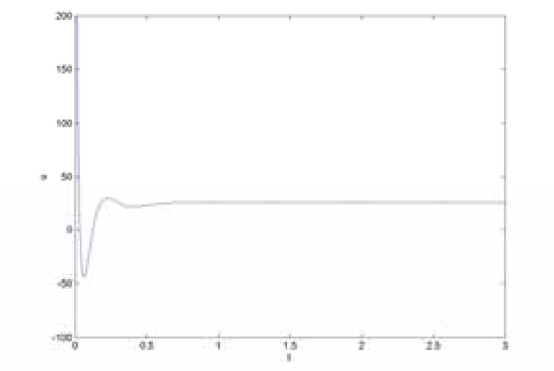


Fig.2. System states: x(–), y(–.), z($\cdots$) .

Fig.3. System parameters identification: $\overline{a}(-),\overline{b}(-.),\overline{c}(\cdots)$.



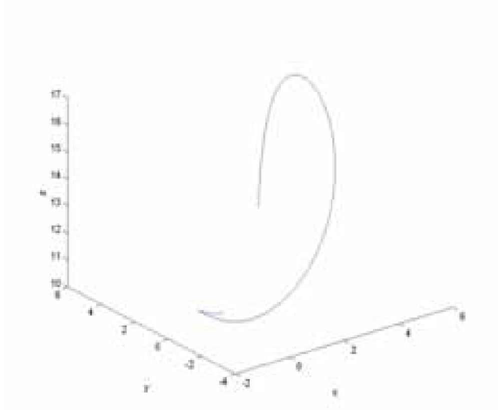


Fig. 4: Control action u.

Fig. 5: Three-dimensional view of the controlled Liu system.

Figure 2 shows that the states *x* and *y* of the controlled Liu system (4) are asymptotically regulated to $x=0$ and $y=0$, and the state z remind bounded.

Figure 3 displays the system parameters identification results.

Figure 4 is the controller *u*. As can be seen from the figure, *u* is also bounded as $t\to+\infty$.

Figure 5 shows that the Liu chaotic system is controlled to a bounded point.

6. Conclusion

This paper has developed a new and effective control law to control the uncertain Liu system with known parameters *k* and *h*. Using the adaptive backstepping design, parameters identification and control can be achieved simultaneously with only one controller within finite steps. Especially, the designed update laws of the unknown parameters can remarkably improve the efficiency of the identification of the unknown parameters by choosing appropriate parameters *m*, *n* and *r*. Numerical simulations show the effectiveness and feasibility of the developed design method.

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