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Void Vertex Genetic Algorithm for Production-Distribution Supply Chain with GTSP Model

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Abstract. In some instances of production supply chain problems, triangular inequality constraint does not hold for the cost functions. This study aims at solving a special case of these problems, where the triangular inequality constraint still remains valid for the delivery cost within districts. After transforming the particular problem to the second kind of Generalized Travelling Salesman Problem (GTSP), an innovative genetic algorithm using generalized chromosomes with void vertices is employed to solve the special GTSP problem. Case study of simulation for benchmark test problems shows that the proposed algorithm is considerably successful.

Keywords: Logistics, SCM, GTSP, TSP, GA.

1. Introduction

The strategic logistic planning is becoming more and more important in the global manufacturing environment. With advanced logistic system, manufacturers can eliminate waste from manufacturing processes, identify and correct weak links in the supply chain, reduce manufacturing lead times and inventories, allow for more flexibility in all operations and manage financial data more effectively. Currently, three types of flows are considered in a Supply Chain: Material flows [1], Financial flows [2] and Information flows [3]. In this paper, only the material flows are considered (refer to Figure 1).

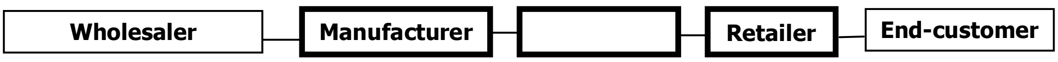

Fig 1. Material flows.

With the increasing complexity of a supply chain, the importance of delivery system between manufacturers and the end customers grows, especially for the manufacturers which provide quantitatively much consumed products such as drinks and foods. To control the market, direct distribution has been the primary type for the product distribution of these firms. For example, Coca Cola Company has made a golden key partner (GKP) strategy [4] to control the drink market. Firstly, the Coca-Cola Company selects one comparatively large-scale dealer for each involved district as the GKP. Secondly, the products are delivered to GKPs directly. Finally, the GKP takes charge of the distribution tasks to the rest retailers in its district. The costs of the logistics are all paid by Coca Cola Company. Compared with the products' value, the delivery cost is much magnitude in this case. As a result, how to minimize the delivery cost of products is the chief target in the production-distribution supply chain management.

This sort of supply chain problems could be generally treated as Generalized Travelling Salesman Problem (GTSP). To solve GTSP, most of the existing researches were focused on the case in which the triangular inequality holds for the cost functions. An innovative distribution model, where the triangular inequality constraint does not hold for the cost functions between manufacturer and the GKPs but still remains valid for that of the internal delivery, is proposed in this paper to optimize the routing of supply chain. A novel genetic algorithm using generalized chromosomes with void vertices, is employed to obtain the solution of this model. In the following, we will firstly present the mathematical model for this problem.

Following the detailed design for the novel genetic algorithm, some computational results of case study are given to validate the proposed algorithm. Finally, we make a conclusion on this study as well as suggestions on future work.

2. Mathematical model

Specifically, the following problem in a production-distribution supply chain is considered: A manufacturer is to distribute its products to n districts from its warehouse or factory. The manufacturer try to minimize the delivery costs by optimizing the transport routing, which must pass through all of the predefined districts to visit at least one wholesaler in each district. Thus, in addition to determining the order in which the districts should be visited, the manufacturer should choose one wholesaler or wholesalers to be visited as distribution centres for each district. Generally, since the delivery cost between districts is much greater than the internal one in each district, the routing passing through each the districts is the main consideration in reality. The problem can be treated as GTSP [5-7].

The GTSP represents a kind of combinatorial optimization problem, which has been introduced by Henry-Labordere [5], Saksena [6], and Srivastava [7] in the context of computer record balancing and of visit sequencing through welfare agencies since 1960s. In the GTSP problem, n cities grouped into p districts are given. A traveling salesman has to find the shortest tour that visits at least one city in each district .The GTSP can be described as the problem of seeking a special Hamiltonian cycle with lowest cost in a complete weighted graph. Let G= (V, E, W) be a complete weighted graph where $\mathrm{V}=\{v_1, v_2, \ldots, v_n\}$, $\mathrm{E}=\{(v_i, v_j) | v_i, v_j \in \mathrm{V}\}$,and $\mathrm{W}=\{w(v_i, v_j) | w(v_i, v_j) \geq 0, w(v_i, v_i)=0, i, j \leq n\}$ are vertex set, edge set, and cost set, respectively. The vertex set V is partitioned into p possibly intersecting groups $\mathrm{V}_1, \mathrm{V}_2, \ldots, \mathrm{V}_p$ and $\mathrm{V}=\cup_{j=1}^{p} \mathrm{V}_j$. The special Hamiltonian cycles required to pass through all of the groups, but not all of the vertices differing from that of TSP *(Travelling Salesman Problem).*

Parameters

1. v_0 denotes the warehouse or factory.

2. v_i $(1 \leq i \leq n)$ denotes the wholesaler i and n is the amount of wholesalers.

3. $\mathrm{V}_j = \{u_{j,1}, \ldots u_{j,|\mathrm{V}_j|}\}$ $(j \leq p)$ denotes the district j and $u_{j,k}$ $(k \leq |\mathrm{V}_j|)$ denotes the *k*th wholesaler in district j. p is the amount of predefined districts.

4. $\mathrm{V}_0 = \{v_0\} = \{u_{0,1}\}$

5. $w(u_{j,k}, u_{j,m}) = f(u_{j,k}, u_{j,m})$ denotes the delivery cost between $u_{j,k}$ and $u_{j,m}$,which represent two different wholesalers in district j.

6. $w(u_{i,k}, u_{j,m}) = F(u_{i,k}, u_{j,m})$ denotes the delivery cost between $u_{i,k}$ and $u_{j,m}$. Specifically, $u_{i,k}$ denotes the *k*th wholesaler in district i and $u_{j,m}$ denotes the *m*th wholesaler in district j.

Decision Variables

$$x(v_i, v_j) = \begin{cases} 1, & \text{if the arc between vertex } v_i \text{ and } v_j \text{ is on the available route} \\ 0, & \text{else} \end{cases}$$

$$y(v_i) = \begin{cases} 1, & \text{if the route enters vertex } v_i \\ 0, & \text{else} \end{cases}$$

$$y'(v_i) = \begin{cases} 1, & \text{if the route leaves vertex } v_i \\ 0, & \text{else} \end{cases}$$

Several assumptions are presented for this model: 1.The cost function f only depends on the length of the routes, and f is much less than F. 2. As a transport cycle, both the origin and destination of the route should be v_0 . 3. Each district is visited at least once.

Using the above notations, the problem is mathematically formulated as follows:

$$\min\left[\sum_{k=1}^{|\mathrm{V}_i|} \sum_{m=1}^{|\mathrm{V}_j|} \sum_{i \neq j}^{p} F(u_{i,k}, u_{j,m}) x(u_{i,k}, u_{j,m}) + \sum_{m \neq k}^{|\mathrm{V}_j|} \sum_{j=1}^{p} f(u_{j,k}, u_{j,m}) x(u_{j,k}, u_{j,m})\right] \tag{1}$$

s.t.

$$\sum_{\substack{j=0 \\ j\neq i}}^{n} x(v_i, v_j) = y(v_i) \qquad \forall i \le n \tag{2}$$

$$\sum_{\substack{j=0 \\ j\neq i}}^{n} x(v_i, v_j) = y'(v_i) \qquad \forall i \le n \tag{3}$$

$$y'(v_i) = y(v_i) \qquad \forall i \le n \tag{4}$$

$$\sum_{k=1}^{|V_i|} \sum_{m=1}^{|V_j|} \sum_{\substack{i=0 \\ i\neq j}}^{p} x(u_{i,k}, u_{j,m}) = 1 \qquad \forall i \le p \tag{5}$$

$$\sum_{k=1}^{|V_i|} \sum_{m=1}^{|V_j|} \sum_{\substack{i=0 \\ i\neq j}}^{p} x(u_{i,k}, u_{j,m}) = 1 \qquad \forall j \le p \tag{6}$$

$$\sum_{k=1}^{|V_i|} y'(u_{i,k}) = \sum_{k=1}^{|V_i|} y(u_{i,k}) \ge 1 \qquad \forall i \le p \tag{7}$$

$$\sum_{\substack{v_i, v_j \in S \\ i\neq j}} x(v_i, v_j) \le |S| - 1 \qquad S \subseteq V \text{and} S \cap V_k = \varnothing \text{ for at least one but not all } k \le p \tag{8}$$

$$x(v_i, v_j), y(v_i), y'(v_i) \in \{0,1\} \qquad \forall i, j \le n \quad i \neq j \tag{9}$$

In this formulation, constraints (2) and (3) indicate $x(v_i, v_j)$ is in terms of the inward flow $y(v_j)$ and of the outward flow $y'(v_i)$; Constraint (4) corresponds to flow conservation equations at the vertices; Constraints (5) (6) (7) ensure that each district is visited at least once. Constraint (8) prohibits the formation of subcycles including vertices from some, but not all districts. Finally, constraint (9) is upper bound and integrality conditions on the variables.

There are two kinds of GTSP corresponding to the different restraint conditions, which the cost functions satisfy. At present, the most studied case of GTSP is the one where the cost functions satisfy the triangle inequality[8-12], which is called the first kind of GTSP (refer to Figure2). Obviously in this case, the optimal route will visit only one vertex in each district. However, in some instances of reality, the triangular inequality may not hold for the cost function, which is called as the second kind of GTSP (refer to Figure 3). Due to the complexity, few relevant studies have been working on it. In the following section, we will propose a novel genetic algorithm to solve a special case of the second kind of GTSP, where triangular inequality constraint still remains valid for cost function of the internal delivery within any district.

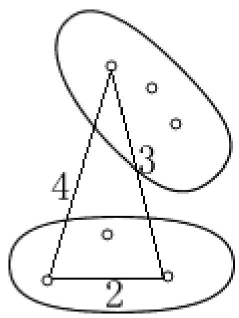

Fig 2. The first kind of GTSP

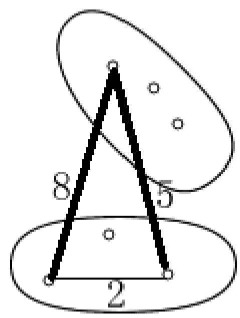

Fig 3. The second kind of GTSP

Before the particular algorithm is introduced, a relevant theorem with proof is presented firstly.

Theorem 1. The optimal route will visit at most two vertices in each district, if cost functions satisfy the triangle inequality in each district:

$$w(v_a, v_b) + w(v_b, v_c) \ge w(v_a, v_c) \text{for } \forall v_a, v_b, v_c \in | \mathrm{V}_k |, k \le p$$

Proof: We suppose that the optimal route visit three vertices v_a, v_b, v_c orderly in a district V_i. Then, $w(v_a, v_b) + w(v_b, v_c) \ge w(v_a, v_c)$ because cost functions satisfy the triangle inequality in the district *i*. Hence, the route $\{...v_a, v_c...\}$ is more optimal. The optimal route will visit at most two vertices in each district.

As the proposed model in this paper satisfies theorem 1, constraints (7) in the model can be replaced by $(7')$:

$$2 \ge \sum_{k=1}^{|\mathrm{V}_i|} y'(u_{i,k}) = \sum_{k=1}^{|\mathrm{V}_i|} y(u_{i,k}) \ge 1, \text{for } \forall i \le p \tag{7'}$$

Consequently, constraints (5) (6) and (7') ensure that each district is visited at least once, and at most two vertices are visited in each district.

3. Design of the algorithm

3.1 algorithms for GTSP

In the previous, simple dynamic programming methods were proposed [5-7] to solve the first kind of GTSP. Laporte [8, 9] used integer programming to solve the instances with 104 vertices. Fischetti *et al.* [10, 11] applied branch-and-cut algorithm to solve the GTSP with 442 vertices. Renaud and Boctor [12] designed a composite heuristic algorithm for GTSP. And some studies on GTSP focused on how to change GTSP into TSP [13, 14].

Genetic algorithm (GA) is one of the most important heuristic algorithms for NP-hard combinatorial optimization problems. Recently, a generalized chromosome genetic algorithm (GCGA), which could be considered as the best available solution for the first kind of GTSP, was proposed by Wu *et al.* [15]. This paper designs a novel chromosome adopting void vertices under the framework of GCGA [15], and then proposes the generalized-chromosome-based genetic algorithm to solve the aforementioned model, i.e., a second kind of GTSP problem.

3.2 Void vertex Genetic Algorithm

There are two parts in the designed chromosome (refer to Figure 4): head part and body part, which is the same as generalized chromosome of GCGA. The void vertex, which is inserted into the body part, can be replaced by an arbitrary vertex in the district which has been indexed by the left side coding of body part.

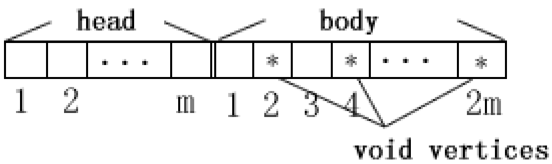

Fig 4. Generalized chromosomes with void vertices

To reserve the updated information of void vertices, info-set A and edge-substitution function S are innovatively defined in this study as follows.

Definition 1.

$$\mathrm{A} = \{a(v_i, v_j) = v_k \text{or} \varnothing \mid v_i, v_k \in V_g, v_j \in V_h \text{and } g \ne h, g, h \le p\}$$

Definition 2.

$$S(v_i, v_j) = \begin{cases} (v_i, v_k) + (v_k, v_j) & \text{if} \quad a(v_i, v_j) = v_k \\ (v_i, v_j) & \text{if} \quad a(v_i, v_j) = \varnothing \end{cases}$$

In the decoding process, the body part defines a GTSP cycle and the head part is used to determine the visited vertices in each district. Edge-substitution function works on every feasible gene segments lying in the body part, while the corresponding information of void vertices are updated and reserved in info-set.

Figure5 shows a GTSP problem, Figure6 shows a generalized chromosome with void vertices and its decoding process.

It is assumed that: $a(v_4, v_{12})=\varnothing, a(v_{12}, v_6)=v_2, a(v_6, v_8)=\varnothing, a(v_8, v_4)=\varnothing$.

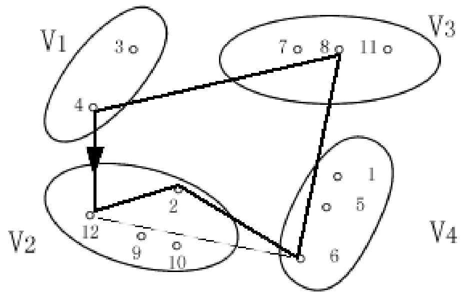

Fig 5.A GTSP problem

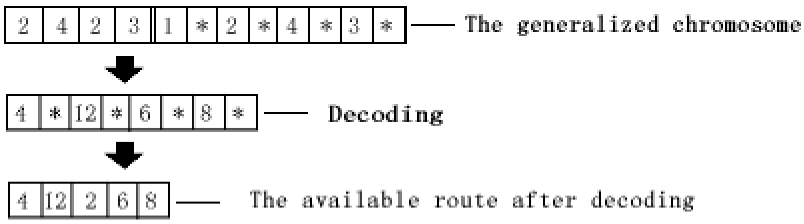

Fig 6. Decoding process

Except for the decoding process, the workflow of the GCGA [15] is adopted for the void vertex genetic algorithm.

4. Computational Results

Using the procedure introduced by Fischetti et al. [16], TSP instances from TSPLIB [17] can be converted to GTSP instances. In this paper, we modify the above partition algorithm so that it can be used to generate test data for the void vertex genetic algorithm.

Firstly, the vertex clustering has been done to simulate geographical regions:

The number of districts is $p=\lceil n/5 \rceil$. Let the distance, i.e. delivery cost, from a vertex v_i to a set S of vertices be the minimum of $w(v_i,u); u \in S$. Then p centres of districts are chosen according to the following procedure. The first centre c_1 is the vertex furthest to the vertex v_0. The kth centre c_k is the vertex furthest to the set $\{c_1, c_2 ... c_{k-1}\}$ .A vertex v_i belongs to the cluster with the centre c_j closest to v_i . If there are several centres equidistant from v_i, the centre c_j of the smallest index is chosen.

Secondly, notice that the vertex set in TSPLIB is a point set with two-dimension coordinate, the cost functions are modified as follows:

$$f(v_i, v_j)=\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \text{for } v_i, v_j \in |V_k|, k \le p$$

$$F(v_i, v_j)=\alpha\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \text{for } \alpha > 1, v_i \in |V_k|, v_j \in |V_m|, k, m \le p, k \ne m$$

Here we set α a preset constant "10" in this paper.Table 1 presents the results of three partitioned problems of TSPLIB using the void vertex genetic algorithm. The instances are all calculated on a PC with 1.5 GHz processor and 256 M memory. The population size is 100, maximal generation is 200, crossover probability is 0.5 and mutation probability is 0.09 in our GA Algorithm.

Table 1. Results for test problems of TSPLIB

Problem	p districts	n vertices	Three runs	Opt	The number of vertices that optimal route passed	Time (s)
krob150	30	150	1	507.5	45	4.2
			2	492.6	50	4.8

			3	507.9	47	1.4
rat99	20	99	1	202.0	26	1.4
			2	199.0	30	2.0
			3	211.4	32	2.0
eil51	11	51	1	89.8	17	0.5
			2	90.6	16	0.3
			3	89.8	17	0.3

Figure 7 shows a solution for the problem eil51 of TSPLIB. To simulate geographical regions, the vertices have been partitioned to 11 districts. The optimal route visits 17 vertices and each district is visited at least once.

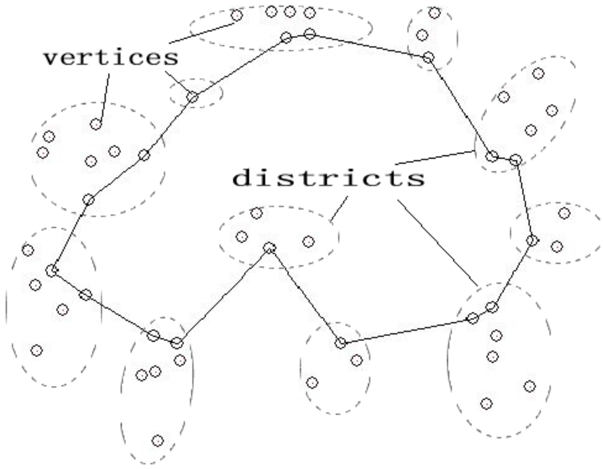

Fig 7. Solution for the Problem eil51

5. Conclusions

In some product distribution problems, delivery cost between districts is much greater than the internal one within each district, which means the triangular inequality constraint for the cost function is broken. This study aims at solving a special case of these problems, where the triangular inequality constraint still remains valid for the delivery cost within districts. After formulating this novel problem mathematically, we transform it into the second GTSP model. Under the framework of GCGA [15], an innovative generalized chromosome-based generic algorithm is employed to solve this special GTSP with the adoption of chromosome with void vertices. Case study of simulation for benchmark test problems shows that the proposed algorithm is considerably successful. Rather than specifically dealing with the delivery routing optimization between manufacturer and the GKPs in districts, this study could be extended, in the future work, to taking into account the strategy of whole product delivery system involving the within-district delivery as well as the between-district delivery.

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