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Heuristic Algorithms for Simultaneously Accepting and Scheduling Advertisements on Broadcast Television

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Abstract. This paper introduces a new optimization problem for a TV station to accept and schedule advertisements. The TV station has its advertising slots with audience ratings, and the advertisers purchase audience ratings. The objective of the TV station is to maximize the total revenue by simultaneously accepting some of the advertisements and finding a feasible schedule for them. An integer programming model is formulated and three heuristic algorithms are proposed. The numerical experiments show that these heuristics are effective.

Keywords: Scheduling, Advertisement, Audience Ratings

1 Introduction

Scheduling problems in the television industry have been studied for many years. A majority of the literature deal with scheduling programs rather than advertisements. The objective of the literature is to optimize some specified criteria, such as the audience ratings. Typical examples are Goodhardt et al. (1975), Headen et al. (1979), Henry and Rinnie (1984), and Rust and Echambadi (1989). Reddy et al. (1998) describe strategies for optimal prime-time TV program scheduling.

Strategies for scheduling advertisements have also been studied by Simon (1982), Mahajan and Muller (1986), and Lilien et al. (1992). These papers are concerned with whether the advertising should be steady or pulsed, so that the effectiveness of the advertising is maximized. Bollapragada et al. (2002) has developed an algorithm to rapidly generate near-optimal sales plans that meet advertisers' requirements. The requirements include budget goals, audience demographics, advertising lengths and the weeks that the client is interested in during the broadcast year. Bollapragada et al. (2004) introduces an algorithm to make the copies for the same advertisement evenly spaced in the advertising slots. Another model is built by Bollapragada and Garbiras (2004), where the first and the last position in a slot get higher audience ratings than those in the middle. The TV station usually promises certain percentages of the first and the last position to each client when the advertising service contract is signed. The TV station is also required to air the clients' advertisements such that the advertisements of competing products are well separated.

All the problems mentioned above have the fixed number of copies for the same advertisement. In this paper, we consider a new optimization problem of accepting and scheduling TV advertisements simultaneously where the number of copies for an advertisement is not fixed in advance.

In our problem, broadcast television stations have some fixed advertising slots and each slot has its audience ratings forecasted by the TV stations. The advertisers purchase audience ratings from the TV stations. The TV

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stations must satisfy the requested audience ratings for each accepted advertisement, no matter how many times it is aired.

For the accepted advertisements, a feasible schedule must satisfy some constraints, e.g., as the following:

(i) For each ad, the total audience ratings actually aired can not be less than that purchased by the advertiser;

(ii) The copies of the same ad must be placed in different slots;

(iii) For all the ads to be aired in the same slot, the sum of their lengths can not be longer than the length of the slot.

The constraints (ii) and (iii) are easy to understand. Now we explain the rationality of the constraint (i). Supposing an advertiser purchases 10% audience ratings and the TV station accepts this advertisement, the TV station will try to schedule the advertisement in some of the advertising slots to make the sum of the total audience ratings aired equal to 10%. However, the placement of this advertisement is influenced by the other ones, so maybe the TV station has to schedule it with more than 10% audience ratings. Although the total audience ratings aired is more than 10%, the TV station can only get the revenue corresponding to 10% audience ratings as the advertiser requests.

As we know, in China, it is popular that the payment from the advertiser to the TV station is related to two parameters: the length of the advertisement and the audience ratings the advertiser requests. Specifically, the payment is proportional to the product of the values of these two parameters. Therefore in this paper, we use the multiplication of these two parameters to represent the payment.

It might be possible to formulate the advertisements accepting and scheduling problem considered in this paper as a multiple knapsack problem (Kellerer et al., 2004), where the slots can be seen as the knapsacks and the advertisements can be seen as the items. In addition to the usual notations used in the knapsack problem, every slot in our problem also has a weight which represents its audience ratings. Furthermore, the copies of the same item must be placed in different slots, and the number of these copies is not fixed in advance. Our problem can also be compared with the advertisement placement problem (Adler et al., 2002), which is concerned with the internet advertising. In the advertisement placement problem, the slot has no weight and the number of the same item is also fixed. The authors solve this problem using the method of bin packing problem (Coffman et al., 1984).

The remainder of this paper is organized as follows. In Section 2, we formulate the problem as an integer programming model. In Section 3, the problem is shown to be an NP-Complete problem. In Section 4, three heuristic algorithms are introduced and the numerical experiments are presented in Section 5. Finally, Section 6 concludes the paper.

2 Formulation

We use the following notations:

$B_j =$ the jth advertising slot, $j = 1, 2, \cdots, M$;

$T_j =$ the length of the advertising slot B_j;

$R_j =$ the audience ratings of the advertising slot B_j;

$a_i =$ the ith advertisement, $i = 1, 2, \cdots, n$;

$t_i =$ the length of the advertisement a_i;

$p_i =$ the requested total audience ratings of the advertisement a_i;

$$x_{ij} = \begin{cases} 1, & \text{if one of the copies of } a_i \text{ is scheduled in } B_j, \\ 0, & \text{if } a_i \text{ is not scheduled in } B_j; \end{cases}$$

$$y_i = \begin{cases} 1, & \text{if } a_i \text{ is accepted by the TV station,} \\ 0, & \text{if } a_i \text{ is rejected by the TV station.} \end{cases}$$

To simplify our expositions, we also use the following symbols:

$$B = \{B_1, B_2, \cdots, B_M\},\ \ T = \{T_1, T_2, \cdots, T_M\},\ \ R = \{R_1, R_2, \cdots, R_M\};$$

$$A = \{a_1, a_2, \cdots, a_n\},\ \ t = \{t_1, t_2, \cdots, t_n\},\ \ p = \{p_1, p_2, \cdots, p_n\}.$$

The advertisements accepting and scheduling problem (AASP hereafter) is to maximize the revenue of the TV station by finding a subset of the advertisements from $A = \{a_1, a_2, \cdots, a_n\}$, which can be scheduled in slots $B = \{B_1, B_2, \cdots, B_M\}$. Mathematically, the problem can be modeled as follows:

$$\max \sum_{i=1}^{n} y_i p_i t_i \tag{1}$$

$$s.t. \quad \sum_{i=1}^{n} x_{ij} t_i \le T_j, \qquad j = 1, 2, \cdots, M \tag{2}$$

$$\sum_{j=1}^{M} x_{ij} R_j \ge p_i y_i, \qquad i = 1, 2, \cdots, n \tag{3}$$

$$y_i \le \sum_{j=1}^{M} x_{ij} \le M y_i, \quad i = 1, 2, \cdots, n \tag{4}$$

$$x_{ij} \in \{0, 1\}, \qquad i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, M \tag{5}$$

$$y_i \in \{0, 1\}, \qquad i = 1, 2, \cdots, n \tag{6}$$

The objective (1) means to maximize the total revenue of the TV station by accepting and scheduling some of the advertisements. Equations (2) and (3) are based on the constraints (iii) and (i) mentioned in Section 1, respectively. The binary variable y_i is introduced here to represent that the payment is related to the audience ratings the advertisement a_i requests, other than the actual audience ratings aired.

3 Computational Complexity

The decision problem corresponding to the optimization problem AASP can be described as following (we denote the decision problem as AASPD hereafter): Given a nonnegative integer z, is there a subset of advertisement $A = \{a_1, a_2, \cdots, a_n\}$ which can be scheduled in slots $B = \{B_1, B_2, \cdots, B_M\}$ with the revenue being at least z?

Theorem 3.1 *AASPD is NP-complete.*

Proof. It is easy to see that AASPD belongs to NP. In fact, given an instance of the problem, our certificate is the matrix $X = (x_{ij})_{n \times M},\ x_{ij} \in \{0, 1\}$ and $Y = (y_i)_{n \times 1},\ y_i \in \{0, 1\}$. The verification algorithm checks whether $\sum_{i=1}^{n} x_{ij} t_i \le T_j,\ \sum_{j=1}^{M} x_{ij} R_j \ge p_i$ and $\sum_{i=1}^{n} y_i p_i t_i \ge z$, for $i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, M$. This process can certainly be done in polynomial time.

We prove that the AASPD is NP-hard by showing that the PARTITION problem can be transformed to AASPD in polynomial time. An instance of PARTITION is defined by a finite set $C = \{1, 2, \cdots, k\}$ and integers $h_i \in Z^+$ for each $i \in C$. The PARTITION problem is to answer whether there is a subset C' of C, such that $\sum_{i \in C'} h_i = \sum_{i \in C - C'} h_i$. Without loss of generality, we suppose $\sum_{i=1}^{k} h_i$ is even.

Given an instance of the PARTITION problem, we can construct an instance of AASPD as follows. Let $n = k,\ z = M = 1/2 \sum_{i=1}^{k} h_i$ and for each i and j , let $t_i = T_j = 1$ (i.e., each advertising slot can only air one advertisement), $R_j = 1,\ p_i = h_i$ (i.e., if a_i is accepted by the TV station, then it must be scheduled in h_i slots).

We now show that the set C has a subset C' satisfying $\sum_{i \in C'} h_i = \sum_{i \in C - C'} h_i$ if and only if the total revenue of all the accepted advertisements is at least $1/2 \sum_{i=1}^{n} h_i$. Suppose for the PARTITION problem, there is a subset C' satisfying $\sum_{i \in C'} h_i = \sum_{i \in C - C'} h_i$. In the AASPD problem, for each $a_i,\ i \in C'$, we accept it and put h_i copies of a_i into h_i slots. For each $a_i,\ i \in C - C'$, we reject them. In this way, the resulted schedule will satisfy the three equations $\sum_{i=1}^{n} x_{ij} t_i = T_j,\ \sum_{j=1}^{M} x_{ij} R_j = p_i$ and $\sum_{i \in C'} p_i t_i = 1/2 \sum_{i=1}^{n} h_i$. This means that the problem AASPD has a 'yes' answer.

On the contrary, suppose there is a schedule of some advertisements for the AASPD problem, and the revenue of these advertisements is at least $1/2 \sum_{i=1}^{n} h_i$. Since the revenue that the TV station can get is at most $\sum_{j=1}^{M} R_j T_j = 1/2 \sum_{i=1}^{n} h_i$, the revenue of all the accepted advertisements is equal to $1/2 \sum_{i=1}^{n} h_i$. Therefore, we find the subset C' satisfying $\sum_{i \in C'} h_i = \sum_{i \in C - C'} h_i$. This completes the proof of Theorem.

4 Heuristic Algorithms

Since the decision problem of AASP is NP-complete, it is unlikely that there is a polynomial-time optimal algorithm for the problem. In this section a few heuristic algorithms are proposed. The main difference between them relates on the initial sequence of the advertisements and the way of scheduling. In order to describe these algorithms clearly, for a given advertisement a_i, we introduce a symbol $z_j,\ j = 1, 2, \cdots, M$, as following:

$$z_j = \begin{cases} 1, & t_i \le T_j, \\ 0, & t_i > T_j. \end{cases}$$

Algorithm 1

Step **0** Store all the advertisements in any specified order.

Step **1** For the advertisements with the same value of R_j, sort them in non-increasing order of T_j (breaking ties arbitrarily).

Step **2** For $i = 1, 2, \cdots n$, we try to put the advertisement a_i into the advertising slots. First, calculate $z_j,\ j = 1, 2, \cdots M$. If $p_i \le \sum_{j=1}^{M} z_j R_j$, we find the first slot B_k, such that $z_k = 1$ and go to Step 3. Otherwise, let $i = i + 1$, repeat Step 2 .

Step **3** From $j = k$ to $j = M$, we find the first slot B_l that $z_l = 1$ and put a_i into B_l, then let $T_l = T_l - t_i,\ p_i = p_i - R_l$. If $p_i \le 0$, let $i = i + 1$ and go to Step 1. Otherwise, let $k = l + 1$ and go to Step 2 .

Algorithm 2

Algorithm 2 differs from Algorithm 1 only in Step 0.

Step **0** Sort the ads in non-increasing order of p_i. For the advertisements with the same value of p_i, sort them in non-increasing order of t_i.

Step **1-3** They are the same as those of Algorithm 1.

Algorithm 3

In order to describe this algorithm easily, we add a dummy slot B_{M+1} into the slots sequence. Let $R_{M+1} = 0$ and $z_{M+1} = 1$ for any advertisement a_i.

Step **0-1** They are the same as those of Algorithm 2.

Step **2** For $i = 1, 2, \cdots, n,$ we try to put the advertisement a_i into the advertising slots. First, calculate $z_j,\ j = 1, 2, \cdots, M.$ If $p_i \le \sum_{j=1}^{M} z_j R_j,$ we find the first slot B_k such that $z_k = 1$ and $p_i \ge R_k,$ then go to Step3. Otherwise, let $i = i + 1$ and repeat Step2.

Step **3** From $j = k$ to $j = M+1,$ we find the first slot B_l that $z_l = 1$ and $p_i \ge R_l.$ If $p_i \le \sum_{j=l}^{M} z_j R_j,$ go to Step4. Otherwise, go to Step 5.

Step **4** Put a_i into $B_l,$ then let $T_l = T_l - t_i,\ p_i = p_i - R_l.$ If $p_i \le 0,$ let $i = i + 1$ and go to Step 1. Otherwise, let $k = l + 1$ and go to Step 3.

Step **5** From $j = l - 1$ to $j = 1,$ we find the first slot B_m such that $z_m = 1$ and there is no any a_i already scheduled in it. Then find all the other slots with the same audience ratings as B_m which have enough room for a_i but there is no any a_i already scheduled in them. Among these slots, we choose $B_{m'}$ with the largest size, put a_i into it and let $T_{m'} = T_{m'} - t_i.$ Let $i = i + 1,$ go to Step 1.

Among the three algorithms, Algorithms 2 and 3 can be used for the off-line problem only, which means that the scheduler knows all the information about the TV station and the advertisements before solving the problem. However, Algorithm 1 can also be used to solve the on-line problem, which means that the advertisements arrive sequentially, and after each advertisement arrives, a decision whether or not to accept the advertisement must be made by the TV station without any knowledge of future advertisements.

5 Numerical Experiments

In China, the TV stations usually make their schedules not so earlier before the advertisements are aired. For example, scheduling 1 day, 3 days, or a week in advance is very popular in practice. Suppose there are 2 or 3 advertising slots during 1 hour in average, then we can approximately calculate the number of slots which should be used for the scheduling horizon. Given a fixed $M,$ we believe that the effectiveness of the algorithms proposed in previous section should be excellent when n is too small or too large comparing to $M.$ That is because for the small $n,$ we may accept almost all of the ads and for the large $n,$ we may have more chances to accept them. Besides the number of the slots and the advertisements, the audience ratings, the lengths of both advertisements and the slots are also not very large in practical cases.

In order to evaluate the efficiency of these algorithms with computer simulations, we generate several groups of random problems as follows:

(1) M is set to be 50, 100, 200, 300, respectively;

(2) For every $M,$ n is set to be $0.4M,\ 0.6M,\ 0.8M,\ 1M,\ 2M,\ 3M,$ respectively;

(3) $T_j \in [30, 12 \times 30],\ T_j \in Z$ is uniformly distributed (the units are in seconds);

(4) $R_j \in [1, 50],\ R_j \in Z$ is uniformly distributed;

(5) $t_i \in [5, T^*],\ t_i \in Z$ is uniformly distributed. Here $T^* = \max_{1 \le j \le M}\{T_j\},$ and T_j is already generated;

(6) $p_i \in [1, R^*],\ p_i \in Z$ is uniformly distributed. Here $R^* = \sum_{j=1}^{M}\{R_j\}$, and R_j is already generated.

The numerical experiments are conducted on an IBM Thinkpad T30. The heuristic algorithms are coded in C++. The computation time for each instance is less than 5 seconds in our numerical experiments. For each of these random problems, a revenue percentage (i.e., $RV = \sum_{i=1}^{n} y_i p_i t_i / \min\{\sum_{i=1}^{n} p_i t_i,\ \sum_{j=1}^{M} R_j T_j\}$) is calculated and used to evaluate the performance of the algorithms.

The results from the numerical experiments are given in Table 1. There are 20 test problems for each problem type. Totally, 480 test problems are generated. Based on the results in Table 1 for the computational experiments, we can see that the performance of Algorithm 3 is usually better than Algorithm 2, and Algorithm 2 usually outperforms Algorithm 1. We can also see that the revenue percentage increases as the number of the slots increases. These results are quite encouraging since the heuristic algorithms provide an efficient procedure for solving large-sized problems.

6 Conclusions

This paper introduces a new optimization problem for a TV station to accept and schedule advertisements. An integer programming model is formulated and three heuristic algorithms are proposed.

Table 1: The Revenue Percentage

		$n = 0.4M$	$n = 0.6M$	$n = 0.8M$	$n = 1M$	$n = 2M$	$n = 3M$
Algorithm 1	$M = 50$	0.62	0.69	0.73	0.81	0.86	0.90
	$M = 100$	0.75	0.80	0.83	0.87	0.90	0.91
	$M = 200$	0.87	0.84	0.88	0.87	0.95	0.96
	$M = 300$	0.89	0.88	0.91	0.93	0.95	0.97
Algorithm 2	$M = 50$	0.71	0.75	0.81	0.86	0.89	0.93
	$M = 100$	0.79	0.88	0.90	0.89	0.92	0.96
	$M = 200$	0.89	0.89	0.92	0.90	0.96	0.98
	$M = 300$	0.91	0.93	0.93	0.96	0.96	0.98
Algorithm 3	$M = 50$	0.71	0.75	0.85	0.85	0.90	0.94
	$M = 100$	0.82	0.90	0.90	0.92	0.95	0.96
	$M = 200$	0.89	0.93	0.93	0.94	0.97	0.97
	$M = 300$	0.94	0.92	0.94	0.95	0.97	0.98

For the future research, better algorithms may be designed and some theoretic analysis should be conducted. For example, one could analyze the worst case performance bounds of the heuristics. It is also challenging to incorporate more practical constraints or objectives into the model and algorithms.

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