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Variable Learning Rate LMS Based Linear Adaptive Inverse Control *

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Abstract. Adaptive inverse control of linear system with fixed learning rate least mean square (LMS) algorithm is improved by varying the learning rate. This variable learning rate LMS algorithm is proved to be convergent by using Lyapunov method. It has better performance especially when there is noise in command input signal. And it is simpler than the Variable Step-size Normalized LMS algorithm. A water box temperature control example is quoted in this paper. Simulation results are carried out and show that the adaptive inverse control with variable learning rate LMS is better than that with the fixed learning rate LMS algorithm and the Variable Step-size Normalized LMS algorithm.

Keywords: Learning Rate, LMS, Adaptive Inverse Control

1. Introduction

The adaptive inverse control approach proposed by B. Widrow in 1986 [1] has more and more applications [2, 3, 4]. In this control scheme the plant model and inverse model are determined by adaptively adjusting the model parameters. So the model can adaptively change for a time-varying plant. It usually uses gradient descent method to get LMS algorithm in linear system and back-propagation (BP) algorithm in nonlinear system. Both the LMS and the BP algorithms use fixed learning rate (FLR).

The adaptive filter based on LMS algorithm proposed by Widrow and Hoff is frequently used in many fields [4, 5]. But the useful signal is almost always disturbed by noises, the LMS algorithm with FLR will produce misadjusted noise. And the misadjustment is proportional to the fixed value of learning rate. So reducing the learning rate can reduce misadjusted noise but meanwhile can also reduce the convergence speed. Therefore, there is a trade-off between the convergence rate and convergence precision of fixed learning rate LMS algorithm.

Many approaches have been proposed to mitigate this contradiction by varying the learning rate (or step-size), such as Variable Step-size LMS algorithm [6], Robust Variable Step-Size LMS algorithm [7], Complementary Pair Variable Step-size LMS [8], and Variable Step-size Normalized LMS (VS NLMS) [9]. And the last one has been proved to be better than the others used in adaptive inverse control systems [10]. But the algorithm is computationally expensive.

In many production fields, the input signal can't avoid being disturbed. The noise therefore affects the control performances and sometimes causes damages even leads to casualties. But there are few literatures considering input signal noises for adaptive inverse control.

In this paper, we proposed a simple variable learning rate (VLR) LMS algorithm and then proved its convergence. After that we used it for adaptive inverse control to improve the performance of system with disturbed command input signal. An example showed the algorithm is robust to the input noise.

The remainder of this paper is structured as follows. Section 2 reviews the adaptive inverse control with FLR LMS algorithm. Section 3 proposes a VLR LMS algorithm and proves its convergence by using Lyapunov stability theorem. An adaptive inverse control example with FLR and VLR LMS and the VS NLMS [9] algorithm and its simulation results will be given in Section 4. Finally, Section 5 concludes this paper.

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2. Adaptive Inverse Control With FLR LMS Algorithm

The basic adaptive inverse control scheme is shown in Fig. 1. Here the plant inverse model is obtained off-line which can also be got on-line [1]. In this paper the off-line modeling method is used because we can select a simple modeling signal r_m, and the adaptive off-line process can more quickly give us an inverse model than on-line. In Fig. 1 P is plant, M is the plant model, C is the controller. r is the command input signal and $dist$ is presented as the disturbance of plant output. r_m is the inverse modeling signal. $Delay$ denotes a suitable number of iterations which can also be replaced by a reference model. An arrow through a rectangular box denotes an adaptive filter. There are two adaptive filters: one in (a) for plant modeling, and the other in (b) for plant inverse modeling. Filters are used as models in adaptive inverse control systems. The process of filter coefficients adjustment is the process of building models. In this paper we discuss linear adaptive inverse control. The inner structure of an adaptive filter in Fig. 1 is shown in Fig. 2.

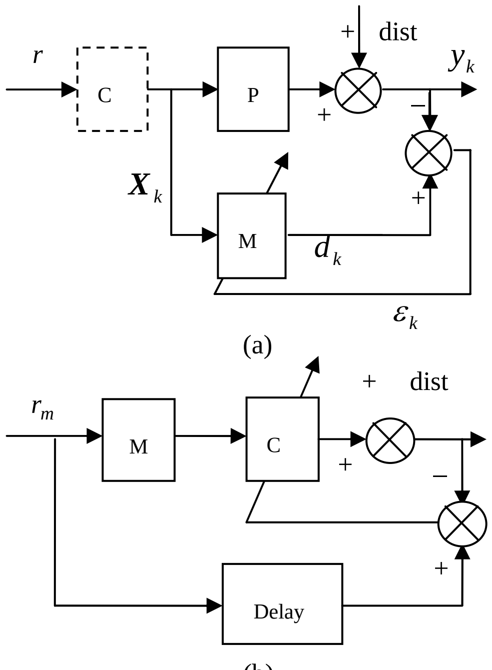

Fig. 1 Adaptive inverse control

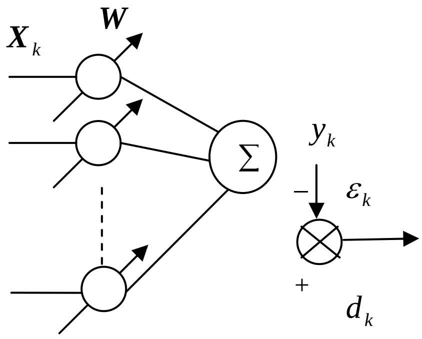

Fig. 2 Linear adaptive filter

The adaptive process in (a) (we don't consider the controller C for the moment) can be described as follows. For a linear causal FIR filter (which in (a) is plant model) shown in Fig. 2, the output of the k th input vector is

$$y_k = \boldsymbol{X}_k{}^T \boldsymbol{W} \tag{1}$$

where $\boldsymbol{X}_k = [x_{1k}, x_{2k}, \cdots, x_{lk}, \cdots, x_{kn}]^T$ and $\boldsymbol{W} = [\omega_1, \omega_2, \cdots, \omega_l, \cdots, \omega_n]^T$ are the k th input vector and the weights vector, respectively. So the plant model output can be described using input signal and adjustable parameters. We can adjust these parameters by minimizing the mean square error which is caused by desire output and real plant output. Then a best least square model to the unknown plant can be produced when the mean square error is minimum. Assume the expect output is d_k, the k th output error is

$$\varepsilon_k = d_k - y_k \tag{2}$$

In order to minimize this error, we define the mean square error (MSE) as

$$J_k = \frac{1}{2} E[\varepsilon_k{}^2]. \tag{3}$$

Then the gradient vector can be obtained by differentiating J_k with respect to weights vector

$$\nabla_k = \frac{\partial J_k}{\partial \boldsymbol{W}} = \frac{1}{2}\frac{\partial E[\varepsilon_k^{\ 2}]}{\partial \boldsymbol{W}} \tag{4}$$

A rough estimation is

$$\nabla_k = -\varepsilon_k \boldsymbol{X}_k \tag{5}$$

In light of the steepest descent algorithm, let the weights update along the gradient descent. So we derive the following as

$$\begin{aligned} \boldsymbol{W}_{k+1} &= \boldsymbol{W}_k + \mu(-\nabla_k) \\ &= \boldsymbol{W}_k + \mu\varepsilon_k \boldsymbol{X}_k \end{aligned} \tag{6}$$

where parameter μ is a positive constant called learning rate or step-size.

We then use (6) to adjust the model coefficients until the MSE gets to minimization. The modeling process has been completed and produced a best least square plant model so that we can use this model for inverse modeling. It cascades with the adaptive filter （That is the inverse model we want to train.） in (b), and the filter’s output compared with the delayed modeling signal which produces an error, too. Undergoing similarly adaptive process as the above, when the error gets to minimization, a best least square inverse model is achieved. After the adaptive process of inverse modeling has been finished, the inverse model can be used as the controller in (a) (the dotted rectangular box) to control the system. It cascades with the plant and this will have a transfer function which will realize our control purpose. This series transfer function can realize tracking desired signal.

The plant used for adaptive inverse control must be stable or else we should first stabilize it using classical control methods. This can ensure the inverse model exists. If the plant is minimum-phase, the delay in (b) is unit one; but if the plant is nonminimum-phase, an appropriate delay is necessary in order to obtain causal inverse model.

Adaptive inverse control not only can realize plant output tracking of command input signal itself but also can track a glided command input. This requires us to use a smooth model which is also called reference model. The control can be achieved by changing the delay in (b) with the expected reference model and use the corresponding “model reference inverse” as a controller to cascade with the plant. Then the plant output will approach the dynamic response of reference model.

The learning rate in (6) is important for the weights update. It determines whether the weights update algorithm converges or not, and affects the convergence rate. If μ is too large, the algorithm maybe not converge; if μ is too small, the weights update very slowly which leads to long training sequence. In FLR LMS adaptive inverse control μ is constant throughout all the adaptive process.

In order to assure the stability of LMS algorithm μ must satisfy the following inequality [11]

$$0 < \mu < \frac{2}{3tr(\boldsymbol{R})} \tag{7}$$

where $\boldsymbol{R} = E[\boldsymbol{X}_k \boldsymbol{X}_k^{\ T}]$ is the input correlation matrix of the adaptive filter.

3. VLR LMS Algorithm

The usual adaptive inverse control discusses disturbances happened to the plant output or the system output. In this paper we mainly focus on the input noises which appear in the command input signal. The usual FLR LMS algorithm is not robust to the input signal noises so it can degrade the system performance, and in some cases drive the system to instability. Here we will use Lyapunov stability theorem to deduce a VLR LMS algorithm which is robust to the input disturbances.

A linear single output adaptive filter satisfies (1) and (2). Still using gradient descent approach, according to the LMS algorithm we should minimize the cost function

$$J_k = \frac{1}{2}(d_k - y_k)^2 \tag{8}$$

by adapting the weights vector $\boldsymbol{W}$. So considering a Lyapunov function as:

$$V(\varepsilon,t)=\frac{1}{2}\varepsilon^2=\frac{1}{2}(d-y)^2 \quad \varepsilon\in R, t>0 \tag{9}$$

where

$$\begin{aligned} y &= \boldsymbol{X}^T\boldsymbol{W} \\ &= [x_1,...,x_n][\omega_1,...,\omega_n]^T \end{aligned} \tag{10}$$

and ε is time-varying along with weights updating. The time derivative of the Lyapunov function V is given by

$$\begin{aligned} \dot{V}_t &= \frac{\partial V}{\partial t}=\varepsilon\cdot\frac{\partial\varepsilon}{\partial y}\cdot\frac{\partial y}{\partial \mathbf{W}}\cdot\dot{\mathbf{W}} \\ &= -\varepsilon\mathbf{X}^T\dot{\mathbf{W}} \end{aligned} \tag{11}$$

Under this condition we can obtain a VLR LMS algorithm in which the weights update according to

$$\boldsymbol{W}_{k+1}=\boldsymbol{W}_k+\eta\cdot\frac{1}{\|\boldsymbol{X}_k\|^2}\varepsilon_k\boldsymbol{X}_k \tag{12}$$

Next we will deduce this algorithm and prove its convergence.

Theorem. If an arbitrary initial weights vector $\boldsymbol{W}(0)$ is updated by

$$\boldsymbol{W}(t)=\boldsymbol{W}(0)+\int_0^t\dot{\boldsymbol{W}}dt \tag{13}$$

where

$$\dot{\boldsymbol{W}}=\frac{1}{\|\boldsymbol{X}\|^2}\varepsilon\boldsymbol{X} \tag{14}$$

Then ε converges to zero under the condition that $\dot{\boldsymbol{W}}$ exists along the convergence trajectory.

Proof: According to the requiring of Lyapunov global stability theorem, for a continuous time-varying system, our Lyapunov function $V(\varepsilon,t)$ has satisfied that it has first continuous partial derivatives for ε and t, respectively, and $V(0,t)=0$. Next we will prove $\varepsilon=0$ is Lyapunov globally uniformly asymptotically stable in the real number area R.

Firstly, we prove that $V(\varepsilon)$ is positive definite and bounded for all $\varepsilon\neq 0$. We know from (9) that

$$V(\varepsilon)=\frac{1}{2}\varepsilon^2\geq 0 \tag{15}$$

where $V(\varepsilon)>0$ for all $\varepsilon\neq 0$ and $V(\varepsilon)=0$ if $\varepsilon=0$. Because the plant is internal stable or has been stabilized, d and y are all stable outputs. So the Lyapunov function $V(\varepsilon)$ is bounded if the input signal is bounded.

Secondly, we prove that the derivative for t of Lyapunov function $\dot{V}_t$ is semi negative definite and bounded. Substitution of (14) into (11), we have

$$\dot{V}_t=-\varepsilon^2\leq 0 \tag{16}$$

where $\dot{V}_t<0$ for all $\varepsilon\neq 0$ and $\dot{V}_t=0$ if $\varepsilon=0$.

Thirdly, we prove that when $\|\varepsilon\|\to\infty$, there exists $V(\varepsilon,t)\to\infty$. Because ε is a scalar for linear adaptive filter, it satisfies

$$\|\varepsilon\|=\varepsilon \tag{17}$$

The condition is satisfied.

So $\varepsilon=0$ is Lyapunov globally asymptotically stable in real number area R which means y is tending to d . That is to say the model is close to the plant so that the weights update algorithm is convergent.

The weights update law in (13) is a batch update law. Analogously we can get the instantaneous gradient descent algorithm for weights updating. For the k th input signal we have

$$\dot{\boldsymbol{W}}_k = \frac{1}{\|\boldsymbol{X}_k\|^2} \varepsilon_k \boldsymbol{X}_k \tag{18}$$

So the update equation is substituted by

$$\begin{aligned} \mathbf{W}_{k+1} &= \mathbf{W}_k + \eta \dot{\mathbf{W}}_k \\ &= \mathbf{W}_k + \eta \cdot \frac{1}{\|\mathbf{X}_k\|^2} \varepsilon_k \mathbf{X}_k \end{aligned} \tag{19}$$

Where η is a constant. Analogously, the instantaneous weights update algorithm is convergent as well as the batch one.

Compared to (6) we have

$$\mu = \frac{\eta}{\|\boldsymbol{X}_k\|^2} \tag{20}$$

The fixed learning rate is replaced by variable learning rate and the VLR LMS algorithm is shown in (19). It is similar to the VS NLMS algorithm where η here is a constant while that of VS NLMS is variable. But the later therefore has three parameters to select heuristically. So it is more complex than the VLR algorithm which also saves many computations.

Here constant η is selected heuristically like μ in FLR LMS algorithm. The excellence of this algorithm will be shown in part 4 where we always select $\eta = \mu$ to detect the differences between FLR LMS and VLR LMS algorithms. The results show that when the input signal is disturbed by noises we can obtain better performances using this VLR LMS algorithm.

4. Simulation Results

Fig. 3 is a water temperature control system [4]. Assume that the length of pipe between mixing valve and tank causes a time delay. So the minimum-phase and nonminimum-phase plant transfer functions have been got as follows

$$H_1(z) = 0.1042 \frac{z + 0.7402}{z^3 - 0.8187 z^2} \tag{21}$$

and

$$H_2(z) = 0.0676 \frac{z + 1.6813}{z^3 - 0.8187 z^2} \tag{22}$$

respectively. We use FLR and VLR LMS and VS NLMS adaptive inverse control methods to control the temperature of water tank between 45 $^\circ C$ and 55 $^\circ C$ and compare their differences.

We use method dither C [1] to build the plant model and select the same filter length and constants η , μ both in FLR and VLR LMS filters. The reference signal is a first-order Markov process disturbed by a white noise with mean 0 and variance 1. The simulation results are shown from Fig. 4 to Fig. 9.

The plant modeling square errors of minimum-phase plant are given in Fig. 4. It can be seen that the convergence rate and precision have been improved by using VLR LMS algorithm. By further simulation, if we increase the learning rate to 0.85, the FLR LMS algorithm becomes not convergent while the VLR LMS algorithm still has small modeling error. The VS NLMS algorithm converges more slowly than VLR algorithm because of computing complexly. Fig. 5 shows the convolution results of the plant and plant inverse model using FLR and VLR LMS and VS NLMS algorithm, respectively. Because the convolution of ideal plant and plant inverse model should be one. It applies us a criterion to observe the precision of plant inverse modeling if we convolute the real plant and the inverse model. In Fig. 5, the convolution amplitude of FLR LMS is approximate 0.6 while that of VLR LMS is nearly 1 and the VS NLMS is a little over 1. So we can see that the second and the third inverse models are almost ideal. Then the inverse model is used as controller to control the water box temperature. The three control results are shown in Fig. 6 respectively. We can observe that the output of FLR LMS control method has been out of the required range 45℃-55℃ because of the command input noise while VLR LMS and VS NLMS get nearly perfect tracking performance. But the VS NLMS is a little worse than VLR in tracking details. The VLR LMS and VS NLMS are robust to the input disturbance.

Similarly, Fig. 7 to Fig. 9 shows us the nonminimum-phase plant simulation results. The performances of VLR LMS are also better than those of FLR LMS and VS NLMS algorithm.

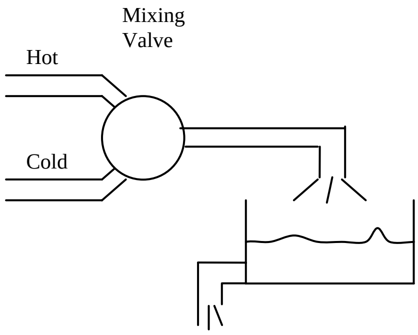

Fig. 3 Tank temperature control

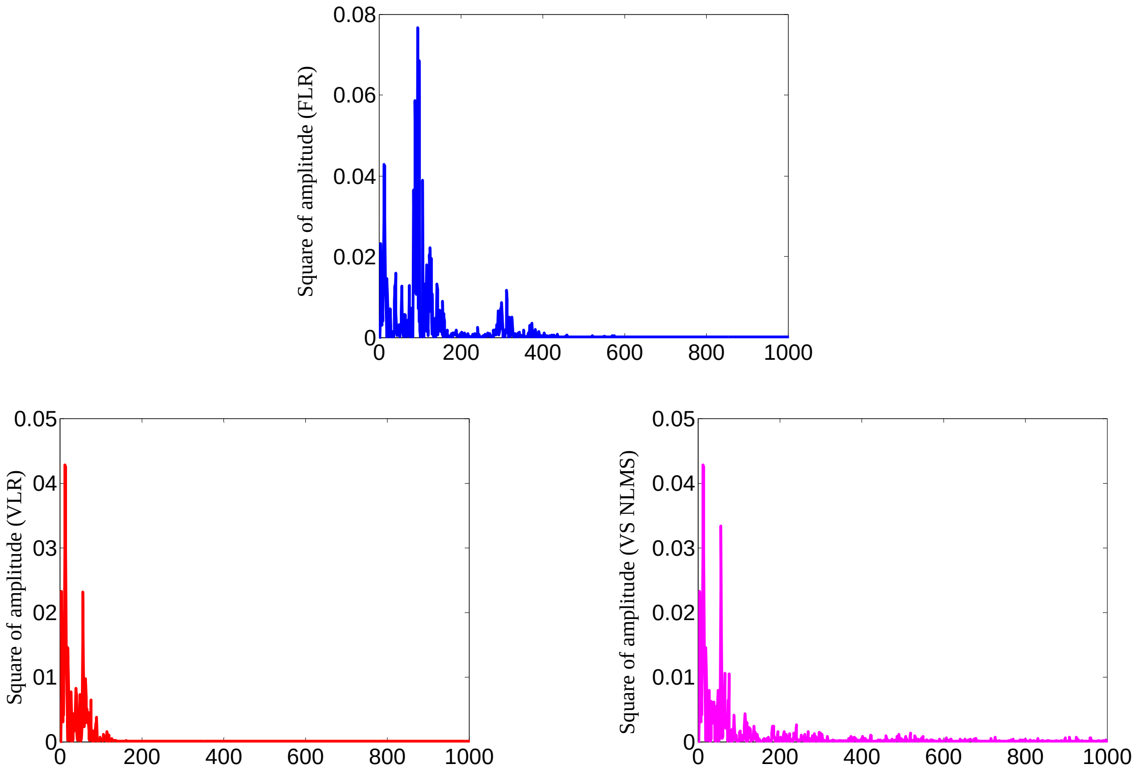

Fig. 4 Minimum-phase plant modeling square error (Time (0.1s))

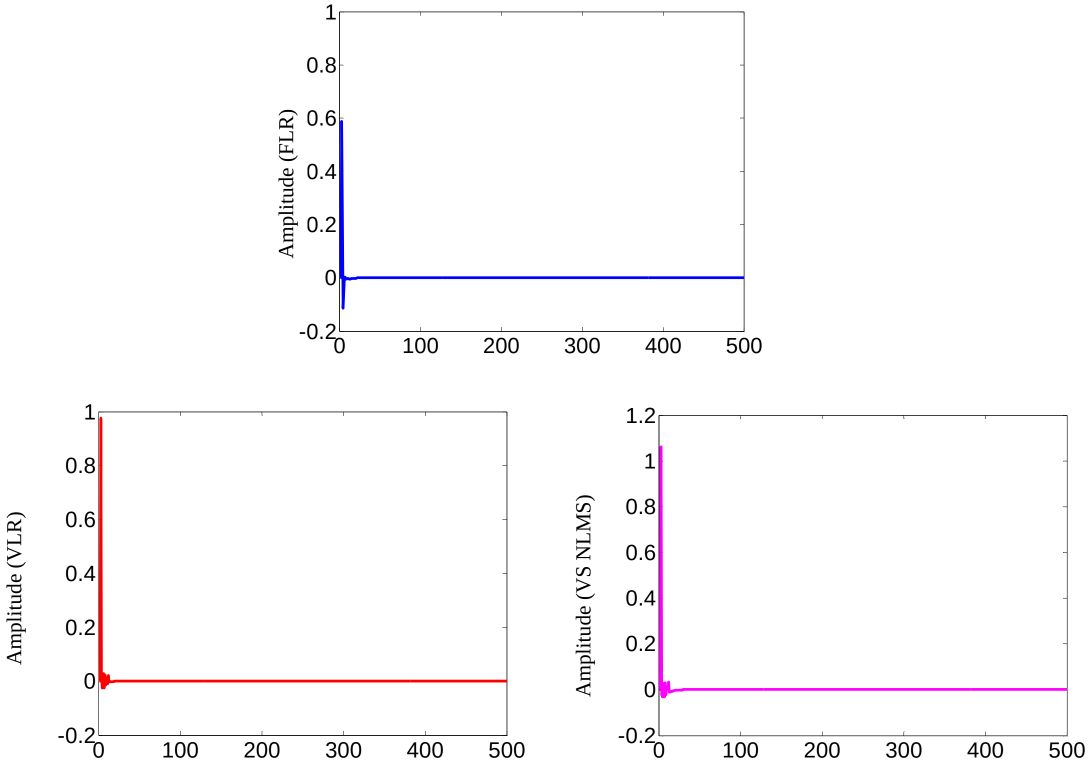

Fig. 5 Precision of minimum-phase plant inverse model (Time (0.1s))

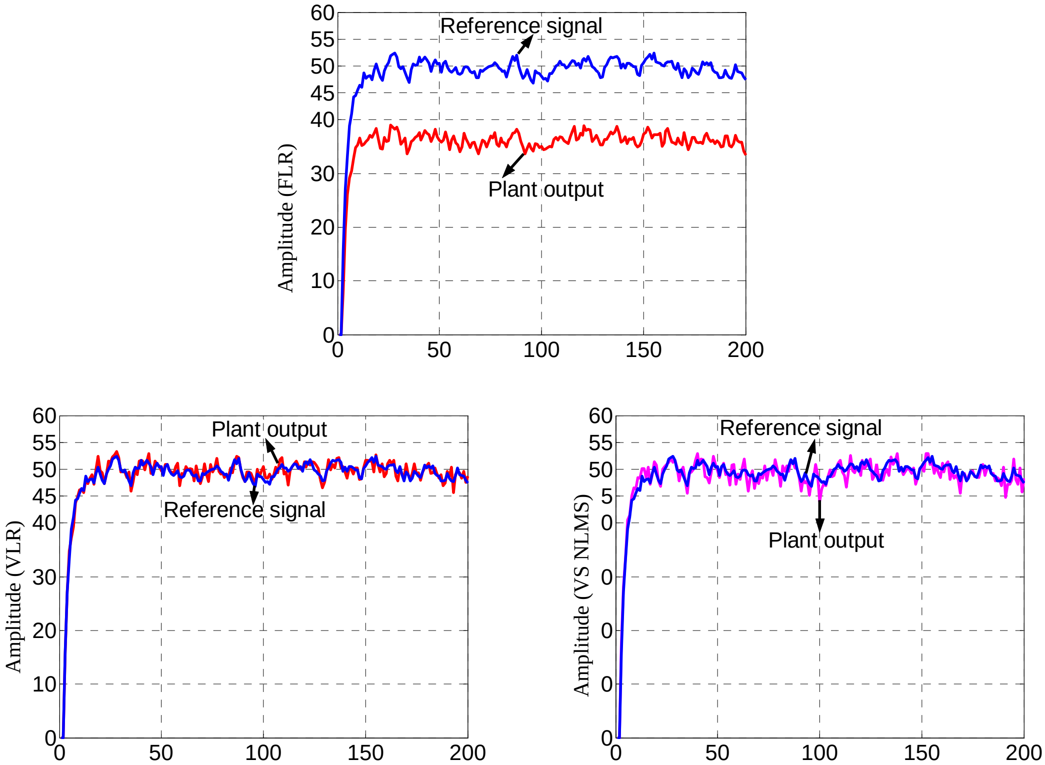

Fig. 6 The tracking ability with white disturbance of minimum-phase plant (Time (0.1s))

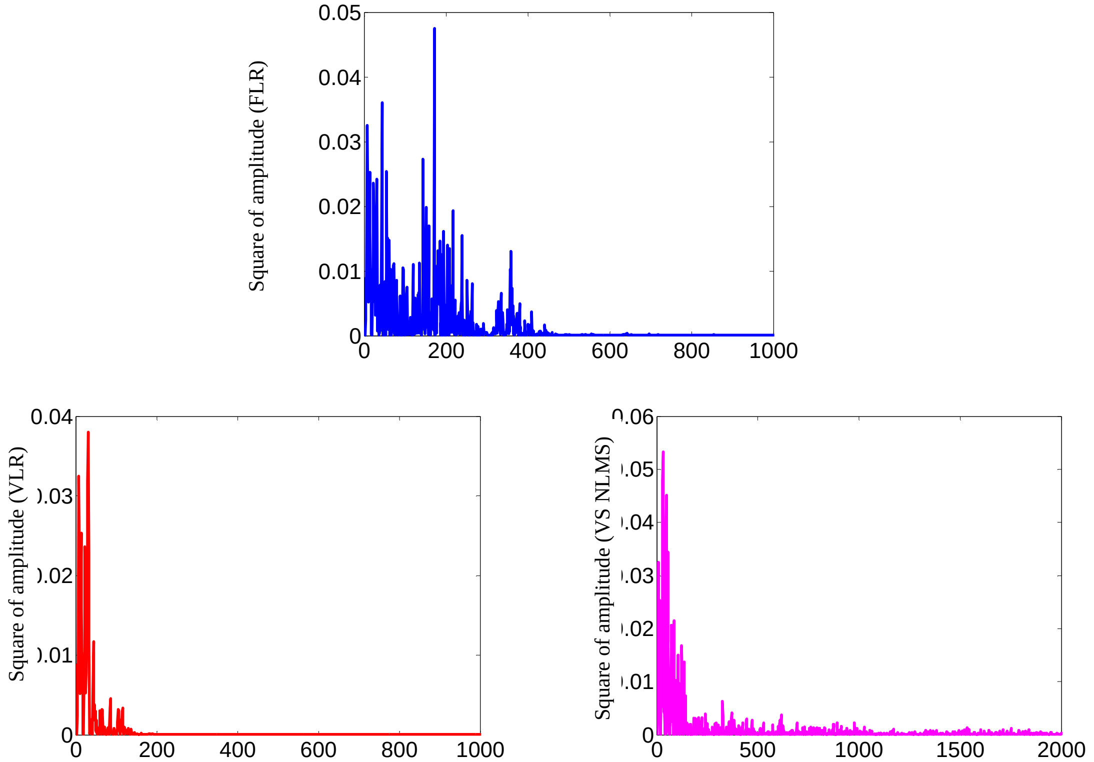

Fig. 7 Nonminimum-phase plant modeling error (Time (0.1s))

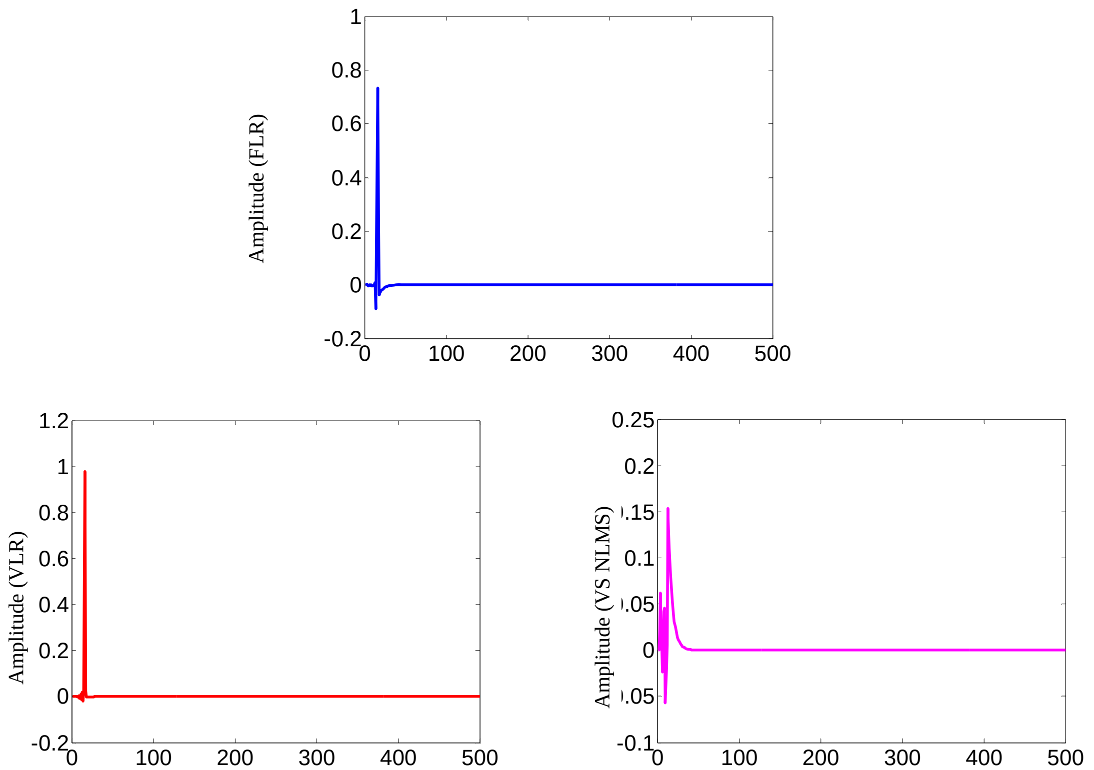

Fig. 8 Precision of nonminimum-phase plant inverse model (Time (0.1s))

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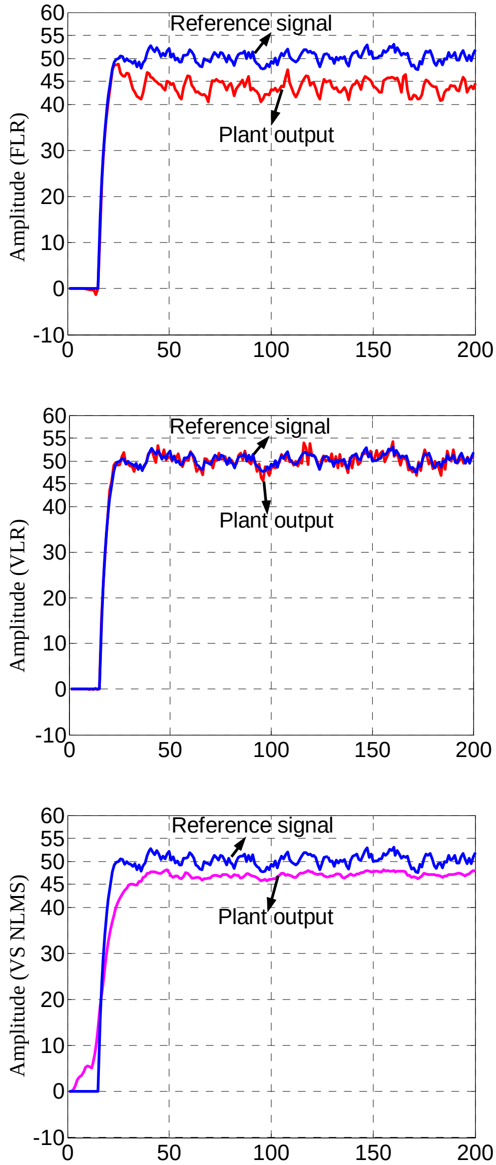

Fig. 9 The tracking ability with white disturbance of nonminimum-phase plant (Time (0.1s))

5. Conclusions

In this paper, a variable learning rate (VLR) LMS algorithm for linear adaptive inverse control was applied. And we have proved it is convergent using Lyapunov global stability theorem. The simulation results show this algorithm uses for adaptive inverse control is effective and has faster modeling rate, smaller error and is more precise compared to fixed learning rate (FLR) LMS and the VS NLMS algorithm. The algorithm is still effective in face of the problem of command input signal disturbed. It is expected to be extended to MIMO nonlinear systems using VLR BP algorithm in adaptive inverse control.

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