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Nonlinear Complementarity Approach to Capacity Allocation Problem in Reserve Markets

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Abstract. In competitive electric reserve markets, the suppliers face the optimal allocation problem of their reserve capacities in order to purse their profit maximizing. Under this background, we develop a capacity allocation model as nonlinear programming. Nonlinear complementarity method is utilized to search for the optimal solution. And smoothing technique is applied to get a system of smooth equations, which can be solved by the Newton method. Numerical result shows the validity of the method.

Keywords: Nonlinear Complementarity, Portfolio Theory, Reserve Market.

1. Introduction

As the deregulation of electricity industry, and with the proposing of " separated from the grid, bidding into the grid", the electricity suppliers are unbundled from the grid and turn into independent utilities that are responsible for their own performances. The reform brings up a competitive electricity market. Like the financial market, the electricity market is also full of risks and uncertainties, and even more complicated because of the nonstorability of electricity. While in a competitive market, all participators should bid for their commodities, and determine their supply according to the demand of the customers. From the uniform dispatch under previously monopolistic mechanism, to competitively bidding into the grid, the suppliers are guaranteed to have the right to purse their own profit maximizing, meanwhile they should deal with the risks due to the uncertainties of the bidding process. In this tightening competitive environment, from the point of the suppliers, we should focus on two kinds of problems: one is how to determine the bidding prices, and should take the rivals' bids into account, which concerns with gaming problems in information economics[5,6]. The other is deciding the allocation of their capacity into various markets, such as electricity market and reserve markets, forward market and spot market. That is to say, it's how to optimize their power allocation[7], which involves the portfolio theory in financial markets.

The risks in the process of bidding can be classified into two kinds, i.e. systematic risk and unsystematic risk. The systematic risk is that derives from the fluctuation of the price. And the unsystematic risk can be defined as the difference between total risk and systematic risk, which derives from the uncertainties of the deviation. The latter can be eliminated by the diversification, while the former be hedged by the derivative instruments (such as forward, future, option and etc.). Diversification means the activities that invest on a series of assets with the character of various risk-profit. Through diversification, the market participators can have higher stability comparing with investing on only one asset. In other words, the diversification of the participators means holding the portfolio of the assets. In this paper, we make researches on how to eliminate unsystematic risk through diversification.

The reserve market can be divided into four independent markets, i.e. regulation, spinning, nonspinning and replacement reserves market. The suppliers should determine the bidding quantity in different markets. Thus forms the problem of capacity allocation optimizing, where the suppliers pursue maximizing their profit and minimizing relative risks. However, in order to make a balance between the profit and the risk, utility function is introduced to realize the utility maximizing. Similar with the mean-variance model in portfolio theory[2], we construct a capacity allocation model as a nonlinear programming. Based upon the KKT optimality conditions, nonlinear complementarity method and a smoothing technique are utilized to transform the inequalities into a system of smooth equations. The classical Newton method is then applied to search for the optimal solution. We also give a numerical test, which proves the validity of the method.

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2. Capacity Allocation Model

Suppose the supplier has the right to allocate among all the four reserve markets. It faces the problem of how to allocate its capacity into the four markets, so as to obtain the maximum profit under acceptable risks. And in mathematics, risk is represented by variance. Let Q denote the total available capacity of the supplier. Firstly, introduce some notations

x_i ratio of reserve capacity allocation into four markets, $i = 1,2,3,4$.

$\tilde{p}_i$ clearing price of the i-th reserve, which is random variable, $i = 1,2,3,4$.

p_i expected value of $\tilde{p}_i$, $i = 1,2,3,4$

σ_i standard value of $\tilde{p}_i$, $i = 1,2,3,4$.

ρ_{ij} correlation coefficient of $\tilde{p}_i$ and $\tilde{p}_j$, ($\rho_{ii} = 1$, $\rho_{ij} = \rho_{ji}$), $i, j = 1,2,3,4$.

The cost of any reserve is expressed as opportunity cost, which means the lost revenue in electricity market. Suppose it linear, and denoted by

$$C_i(Qx_i) = a_i x_i Q, \ i = 1,2,3,4$$

where $a_i > 0$ ($i = 1,2,3,4$) is the coefficient of the i-th reserve's capacity cost.

The supplier's profit equals revenue minus cost $\pi = \sum_{i=1}^{4} \tilde{p}_i x_i Q - \sum_{i=1}^{4} C_i(x_i Q)$, and the expected value is

$$f(x) = E(\pi) = (\sum_{i=1}^{4} p_i x_i - \sum_{i=1}^{4} a_i x_i) Q,$$

where $x = (x_1, x_2, x_3, x_4)$, and the variance

$$g(x) = E[\pi - E(\pi)]^2 = Q^2 \sum_{i=1}^{4} \sum_{j=1}^{4} \rho_{ij} \sigma_i \sigma_j x_i x_j .$$

The supplier hopes maximize its expected profit and minimize corresponding variance, so we construct biobjective problem as $\max\{f(x), -g(x)\}$.

As higher profit is sometimes accompanied by higher risk, the supplier tries to find an equilibrium between the variance and the expected profit. In other words, it must be satisfied. However different suppliers have different satisfaction degrees on the same result. So the utility function is utilized to pursue the utility maximization. Utility means the accepted degree for the potent profit of the suppliers under risk conditions. The utility function is defined as

$$U = \gamma f(x) - g(x)$$

where γ is the preference factor of risk, which means the preferring degree for the risk of the supplier. And the bigger γ is, the higher preferring degree is. When γ is large enough to make $g(x)$ less important, we can say that the supplier extremely pursues profit maximizing without considering its risk. When γ =0, it shows the other extremeness that the supplier is hating risks and hopes minimizing its risks, no matter what profit is.

After introducing utility function, the optimal solution can be obtained through solving the following nonlinear programming

$$\max \gamma f(x) - g(x) = \gamma Q \sum_{i=1}^{4} (p_i - a_i) x_i - Q^2 \sum_{i=1}^{4} \sum_{j=1}^{4} \rho_{ij} \sigma_i \sigma_j x_i x_j ,$$

$$s.t. \ q_{i\min} \le Q x_i \le q_{i\max}, i = 1,2,3,4, \qquad (P_1)$$

$$\sum_{i=1}^{4} x_i = 1,$$

where $q_{i\min}$ and $q_{i\max}$ （$i = 1,2,3,4$） are the lower and upper constrains respectively of the i -th reserve.

In order to realize the utility maximization, each supplier could choose its preferring factor γ, and each γ results in different optimal capacity allocation.

3. Numerical Method

3.1. Nonlinear complementarity method

There are many methods for solving (P_1), and in this paper, nonlinear complementarity method is utilized. The general form of nonlinear programming is

$$\begin{aligned} &\min_{x\in R^n} \quad f(x), \\ &s.t. \quad g_i(x) \geq 0, \quad i = 1,\dots,m, \\ &\qquad h(x) = 0, \end{aligned} \tag{P_2}$$

where, $f, g_i (i = 1,\dots,m), h$: $\Re \to \Re$ are continuously differentiable. Construct Lagrange function as

$$L(x, \lambda, \mu) = f(x) - \sum_{i=1}^{m} \lambda_i g_i(x) - \mu h(x),$$

where $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)$. The KKT condition for (P_2) is

$$\nabla_x L(x^*, \lambda^*, \mu^*) = \nabla f(x^*) - \sum_{i=1}^{m} \lambda_i^* \nabla g_i(x^* - \mu^* \nabla h(x^*) = 0, \tag{3.1a}$$

$$\lambda_i^* g_i(x^*) = 0, \; i = 1,2,\dots,m, \tag{3.1b}$$

$$\lambda_i^* \geq 0, i = 1,2,\dots,m. \tag{3.1c}$$

Under some constraint qualifications, problem (P_2) is equivalent to the system (3.1). For solving (3.1), we first introduce nonlinear complementarity function to transform the inequalities in (3.1) into nonsmooth equations. Nonlinear complementarity function has the following form[1]

$$\phi(a,b) = a + b - \sqrt{a^2 + b^2}, \tag{3.2}$$

where $a, b \in \Re^n$. $\phi(a,b)$ has the feature of

（1） $\phi(a,b) = 0 \Leftrightarrow a \geq 0, b \geq 0, ab = 0$,

（2） $\phi^2(a,b)$ is continuously differentiable,

（3） $\phi(a,b)$ is nonsmooth at (0,0).

Substituting (3.1b) and (3.1c) into (3.2), we get

$$\phi(\lambda_i^*, g_i(x^*)) = \lambda_i^* + g_i(x^*) - \sqrt{(\lambda_i^*)^2 + (g_i(x^*))^2} = 0, \; i = 1,2,\dots,m. \tag{3.3}$$

Equation (3.3) is nonsmooth, which cannot apply the classical Newton method. Then, a parameter is introduced to develop the nonlinear complementarity function (3.2) into a smooth function [4]

$$\phi_\varepsilon(a,b) = a + b - \sqrt{a^2 + b^2 + \varepsilon}, \tag{3.4}$$

where $\varepsilon > 0$ is small enough. It has been proved that $\phi_\varepsilon(a,b) \to \phi(a,b)$, $\forall \; \varepsilon \to 0^+$.

By (3.4), we have

$$\phi_\varepsilon(\lambda_i^*, g_i(x^*)) = \lambda_i^* + g_i(x^*) - \sqrt{(\lambda_i^*)^2 + (g_i(x^*))^2 + \varepsilon} = 0, \tag{3.5a}$$

Combining with

$$\nabla_x L(x^*, \lambda^*, \mu^*) = \nabla f(x^*) - \sum_{i=1}^{m} \lambda_i^* \nabla g_i(x^*) - \mu^* \nabla h(x^*) = 0, \tag{3.5b}$$

$$h(x^*) = 0, \tag{3.5c}$$

we get a system of smooth equations, which can be solved by the classical Newton method.

However, for solving (P_2), we turn to solve (3.5). When $\varepsilon \to 0^+$, the optimal solution of (3.5) is the approximative solution of (P_2).

After transforming the continuously nonsmooth functions into continuously differentiable ones, we can utilize Newton method to solve it.

3.2. Solution to Capacity Allocation Model

The capacity allocation model (P_1) can be solved using above method. The KKT optimality condition for (P_1) is.

$$(x_i^*, \forall i):\ [\gamma(p_i - a_i)Q - 2Q^2\sum_{j=1}^{4}\rho_{ij}\sigma_i\sigma_j x_j] + (-\lambda_{1i}^* Q + \lambda_{2i}^* Q) - \mu^* = 0, \tag{3.6a}$$

$$(\lambda_{1i}^*, \forall i):\ \lambda_{1i}^* \ge 0, (-Qx_i^* + q_{i\max}) \ge 0,\ \lambda_{1i}^* \cdot (-Qx_i^* + q_{i\max}) = 0, \tag{3.6b}$$

$$(\lambda_{2i}^*, \forall i):\ \lambda_{2i}^* \ge 0, (Qx_i^* - q_{i\min}) \ge 0, \lambda_{2i}^* \cdot (Qx_i^* - q_{i\min}) = 0, \tag{3.6c}$$

$$(\mu^*):\ \sum_{i=1}^{4} x_i - 1 = 0, \tag{3.6d}$$

where the Lagrange multipliers $\lambda_{1i}^*, \lambda_{2i}^*$ are the shadow prices of the capacity limits respectively, μ^* the equilibrium price of the reserve market.

Substituting (3.6b)-(3.6c) to (3.5a), then

$$(\lambda_{1i}^*, \forall i):\ \phi_\varepsilon(\lambda_{1i}^*, -Qx_i^* + q_{i\max}) = \lambda_{1i}^* + (-Qx_i^* + q_{i\max}) - \sqrt{(\lambda_{1i}^*)^2 + (-Qx_i^* + q_{i\max})^2 + \varepsilon} = 0, \tag{3.7a}$$

$$(\lambda_{2i}^*, \forall i):\ \phi_\varepsilon(\lambda_{2i}^*, Qx_i^* - q_{i\min}) = \lambda_{2i}^* + (Qx_i^* - q_{i\min}) - \sqrt{(\lambda_{2i}^*)^2 + (Qx_i^* - q_{i\min})^2 + \varepsilon} = 0. \tag{3.7b}$$

Theorem 3.1. $\phi_\varepsilon(\lambda_{1i}^*, -x_i^* Q + q_{i\max})$ approximates $\phi(\lambda_{1i}^*, -x_i^* Q + q_{i\max})$ by accuracy $\dfrac{\varepsilon}{2\lambda_{1i}^*}$.

Proof. By calculating, we get

$$\begin{aligned}
&\left|\phi_\varepsilon(\lambda_{1i}^*, -x_i^* Q + q_{i\max}) - \phi(\lambda_{1i}^*, -x_i^* Q + q_{i\max})\right| \\
&= \left|\sqrt{(\lambda_{1i}^*)^2 + (-x_i^* Q + q_{i\max})^2 + \varepsilon} - \sqrt{(\lambda_{1i}^*)^2 + (-x_i^* Q + q_{i\max})^2}\right| \\
&= \left|\frac{\varepsilon}{\sqrt{(\lambda_{1i}^*)^2 + (-x_i^* Q + q_{i\max})^2 + \varepsilon} + \sqrt{(\lambda_{1i}^*)^2 + (-x_i^* Q + q_{i\max})^2}}\right| \\
&\le \frac{\varepsilon}{2\sqrt{(\lambda_{1i}^*)^2 + (-x_i^* Q + q_{i\max})^2}} \le \frac{\varepsilon}{2\lambda_{1i}^*}
\end{aligned}$$

This completes the proof of the theorem.

Similarly, we can obtain Theorem 3.2.

Theorem 3.2. $\phi_\varepsilon(\lambda_{2i}^*, x_i^* Q - q_{i\min})$ approximates $\phi(\lambda_{2i}^*, x_i^* Q - q_{i\min})$ by accuracy $\dfrac{\varepsilon}{2\lambda_{2i}^*}$.

Combining (3.7) with (3.6a) and (3.6d), we get a system of smooth equations, which can be solved by the classical Newton method. The optimal solution to the smooth equations approximates the initial problem (P_1) under the condition $\varepsilon \to 0^+$.

Consider the following nonlinear equations

$$H(x) = 0, \tag{3.8}$$

where, $H(x): \Re^n \to \Re$ is locally Lipschitzian. The Newton method for solving the smooth equations is given by

$$x^{k+1} = x^k - J_k^{-1} H(x^k), \tag{3.9}$$

where, J_k is Jacobian of H at x^k [3].

We now give the algorithm of smoothing Newton method for solving equations (3.6a),(3.6d) and (3.7).

Algorithm 3.1. Step 0. Given $\varepsilon > 0$, Set $k = 0$, and choose an initial point x^0. Given termination accuracy ξ.

Step 1. Calculate the value of the function in the smooth equations and J_k^{-1} at x^k, where,

$$J_k = \begin{vmatrix} 2Q^2 \rho_{ij} \sigma_i \sigma_j & -1 & 1 & -1 \\ -Q + \dfrac{Q(-x_i^* Q + p_{i\max})}{\Theta(1)} & 1 - \dfrac{\lambda_{1i}^*}{\Theta(1)} & 0 & 0 \\ Q - \dfrac{Q(x_i^* Q - p_{i\min})}{\Theta(2)} & 0 & 1 - \dfrac{\lambda_{2i}^*}{\Theta(2)} & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix},$$

with

$$\Theta = \begin{pmatrix} \Theta(1) \\ \Theta(2) \end{pmatrix} = \begin{pmatrix} \sqrt{(\lambda_{1i}^*)^2 + \left(-x_i Q + p_{i\max}\right)^2 + \varepsilon} \\ \sqrt{(\lambda_{2i}^*)^2 + \left(x_i Q - p_{i\min}\right)^2 + \varepsilon} \end{pmatrix}.$$

Step 2. Compute x^{k+1} by (3.9).

Step 3. If $\left| x^{k+1} - x^k \right| \le \xi$, then stop. Otherwise, set $k = k + 1$ and go to Step 1.

4. Numerical Example

Suppose at some period, the supplier has the total capacity Q being 30MW. The relevant parameters are listed in Tab.4.1. Set termination accuracy $\xi \le 10^{-20}$.

Tab.4.1: Data of units

Reserve i	a_i	$q_{i\max}$ (MW)	$q_{i\min}$ (MW)	p_i (\$/MW)	σ_i^2	ρ_{1i}	ρ_{2i}	ρ_{3i}	ρ_{4i}
1	0.9	15	0	2.4	192.24	1	0.7	0.4	0.3
2	0.5	10	0	2.5	229.08	0.7	1	0.6	0.4
3	0.35	5	0	2.6	402.16	0.4	0.6	1	0.5
4	0.25	5	0	3.0	634.90	0.3	0.4	0.5	1

Setting $\varepsilon \le 10^{-20}$, we can compute the optimal result of capacity allocation under various γ, which is listed in Tab.4.2.

Tab.4.2: Risk factor γ versus x_i

γ	x_1	x_2	x_3	x_4	$f(x)$	$g(x)$
0	0.5000	0.3109	0.1126	0.0765	55.066	152629
54	0.4980	0.2719	0.1189	0.1112	55.924	153351
100	0.3977	0.3305	0.1251	0.1466	58.265	158742
102	0.3934	0.3331	0.1254	0.1481	58.372	159079
103	0.3919	0.3333	0.1259	0.1489	58.416	159215
124	0.3640	0.3333	0.1363	0.1663	59.298	162205
125	0.3629	0.3333	0.1371	0.1667	59.335	162355
154	0.3336	0.3333	0.1664	0.1667	59.995	165115
155	0.3333	0.3333	0.1667	0.1667	60.002	165146

Tab.4.2 also gives the expected value and the variance, which shows that the higher the profit is, the higher the risk is. From Tab.4.2, we can conclude that as the increasing of γ, the supplier becomes venturesome, who prefers allocating its capacity into replacement reserve, which has the higher clearing price and also higher variance. In other words, the supplier hopes maximize its profit, though it is accompanied by higher risk.

5. Conclusions

In the competitive electricity market, the bidding process of the suppliers is full of risks. In order to eliminate and hedge all these risks, the suppliers could take some strategies that have been used successfully in the financial market, such as diversification and utilizing derivative instruments. In this paper, we focus on how to eliminate the nonsystematic risk in the process of supplier' bidding. By constructing mean-variance model, the supplier can disperse the nonsystematic risk， and obtain the optimal capacity allocation among various reserves.

For solving the capacity allocation problem, the nonlinear complementarity method is utilized to transform the inequalities constraints under KKT conditions into continuously nonsmooth equations. And some smoothing techniques is used to obtain a system of smooth equations, which can be solved by Newton method. The optimal solution to the equations approximates the initial nonlinear problem. The numerical example shows the validity of this method. We can conclude from the result that the supplier has different optimal strategies of capacity allocation with different preference factors, while the final objective is to realize its utility maximizing.

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