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The Existence and Global *p*-Exponential Stability of Periodic Solution for Stochastic BAM Neural Networks with Delays*

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Abstract In this paper, we consider a class of stochastic BAM neural networks with delays. By establishing new integral inequalities and using the properties of spectral radius of nonnegative matrix, some sufficient conditions for the existence and global p-exponential stability of periodic solution for stochastic BAM neural networks with delays are given. An example is provided to show the effectiveness of the theoretical results.

Keywords Global p-exponential stability, periodic solution, BAM neural networks, stochastic

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1. Introduction

A class of two-layer interassociative networks called bidirectional associative memory (BAM) neural networks is an important model with the ability of information memory and information association, which is crucial for application in pattern recognition, solving optimization problems and automatic control engineering [11, 14, 18]. In such applications, the dynamical characteristics of networks play an important role.

As is well known, in both biological and man-made neural networks, delays occur due to finite switching speed of the amplifiers and communication time. They slow down the transmission rate and can influence the stability of designed neural networks by creating oscillatory or unstable phenomena. Many authors have obtained interesting results on the stability of neural networks in [4, 20, 25], and synchronization in [5], so it is more important in accordance with this fact to study the BAM neural networks with delays. The circuits diagram and connection pattern implementing for the delayed BAM neural networks can be found in [3]. In recent years, some useful results on the dynamical behaviors of the delayed BAM neural networks have been given, for example, see [12, 16, 21, 27, 28] for stability, see [23] for the synchronization, and see [2, 3, 17, 19] for the periodic oscillatory behavior. Of those, since it has been found applications in learning theory [22], which is moti-

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vated by the fact that learning usually requires repetition, it is of prime importance to study periodic oscillatory solutions of neural networks.

However, a real system is usually affected by external perturbations which in many cases are of great uncertainty and hence may be treated as random, as pointed out by [8] that in real nervous systems synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters, and other probabilistic causes. Therefore, it is significant and of prime importance to consider stochastic effects to the dynamics behavior of stochastic BAM neural networks with delays. Many interesting results on stability of stochastic BAM neural networks with delays have been reported, see [1,6,15,24,29,30].

To the best of our knowledge, some authors have considered the stability of trivial solution to the stochastic BAM neural networks, see [6,15,24,29]. However, few authors have considered the existence of periodic solution to stochastic BAM neural networks with delays. Motivated by the above discussions, we will study the existence and global p-exponential stability of periodic solution for stochastic BAM neural networks with delays. By establishing new integral inequalities and using the properties of spectral radius of nonnegative matrix, some sufficient conditions for the existence and global p-exponential stability of periodic solution for stochastic BAM neural networks with delays are given. An example is provided to show the effectiveness of the theoretical results.

2. Model description and preliminaries

For the sake of convenience, we introduce several notations and recall some basic definitions.

Let R^l (R^l_+) be the space of l-dimensional (nonnegative) real column vectors, and $R^{m\times l}$ ($R^{m\times l}_+$) denotes the set of $m\times l$ (nonnegative) real matrices. Usually I denotes an $l\times l$ unit matrix. For $A,\, B\in R^{m\times l}$ or $A,\, B\in R^l$, the notation $A\geq B$ ($A>B$) means that each pair of corresponding elements of A and B satisfies the inequality "$\geq (>)$". Especially, $A\in R^{m\times l}$ is called a nonnegative matrix if $A\geq 0$, and $z\in R^l$ is called a positive vector if $z>0$. Let $\rho(A)$ denote the spectral radius of nonnegative square matrix A.

$C(X,Y)$ denotes the space of continuous mappings from the topological space X to the topological space Y. Especially, let $C\triangleq C([-\tau,0],R^l)$ with a norm $\|\varphi\|=\sup\limits_{-\tau\leq s\leq 0}|\varphi(s)|$ and let $|\cdot|$ be the Euclidean norm of a vector $x\in R^l$, where τ is a positive constant. Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfy the usual conditions ($i.e$, it is right continuous and $\mathcal{F}_0$ contains all P-null sets). If $x(t)$ is an R^l-valued stochastic process on $t\in[-\tau,\infty)$, we let $x_t=x(t+s):-\tau\leq s\leq 0$, which is regarded as a C-valued stochastic process for $t\geq 0$. Denote by $BC^b_{\mathcal{F}_0}\left([-\tau,0],R^l\right)$ the family of all bounded $\mathcal{F}_0$-measurable, C-valued random variables ϕ, satisfying $\|\phi\|^p_{L^p}=\sup\limits_{-\tau\leq s\leq 0}E|\phi(s)|^p<\infty$, where Ef means the mathematical expectation of f.

For any $x\in R^l$, $\phi\in C$, we define $[x]^+=(|x_1|,\ldots,|x_l|)^T$,and $[\phi(t)]^+_\tau=(|\phi_1|_\tau,\ldots,|\phi_l|_\tau)^T$, where $|\phi_i|_\tau=\sup_{-\tau\leq s\leq 0}|\phi_i(t+s)|$, $i=1,2,\ldots,l$.

We consider stochastic BAM neural networks with delays as follows:

$$\begin{cases} dx_i(t) = \left[-c_i x_i(t) + \sum\limits_{j=1}^{n} a_{ij} f_j\left(y_j\left(t-\tau_i\right)\right) + I_i(t)\right] dt \\ \qquad\qquad + \sum\limits_{j=1}^{n} \sigma_{ij}\left(y_j(t)\right) dw(t), \quad t \geq t_0 \geq 0, \\ dy_i(t) = \left[-\bar{c}_i y_i(t) + \sum\limits_{j=1}^{n} \bar{a}_{ij} g_j\left(x_j\left(t-\tau_i\right)\right) + \bar{I}_i(t)\right] dt \\ \qquad\qquad + \sum\limits_{j=1}^{n} \bar{\sigma}_{ij}\left(x_j(t)\right) dw(t), \quad t \geq t_0 \geq 0, \\ x_i(t) = \phi_i(t), \quad y_i(t) = \varphi_i(t), \quad t_0 - \tau \leq t \leq t_0, \end{cases} \tag{2.1}$$

in which $i = 1, \ldots, n$; $c_i > 0$ and $\overline{c}_i > 0$ denote the passive decay rates; time delays $0 \leq \tau_i < \tau$ correspond to finite speed of axonal signal transmission; a_{ij} and $\overline{a}_{ij}$ are the synaptic connection strengths; f_j and g_j represent the signal propagation functions; $I_i(t)$, $\overline{I}_i(t)$ are the exogenous inputs and are periodic continuous functions with periodic $\omega > 0$ for $t \geq t_0$; $\sigma(\cdot) = (\sigma_1(\cdot), \ldots, \sigma_n(\cdot))$ and $\overline{\sigma}(\cdot) = (\overline{\sigma}_1(\cdot), \ldots, \overline{\sigma}_n(\cdot)) : R^n \to R^n$ are the diffusion coefficient vectors; $w(t)$ is a scalar Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. The initial condition $\phi = col\{\phi_i\}$,and $\varphi = col\{\varphi_i\} \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$.

For convenience in the following we shall rewrite equation(2.1) in the form:

$$\begin{cases} dx(t) = \left[-Cx(t) + Af(y(t-\tau)) + I(t)\right] dt + \sigma(y(t))\, dw(t), \quad t \geq t_0 \geq 0, \\ dy(t) = \left[-\bar{C}y(t) + \bar{A}g(x(t-\tau)) + \bar{I}(t)\right] dt + \bar{\sigma}(x(t))\, dw(t), \quad t \geq t_0 \geq 0, \\ x(t) = \phi(t), \quad y(t) = \varphi(t), \quad t_0 - \tau \leq t \leq t_0, \end{cases}$$

where $x(t) = col\{x_i(t)\}$, $y(t) = col\{y_i(t)\}$, $C = diag\{c_i\}$, $A = (a_{ij})_{n\times n}$, $\overline{C} = diag\{\overline{c}_i\}$, $\overline{A} = (\overline{a}_{ij})_{n\times n}$, $f(y(t-\tau)) = col\{f_j(y_j(t-\tau_i))\}$, $g(x(t-\tau)) = col\{g_j(x_j(t-\tau_i))\}$, $I(t) = col\{I_i(t)\}$, $\overline{I}(t) = col\{\overline{I}_i(t)\}$, $\sigma(y(t)) = col\{\sum\limits_{j=1}^{n} \sigma_{ij}(y_j(t))\}$, and $\overline{\sigma}(x(t)) = col\{\sum\limits_{j=1}^{n} \overline{\sigma}_{ij}(x_j(t))\}$, $\tau = (\tau_i)$.

As a standing hypothesis, we assume that for any $\phi, \varphi \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$, there exists a solution of equation(2.1). Later on we shall often denote the solution of equation(2.1) by $z(t) = z(t, t_0, \phi, \varphi) = \left(x^T(t, t_0, \phi), y^T(t, t_0, \varphi)\right)^T$, or $z_t(t_0, \phi, \varphi)$ for all $t_0 \geq 0$ and $\phi, \varphi \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$.

Definition 2.1. A stochastic process $z_t(s)$ is said to be periodic with period ω if its finite dimensional distributions are periodic with periodic ω, i.e., for any positive integer m and any moments of time $t_1, \ldots, t_m$, the joint distributions of the random variables $z_{t_{1+k\omega}}(s), \ldots, z_{t_{m+k\omega}}(s)$ are independent of k $(k = \pm 1, \pm 2, \ldots)$.

Remark 2.1. By the definition of periodicity, if $z(t)$ is a ω-periodic stochastic process, then its mathematical expectation and variance are ω-periodic [7, p49].

Definition 2.2. The periodic solution $z(t, t_0, \phi, \varphi)$ with the initial condition $\phi, \varphi \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$ of equation(2.1) is called global p-exponential stability if there are constants $\lambda > 0$ and $L \geq 1$ such that for any solution $z(t, t_0, \phi_1, \varphi_1)$ with the initial condition $\phi_1, \varphi_1 \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$ of equation(2.1), we have

$$E|z(t, t_0, \phi, \varphi) - z(t, t_0, \phi_1, \varphi_1)|^p \le L \left\|(\phi^T, \varphi^T)^T - (\phi_1^T, \varphi_1^T)^T\right\|^p_{L^p} e^{-\lambda(t-t_0)}, t \ge t_0.$$

Here λ is called the exponential convergence rate.

Definition 2.3. The set $S \subset BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$ is called a global attracting set of equation(2.1), if for any initial value $\phi, \varphi \subset BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$, we have

$$\text{dist}(x_t(t_0, \phi), S) \to 0 \quad \text{and} \quad \text{dist}(y_t(t_0, \varphi), S) \to 0 \quad \text{as} \quad t \to \infty,$$

where

$$\text{dist}(\eta, S) = \inf_{\gamma \in S} \rho(\eta, \gamma) \quad \text{for} \quad \eta \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n),$$

where $\rho(\cdot, \cdot)$ is any distance in $BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$.

Definition 2.4. The solutions $z_t(t_0, \phi, \varphi)$ of equation(2.1) are said to be

(i) p-uniformly bounded, if for each $\alpha > 0$, $t_0 \ge 0$, there exists a positive constant $\theta = \theta(\alpha)$ which is independent of t_0 such that $E\left\|(\phi^T, \varphi^T)^T\right\|^p \le \alpha$ implies $E[\|z_t(t_0, \phi, \varphi)\|^p] \le \theta$, $t \ge t_0$;

(ii) p-point dissipative, if there is a constant $N > 0$, for any point $\phi, \varphi \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n)$, there exists $T(t_0, \phi, \varphi)$ such that

$$E[\|z_t(t_0, \phi, \varphi)\|^p] \le N, \quad t \ge t_0 + T(t_0, \phi, \varphi).$$

We recall the following result [26, Theorem 3.5] which lays the foundation for the existence of a periodic solution to equation (2.1).

Lemma 2.1 ([26]). *Assume that the solutions of equation (2.1) are globally existent, p-uniformly bounded and p-point dissipative for $p > 2$, then there is an ω-periodic solution.*

For $A \in R^{n\times n}_+$, the spectral radius $\rho(A)$ is an eigenvalue of A and its eigenspace is denoted by

$$W_\rho(A) \triangleq \{z \in R^n \,|Az = \rho(A) z\},$$

which includes all positive eigenvectors of A provided that the nonnegative matrix A has at least one positive eigenvector(see [10]).

Lemma 2.2 ([13]). *Suppose that $M \in R^{n\times n}_+$ and $\rho(M) < 1$, then there exists a positive vector z such that*

$$(I - M)z > 0.$$

For $M \in R^{n\times n}_+$ and $\rho(M) < 1$, we denote

$$\Omega_\rho(M) = \{z \in R^n | (I - M)z > 0, z > 0\},$$

which is a nonempty set by Lemma 2.2, and which satisfies that $k_1 z_1 + k_2 z_2 \in \Omega_\rho(M)$ for any scalars $k_1 > 0$, $k_2 > 0$ and vectors $z_1, z_2 \in \Omega_\rho(M)$. So $\Omega_\rho(M)$ is a cone without a vertex in R^n, which we refer to as a "ρ-cone ".

Lemma 2.3. *Let $u(t), v(t) \in C(R, R_+^n)$ be a solution of the delay integral inequality*

$$\begin{cases} u(t) \le M_1 e^{-\delta_1(t-t_0)} \left[\phi_1(t_0)\right]_\tau^+ + \int_{t_0}^t e^{-C_1(t-t_0)} A_1 v\left(s\right) ds \\ \qquad + \int_{t_0}^t e^{-C_1(t-t_0)} B_1 \left[v\left(s\right)\right]_\tau^+ ds + J_1, \quad t \ge t_0, \\ v(t) \le \bar{M}_1 e^{-\bar{\delta}_1(t-t_0)} \left[\varphi_1(t_0)\right]_\tau^+ + \int_{t_0}^t e^{-\bar{C}_1(t-t_0)} \bar{A}_1 u\left(s\right) ds \\ \qquad + \int_{t_0}^t e^{-\bar{C}_1(t-t_0)} \bar{B}_1 \left[u\left(s\right)\right]_\tau^+ ds + \bar{J}_1, \quad t \ge t_0, \\ u(t) \le \phi_1\left(t\right), \quad v(t) \le \varphi_1\left(t\right), \quad \forall t \in \left[t_0 - \tau, t_0\right], \end{cases} \tag{2.2}$$

where $A_1, B_1, M_1, \overline{A}_1, \overline{B}_1, \overline{M}_1 \in R_+^{n\times n}$, C_1 =diag$\{c_{1i}\}$ and $\overline{C}_1$ =diag$\{\overline{c}_{1i}\}$, where $c_{1i}, \overline{c}_{1i} > 0$, $\delta_1, \overline{\delta}_1 > 0$ are constants, $J_1, \overline{J}_1 \ge 0$ are constant vectors, $\phi_1(t), \varphi_1(t) \in C([t_0-\tau, t_0], R_+^n)$. If $\rho(K_1^{-1}(A_1 + B_1 + \overline{A}_1 + \overline{B}_1)) < 1$, then there are constants $0 < \lambda$ and $N > 0$ such that

$$u\left(t\right) + v(t) \le Nze^{-\lambda(t-t_0)} + \left(I - \hat{\Pi}\right)^{-1}(J_1 + \overline{J}_1), \quad t \ge t_0, \tag{2.3}$$

for any $\phi_1(t), \varphi_1\left(t\right) \in Q = \left\{ \left[\phi_1(t_0)\right]_\tau^+ + \left[\varphi_1(t_0)\right]_\tau^+ \le z \,\middle|\, z > 0,\ z \in \Omega_\rho\left(e^{\lambda\tau}\hat{\Pi} + \frac{M_1 + \overline{M}_1}{N}\right)\right\}$, where K_1, $\hat{\Pi}$, λ and N are determined by

$$K_1 = diag\{k_1\} \text{ with } k_i = \min\{c_{1i}, \overline{c}_{1i}\}, \quad \rho\left(e^{\lambda\tau}\hat{\Pi} + \frac{M_1 + \overline{M}_1}{N}\right) < 1,$$

$$\lambda < \min\left\{\min_{1\le i\le n}\left\{k_i\right\}, \delta_1, \overline{\delta}_1\right\}, \quad \hat{\Pi} = (K_1 - \lambda I)^{-1}(A_1 + B_1 + \overline{A}_1 + \overline{B}_1).$$

Proof. From the condition $\rho(K_1^{-1}(A_1+B_1+\overline{A}_1+\overline{B}_1)) < 1$, by using continuity, we obtain that there exist positive constants λ and N such that $\rho(e^{\lambda\tau}\hat{\Pi} + \frac{M_1+\overline{M}_1}{N}) < 1$. From Lemma 2.2, we know

$$(e^{\lambda\tau}\hat{\Pi} + \frac{M_1 + \overline{M}_1}{N})z < z. \tag{2.4}$$

In order to prove (2.3), we first prove for any $d > 1$,

$$u\left(t\right) + v(t) < dNze^{-\lambda(t-t_0)} + \left(I - \hat{\Pi}\right)^{-1}(J_1 + \overline{J}_1), \quad t \ge t_0. \tag{2.5}$$

If (2.5) is not true, from the fact that $\left[\phi_1(t_0)\right]_\tau^+ + \left[\varphi_1(t_0)\right]_\tau^+ \le z$ and $u(t), v(t)$ is continuous, then there must be a $t_1 > t_0$ and $1 \le i \le n$ such that

$$e^i\left(u(t_1) + v(t_1)\right) = e^i\left(dNze^{-\lambda(t_1-t_0)} + \left(I - \hat{\Pi}\right)^{-1}(J_1 + \overline{J}_1)\right), \tag{2.6}$$

$$u(t) + v(t) \le dNze^{-\lambda(t-t_0)} + \left(I - \hat{\Pi}\right)^{-1}(J_1 + \overline{J}_1), \quad t \le t_1, \tag{2.7}$$

where $e^i = (\underbrace{0, \ldots, 0, 1}_{i}, 0, \ldots, 0)$. Hence, it follows from (2.2), (2.4) and (2.7) that

$u(t_1) + v(t_1)$

$$
\begin{aligned}
&\leq \left(M_1+\overline{M}_1\right) e^{-\lambda(t-t_0)} \left([\phi_1(t_0)]_\tau^+ + [\varphi_1(t_0)]_\tau^+\right) + \int_{t_0}^{t_1} e^{-C_1(t_1-s)} A_1 v(s)\, ds \\
&\quad + \int_{t_0}^{t_1} e^{-C_1(t_1-s)} B_1 [v(s)]_\tau^+ ds + \int_{t_0}^{t_1} e^{-\overline{C}_1(t_1-s)} \overline{A}_1 u(s)\, ds \\
&\quad + \int_{t_0}^{t_1} e^{-\overline{C}_1(t_1-s)} \overline{B}_1 [u(s)]_\tau^+ ds + J_1 + \overline{J}_1 \\
&\leq \left(M_1+\overline{M}_1\right) e^{-\lambda(t_1-t_0)} z + \int_{t_0}^{t_1} e^{-C_1(t_1-s)} A_1 \left(dNze^{-\lambda(s-t_0)} + \left(I-\hat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right)\right) ds \\
&\quad + \int_{t_0}^{t_1} e^{-C_1(t_1-s)} B_1 \left(dNze^{\lambda\tau} e^{-\lambda(s-t_0)} + \left(I-\hat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right)\right) ds \\
&\quad + \int_{t_0}^{t_1} e^{-\overline{C}_1(t_1-s)} \overline{A}_1 \left(dNze^{-\lambda(s-t_0)} + \left(I-\hat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right)\right) ds \\
&\quad + \int_{t_0}^{t_1} e^{-\overline{C}_1(t_1-s)} \overline{B}_1 \left(dNze^{\lambda\tau} e^{-\lambda(s-t_0)} + \left(I-\hat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right)\right) ds + J_1 + \overline{J}_1 \\
&\leq \left(M_1+\overline{M}_1\right) e^{-\lambda(t_1-t_0)} z + \int_{t_0}^{t_1} e^{-K_1(t_1-s)} A_1 dNze^{-\lambda(s-t_0)} ds \\
&\quad + \int_{t_0}^{t_1} e^{-K_1(t_1-s)} B_1 dNze^{\lambda\tau} e^{-\lambda(s-t_0)} ds + \int_{t_0}^{t_1} e^{-K_1(t_1-s)} \overline{A}_1 dNze^{-\lambda(s-t_0)} ds \\
&\quad + \int_{t_0}^{t_1} e^{-K_1(t_1-s)} \overline{B}_1 dNze^{\lambda\tau} e^{-\lambda(s-t_0)} ds \\
&\quad + K_1^{-1} \left(A_1+B_1+\overline{A}_1+\overline{B}_1\right) \left(I-\hat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right) + J_1 + \overline{J}_1 \\
&\leq \frac{\left(M_1+\overline{M}_1\right)}{N} e^{-\lambda(t_1-t_0)} Nz + \int_{t_0}^{t_1} e^{-K_1(t_1-s)} \left(A_1+\overline{A}_1\right) dNze^{-\lambda(s-t_0)} ds \\
&\quad + \int_{t_0}^{t_1} e^{-K_1(t_1-s)} \left(B_1+\overline{B}_1\right) dNze^{\lambda\tau} e^{-\lambda(s-t_0)} ds + \hat{\Pi}\left(I-\hat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right) + J_1 + \overline{J}_1 \\
&= e^{-\lambda(t_1-t_0)} \left(\frac{\left(M_1+\overline{M}_1\right)}{N} + \int_{t_0}^{t_1} e^{-K_1(t_1-s)} \left(A_1+\overline{A}_1\right) e^{\lambda(t_1-s)} ds \right. \\
&\quad \left. + \int_{t_0}^{t_1} e^{\lambda\tau} e^{-K_1(t_1-s)} \left(B_1+\overline{B}_1\right) e^{\lambda(t_1-s)} ds\right) dNz + \left(I-\hat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right) \\
&\leq e^{-\lambda(t_1-t_0)} \left(\frac{\left(M_1+\overline{M}_1\right)}{N} + e^{\lambda\tau}\widehat{\Pi}\right) dNz + \left(I-\hat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right) \\
&< dNze^{-\lambda(t_1-t_0)} + \left(I-\widehat{\Pi}\right)^{-1} \left(J_1+\overline{J}_1\right),
\end{aligned}
$$

which contradicts to the equality (2.6). So (2.5) holds for all $t \geq t_0$. Letting $d \to 1$ in (2.5), we have (2.3). The proof is complete. □

If $J_1 = 0$ and $\overline{J}_1 = 0$, we can easily get the following corollary.

Corollary 2.1. *Assume that all conditions of Lemma 2.3 hold. Then all solutions of the inequality (2.2) exponentially convergence to zero.*

3. Main results

To obtain the existence and global p-exponential stability of periodic solution of equation(2.1), we introduce the following assumptions.

(H_1) $f_j(0) = g_j(0) = \sigma_{ij}(0) = \overline{\sigma}_{ij}(0) = 0$, f_j, g_j, σ_{ij} and $\overline{\sigma}_{ij}$ are Lipschitz-continuous with Lipschitz constants $\alpha_j > 0$, $\beta_j > 0$, $L_{ij} > 0$ and $\overline{L}_{ij} > 0$, respectively, for $i, j = 1, 2, \ldots, n$.

(H_2) Set $\Upsilon = K_2^{-1}(A_2H + B_2H + \overline{A}_2H + \overline{B}_2H)$, and there exists an integral $p > 2$ such that $\rho(\Upsilon) < 1$, where

$$A_2 = diag\left\{4^{p-1}(p\,(p-1)\,/2)^{p/2}(2c_i\,(p-1)\,/\,(p-2))^{1-p/2}\left(\sum_{j=1}^{n}|L_{ij}|^{\frac{p}{p-1}}\right)^{p-1}\right\},$$
$$B_2 = diag\left\{4^{p-1}c_i^{1-p}\left(\sum_{j=1}^{n}|a_{ij}\alpha_j|^{\frac{p}{p-1}}\right)^{p-1}\right\},$$
$$\overline{A}_2 = diag\left\{4^{p-1}(p\,(p-1)\,/2)^{p/2}(2\overline{c}_i\,(p-1)\,/\,(p-2))^{1-p/2}\left(\sum_{j=1}^{n}|\overline{L}_{ij}|^{\frac{p}{p-1}}\right)^{p-1}\right\},$$
$$\overline{B}_2 = diag\left\{4^{p-1}\overline{c}_i^{1-p}\left(\sum_{j=1}^{n}|\overline{a}_{ij}\beta_j|^{\frac{p}{p-1}}\right)^{p-1}\right\},$$
$$K_2 = diag\{k_{2i}\} \text{ with } k_{2i} = \min\{c_i, \overline{c}_i\}, \quad H = (h_{ij})_{n\times n}, \quad h_{ij} = 1, \quad i, j = 1, \ldots, n.$$

Theorem 3.1. *Suppose that $(H_1) - (H_2)$ hold, then the system (2.1) must have a periodic solution, which is globally p-Exponentially stable and in the attracting set* $S = \{\phi \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n) \,|\, (\|\phi_1\|^p_{L^p}, \ldots, \|\phi_n\|^p_{L^p}) < (I - \Upsilon_1)^{-1}(J_2 + \overline{J}_2)\}$, *where* $J_2 = col\left\{\left(\frac{\hat{I}_i}{c_i}\right)^p\right\}$, $\overline{J}_2 = col\left\{\left(\frac{\hat{\overline{I}}_i}{\overline{c}_i}\right)^p\right\}$, $\hat{I}_i = \sup_{0\le t\le\omega} I_i(t)$, $\hat{\overline{I}}_i = \sup_{0\le t\le\omega} \overline{I}_i(t)$, $\Upsilon_1 = (K_2 - \lambda I)^{-1}(A_2H + B_2H + \overline{A}_2H + \overline{B}_2H)$ *and $\lambda > 0$ is determined by*

$$\rho(\Upsilon_1) < 1 \quad and \quad \lambda < \min_{1\le i\le n}\{k_{2i}\}. \tag{3.1}$$

Proof. By using continuity and condition (H_2), we know that (3.1) has at least one positive solution.

By the method of variation parameter, we have for $t \ge t_0$, $i = 1, \ldots, n$,

$$\begin{cases} x_i(t) = x_i(t_0)\,e^{-c_i(t-t_0)} + \int_{t_0}^{t} e^{-c_i(t-s)}\sum\limits_{j=1}^{n} a_{ij} f_j(y_j(s-\tau_i))\,ds + \int_{t_0}^{t} e^{-c_i(t-s)} \\ \qquad \times \sum\limits_{j=1}^{n}\sigma_{ij}(y_j(s))\,dw(s) + \int_{t_0}^{t} e^{-c_i(t-s)} I_i(s)\,ds =: I_{1i} + I_{2i} + I_{3i} + I_{4i}, \\ y_i(t) = y_i(t_0)\,e^{-\bar{c}_i(t-t_0)} + \int_{t_0}^{t} e^{-\bar{c}_i(t-s)}\sum\limits_{j=1}^{n} \bar{a}_{ij} g_j(x_j(s-\tau_i))\,ds + \int_{t_0}^{t} e^{-\bar{c}_i(t-s)} \\ \qquad \times \sum\limits_{j=1}^{n}\bar{\sigma}_{ij}(x_j(s))\,dw(s) + \int_{t_0}^{t} e^{-\bar{c}_i(t-s)} \bar{I}_i(s)\,ds =: \bar{I}_{1i} + \bar{I}_{2i} + \bar{I}_{3i} + \bar{I}_{4i}. \end{cases}$$

By using the inequality $(a+b+c+d)^p \le 4^{p-1}(a^p + b^p + c^p + d^p)$ for any positive real numbers a, b, c and d, taking expectations, we find for all $t \ge t_0$,

$$E|x_i(t)|^p \le 4^{p-1}E\left(|I_{1i}|^p + |I_{2i}|^p + |I_{3i}|^p + |I_{4i}|^p\right). \tag{3.2}$$

First, we evaluate the first term of the right-hand side as follows:

$$E|I_{1i}|^p = E\left|x_i(t_0)\,e^{-c_i(t-t_0)}\right|^p$$

$$\leq e^{-pc_i(t-t_0)} \|\phi\|_{L^p}^p . \tag{3.3}$$

As to the second term, by hölder inequality, we have

$$\begin{aligned}
E|I_{2i}|^p &= E\left|\int_{t_0}^t e^{-c_i(t-s)} \sum_{j=1}^n a_{ij} f_j\left(y_j\left(s-\tau_i\right)\right) ds\right|^p \\
&\leq E\left|\int_{t_0}^t e^{-c_i(t-s)} \sum_{j=1}^n a_{ij}\alpha_j \left|y_j\left(s-\tau_i\right)\right| ds\right|^p \\
&\leq E\left[\int_{t_0}^t e^{-c_i(t-s)} ds\right]^{p-1} \left[\int_{t_0}^t e^{-c_i(t-s)} \left(\sum_{j=1}^n |a_{ij}\alpha_j| \left|y_j\left(s-\tau_i\right)\right|\right)^p ds\right] \\
&\leq c_i^{1-p} \left[\int_{t_0}^t e^{-c_i(t-s)} \left(\sum_{j=1}^n |a_{ij}\alpha_j|^{\frac{p}{p-1}}\right)^{p-1} \sum_{j=1}^n E\left|y_j\left(s-\tau_i\right)\right|^p ds\right]. \qquad (3.4)
\end{aligned}$$

As to the third term, using an estimate on the It$\hat{o}$ integral established in [9, Proposition 1.9] and hölder inequality, we obtain:

$$\begin{aligned}
E|I_{3i}|^p &= E\left|\int_{t_0}^t e^{-c_i(t-s)} \sum_{j=1}^n \sigma_{ij}\left(y_j\left(s\right)\right) dw\left(s\right)\right|^p \\
&\leq c_p\left[\int_{t_0}^t \left(e^{-c_i p(t-s)} E\left|\sum_{j=1}^n \sigma_{ij}\left(y_j\left(s\right)\right)\right|^p\right)^{2/p} ds\right]^{p/2} \\
&\leq c_p\left[\int_{t_0}^t \left(e^{-c_i p(t-s)} E\left|\sum_{j=1}^n L_{ij}\left|y_j\left(s\right)\right|\right|^p\right)^{2/p} ds\right]^{p/2} \\
&= c_p\left[\int_{t_0}^t \left(e^{-c_i(p-1)(t-s)} e^{-c_i(t-s)} E\left|\sum_{j=1}^n L_{ij}\left|y_j\left(s\right)\right|\right|^p\right)^{2/p} ds\right]^{p/2} \\
&\leq c_p\left[\int_{t_0}^t e^{-c_i\frac{2(p-1)}{p-2}(t-s)} ds\right]^{p/2-1} \left[\int_{t_0}^t e^{-c_i(t-s)} E\left|\sum_{j=1}^n L_{ij}\left|y_j\left(s\right)\right|\right|^p ds\right] \\
&\leq c_p (2c_i\left(p-1\right)/\left(p-2\right))^{1-p/2} \left[\int_{t_0}^t e^{-c_i(t-s)} E\left|\sum_{j=1}^n L_{ij}\left|y_j\left(s\right)\right|\right|^p ds\right] \\
&\leq c_p\left(\frac{2c_i\left(p-1\right)}{\left(p-2\right)}\right)^{1-\frac{p}{2}} \left[\int_{t_0}^t e^{-c_i(t-s)} \left(\sum_{j=1}^n |L_{ij}|^{\frac{p}{p-1}}\right)^{p-1} \sum_{j=1}^n E|y_j\left(s\right)|^p ds\right] (3.5)
\end{aligned}$$

where $c_p = \left(p\left(p-1\right)/2\right)^{p/2}$. As far as the last term is concerned, we have

$$E|I_{4i}|^p = \left|\int_{t_0}^t e^{-c_i(t-s)} I_i\left(s\right) ds\right|^p \leq \left(\frac{\hat{I}_i}{c_i}\right)^p . \tag{3.6}$$

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It follows from (3.2)-(3.6) that

$$E|x_i(t)|^p \le 4^{p-1}\left\{e^{-pc_i(t-t_0)}\|\phi\|_{L^p}^p + c_i^{1-p}\left[\int_{t_0}^t e^{-c_i(t-s)}\left(\sum_{j=1}^n |a_{ij}\alpha_j|^{\frac{p}{p-1}}\right)^{p-1}\right.\right.$$
$$\left.\times\sum_{j=1}^n E|y_j(s-\tau_i)|^p ds\right] + c_p(2c_i(p-1)/(p-2))^{1-p/2}$$
$$\left.\times\left[\int_{t_0}^t e^{-c_i(t-s)}\left(\sum_{j=1}^n |L_{ij}|^{\frac{p}{p-1}}\right)^{p-1}\sum_{j=1}^n E|y_j(s)|^p ds\right] + \left(\frac{\hat{I}_i}{c_i}\right)^p\right\}.$$

Proceeding as the proof above, we have

$$E|y_i(t)|^p \le 4^{p-1}\left\{e^{-p\bar{c}_i(t-t_0)}\|\varphi\|_{L^p}^p + \bar{c}_i^{1-p}\left[\int_{t_0}^t e^{-\bar{c}_i(t-s)}\left(\sum_{j=1}^n |\bar{a}_{ij}\beta_j|^{\frac{p}{p-1}}\right)^{p-1}\right.\right.$$
$$\left.\times\sum_{j=1}^n E|x_j(s-\tau_i)|^p ds\right] + c_p(2\bar{c}_i(p-1)/(p-2))^{1-p/2}$$
$$\left.\times\left[\int_{t_0}^t e^{-\bar{c}_i(t-s)}\left(\sum_{j=1}^n |\bar{L}_{ij}|^{\frac{p}{p-1}}\right)^{p-1}\sum_{j=1}^n E|x_j(s)|^p ds\right] + \left(\frac{\hat{\bar{I}}_i}{\bar{c}_i}\right)^p\right\}.$$

Set $V_i(t) = E|x_i(t)|^p$, $\overline{V}_i(t) = E|y_i(t)|^p$, $i = 1, \ldots, n$. It follows from (H_2) that

$$\begin{cases} V(t) \le 4^{p-1}\|\phi\|_{L^p}^p e^{-c(t-t_0)} + \int_{t_0}^t e^{-C(t-s)} A_2 \overline{V}(s)\, ds \\ \qquad + \int_{t_0}^t e^{-C(t-s)} B_2 \left[\overline{V}(s)\right]_\tau^+ ds + J_2, \\ \overline{V}(t) \le 4^{p-1}\|\varphi\|_{L^p}^p e^{-\overline{c}(t-t_0)} + \int_{t_0}^t e^{-\overline{C}(t-s)} \overline{A}_2 V(s)\, ds \\ \qquad + \int_{t_0}^t e^{-\overline{C}(t-s)} \overline{B}_2 [V(s)]_\tau^+ ds + \overline{J}_2, \end{cases}$$

where $V(t) = (V_1(t), \ldots, V_n(t))^T$, $\overline{V}(t) = (\overline{V}_1(t), \ldots, \overline{V}_n(t))^T$, $c = \min_{1\le i\le n} c_i$ and $\overline{c} = \min_{1\le i\le n} \overline{c}_i$.

From Lemma 2.3 and Condition (H_2), the solutions of equation(2.1) are p-uniformly bounded and $S = \{\phi \in BC^b_{\mathcal{F}_0}([-\tau, 0], R^n) \,|\, (\|\phi_1\|_{L^p}^p, \ldots, \|\phi_n\|_{L^p}^p) < (I - \Upsilon_1)^{-1}(J_2 + \overline{J}_2)\}$ is an attracting set of equation (2.1) (i.e., the family of all solutions of equation (2.1) is p-point dissipative). From Lemma 2.1, there must exist an ω-periodic solution.

Denote $z^*(t) = (x_1^*(t), \ldots, x_n^*(t), y_1^*(t), \ldots, y_n^*(t))^T$ with the initial condition $\left(\phi^{*T}, \varphi^{*T}\right)^T$ be the ω-periodic solution and $z(t) = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_n(t))^T$ with initial condition $(\phi^T, \varphi^T)^T$ be an arbitrary solution of equation(2.1).

We rewrite equation (2.1) by

$$\begin{cases} d\left[x_i(t)-x_i^*(t)\right]=\left[-c_i\left(x_i(t)-x_i^*(t)\right)+\sum\limits_{j=1}^{n}a_{ij}\left(f_j\left(y_j\left(t-\tau_i\right)\right)\ -f_j\left(y_j^*\left(t-\tau_i\right)\right)\right)\right]dt \\ \qquad\qquad +\sum\limits_{j=1}^{n}\left(\sigma_{ij}\left(y_j(t)\right)-\sigma_{ij}\left(y_j^*(t)\right)\right)dw(t), t\geq t_0, \\ d\left[y_i(t)-y_i^*(t)\right]=\left[-\bar{c}_i\left(y_i(t)-y_i^*(t)\right)+\sum\limits_{j=1}^{n}\bar{a}_{ij}\left(g_j\left(\bar{x}_j\left(t-\tau_i\right)\right)\ -g_j\left(\bar{x}_j^*\left(t-\tau_i\right)\right)\right)\right]dt \\ \qquad\qquad +\sum\limits_{j=1}^{n}\left(\bar{\sigma}_{ij}\left(x_j(t)\right)-\bar{\sigma}_{ij}\left(x_j^*(t)\right)\right)dw(t), t\geq t_0, \\ x_i(t)-x_i^*(t)=\phi_i(t)-\phi_i^*(t),\quad y_i(t)-y_i^*(t)=\varphi_i(t)-\varphi_i^*(t),\quad t_0-\tau\leq t\leq t_0. \end{cases}$$

Let $U_i(t)=E|x_i(t)-x_i^*(t)|^p$ and $\overline{U}_i(t)=E|y_i(t)-y_i^*(t)|^p,\ i=1,\dots,n$. Proceeding as the proof of the existence of periodic solution of equation(2.1), we have

$$\begin{cases} U(t)\leq 4^{p-1}\|\phi\|_{L^p}^p e^{-c(t-t_0)}+\int_{t_0}^t e^{-C(t-s)}A_2\overline{U}(s)\,ds \\ \qquad +\int_{t_0}^t e^{-C(t-s)}B_2\left[\overline{U}(s)\right]_\tau^+ds, \\ \overline{U}(t)\leq 4^{p-1}\|\varphi\|_{L^p}^p e^{-\bar{c}(t-t_0)}+\int_{t_0}^t e^{-\overline{C}(t-s)}\overline{A}_2U(s)\,ds \\ \qquad +\int_{t_0}^t e^{-\overline{C}(t-s)}\overline{B}_2\left[U(s)\right]_\tau^+ds, \end{cases}$$

where $U(t)=(U_1(t),\dots,U_n(t))^T$ and $\overline{U}(t)=(\overline{U}_1(t),\dots,\overline{U}_n(t))^T$.

From Corollary 2.1, we get that the periodic solution is globally p-exponentially stable, and the proof is completed.

□

4. Example

Example 4.1. Consider the periodic stochastic BAM neural networks with delays:

$$\begin{cases} d\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}=-\begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}dt+\begin{pmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{pmatrix}\begin{pmatrix} f_1(y_1(t-1)) \\ f_2(y_2(t-2)) \end{pmatrix}dt \\ \qquad +\begin{pmatrix} \sin t \\ \sin t \end{pmatrix}dt+\begin{pmatrix} 0.1y_1(t)+0.1y_2(t) \\ 0.1y_1(t)+0.1y_2(t) \end{pmatrix}dw(t), \\ d\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}=-\begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}dt+\begin{pmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{pmatrix}\begin{pmatrix} g_1(x_1(t-1)) \\ g_2(x_2(t-2)) \end{pmatrix}dt \\ \qquad +\begin{pmatrix} \sin t \\ \sin t \end{pmatrix}dt+\begin{pmatrix} 0.1x_1(t)+0.1x_2(t) \\ 0.1x_1(t)+0.1x_2(t) \end{pmatrix}dw(t), \end{cases} \tag{4.1}$$

where $f(x)=\arctan x,\ g(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}$.

It is obvious that

$$A=\overline{A}=\begin{pmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{pmatrix},\quad C=\overline{C}=\begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix},$$

$\alpha_j = \beta_j = 1,\ L_{ij} = \overline{L}_{ij} = 0.1,\ i, j = 1, 2,\ \tau = 2$ and $I_1(t) = I_2(t) = \overline{I}_1(t) = \overline{I}_2(t) = \sin t$. Taking $p = 3$, we have

$$A_2 = \bar{A}_2 = \begin{pmatrix} (16 \times \sqrt{27}) \times 10^{-3} & 0 \\ 0 & (8\sqrt{2} \times \sqrt{27}) \times 10^{-3} \end{pmatrix},$$

$$B_2 = \bar{B}_2 = \begin{pmatrix} 81 \times 10^{-3} & 0 \\ 0 & 81/4 \times 10^{-3} \end{pmatrix},$$

$$K_2 = \begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix}, \qquad H = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Therefore,

$$\begin{aligned} &K_2^{-1}(A_2H + B_2H + \bar{A}_2H + \bar{B}_2H) \\ = &\begin{pmatrix} \left(8 \times \sqrt{27} + 162\right) \times 10^{-3} & \left(8 \times \sqrt{27} + 162\right) \times 10^{-3} \\ \left(2\sqrt{2} \times \sqrt{27} + 81/16\right) \times 10^{-3} & \left(2\sqrt{2} \times \sqrt{27} + 81/16\right) \times 10^{-3} \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} &\rho\left(K_2^{-1}(A_2H + B_2H + \bar{A}_2H + \bar{B}_2H)\right) \\ = &\left[\left(8 + 2\sqrt{2}\right) \times \sqrt{27} + 162 + 81/16\right] \times 10^{-3} < 1. \end{aligned}$$

It follows from Theorem 3.1 that this equation has a 2π-periodic solution, which is globally exponentially stable.

5. Conclusion

Some sufficient conditions for the existence and global p-exponential stability of ω-periodic solutions for stochastic BAM neural networks with delays are given by establishing new integral inequalities and using the properties of spectral radius of nonnegative matrice. At the same time, the solution satisfies the p-point dissipation and p-uniform boundedness, the BAM neural network under random disturbance can also obtain the existence of periodic solution. In the future, we plan to explore the influence of fractional Brownian motion on the existence of periodic solutions of BAM neural network through the demonstration which is similar to this paper.

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