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Traveling Wave Solutions in a Chemotaxis Model with Two Chemoattractants*

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Abstract In this work, we investigate the existence and non-existence of traveling wave solutions for a chemotaxis model with two chemoattractants. To prove our main results, we apply the dynamical systems theory by constructing a positively invariant set in the four-dimensional space. Particularly, we analyze the monotonicity of traveling wave solutions.

Keywords Chemotaxis model, traveling wave solutions, dynamical systems theory

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1. Introduction

Our purpose in this work is to investigate the traveling wave solutions of the following chemotaxis model with two chemoattractants,

$$\begin{cases} u_t = u_{xx} - (\chi_1(v)uv_x)_x - (\chi_2(w)uw_x)_x + ru(1-u), \\ v_t = v - \alpha u, \\ w_t = w - \beta u, \end{cases} \tag{1.1}$$

for $x \in \mathbb{R}$ and $t \geq 0$. Here $u(x,t)$ represents the density of cell population, while $v(x,t)$ and $w(x,t)$ represent the densities of chemical concentrations of two different chemicals. The parameter $r > 0$ denotes the rate of logistic cell growth, while $\alpha > 0$ and $\beta > 0$ mean that the cells consume the chemoattractants. The chemotactic sensitivity $\chi_i(\cdot)$ $(i = 1, 2)$ describes the measure of the strength of chemotaxis and is referred as the Chemotactic Coefficient. The cells move towards where the concentration of chemical v and w increase. This motion is represented by $-(\chi_1(v)uv_x)_x$ and $-(\chi_2(w)uw_x)_x$ respectively. To simplify the calculation, we consider that the chemical growth rate of v or w is 1.

In recent years, many experts have focused their attention on the research of biomathematics. Scholars have constructed a series of biological mathematical models based on the behavior of individual organisms seeking benefits and avoiding

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harm, and have provided objective and reasonable explanations and predictions for many phenomena in biology by studying relevant partial differential equations.

One such phenomenon is chemotaxis, which causes species to move in specific directions in response to the attraction of certain chemical signals [1]. This chemotaxis leads to the formation of diverse patterns in nature, thereby creating a rich and colorful world. The reaction diffusion equation system proposed by Turing [2] in 1952 successfully explained the mechanism of speckle phenomenon, also known as the Turing mode. Mathematically speaking, when the parameters change, the stability of the constant equilibrium solution changes, from stable to unstable, and the process of generating non-homogeneous non-constant equilibrium solutions in space is called Turing mode. The mechanism of chemotaxis has been widely applied in daily life, such as trapping and killing pests, infecting bacteria, cultivating microorganisms, and treating wounds. Through theoretical and experimental observations, the morphological generation phenomena of chemotaxis exhibit a rich and colorful structure, including aggregation, finite time explosion, spot patterns, spikes, stripes, rings, etc. Due to the different principles and mechanisms of chemotaxis, simulation of specific systems, and mathematical explanations, a large number of different forms of chemotaxis models have emerged.

Keller and Segel [3] established a chemotaxis model for the first time in the 1970s.

$$\begin{cases} b_t = (\mu(s) b_x)_x - (b\chi_s s_x)_x, \\ s_t = D s_{xx} - bk(s), \end{cases} \tag{1.2}$$

which was proposed to explicate the phenomenon of aggregation observed in the celebrated experiment of Adler [4,5] in the cellular slime mold Dictyostelium. They showed that the model (1.2) can reproduce the traveling bands whose speeds are consistent with Adler's experimental observation in [7]. After Keller-Segel's work, traveling wave solutions of the chemotaxis models have been widely studied by many other scholars; see [6–15] and the reference therein.

Motivated by Li [14], we consider the chemotaxis model (1.1) with two chemoattractants which no one has studied before. We prove the existence and non-existence of traveling wave solutions by using the dynamical systems theory. Firstly, system (1.1) is transformed into an ODE system. Next, the existence of traveling wave solutions connecting two different equilibria is equivalent to the existence of heteroclinic orbits of the transformed ODE system. Then we construct a positively invariant set of the corresponding ODE system in the four-dimensional phase space that guarantees the existence of the desired heteroclinic orbits.

Our main results for the existence of traveling wave solutions are under the assumptions,

- (H1) $0 \le \chi_1(v) \le k_1$ and $\chi_1(\alpha) = 0$,
- (H2) $0 \le \chi_2(w) \le k_2$ and $\chi_2(\beta) = 0$,
- (H3) $r > \alpha k_1 + \beta k_2$.

where k_i $(i = 1, 2)$ is a constant. Now we present the results on the existence and non-existence of traveling wave solutions.

Theorem 1.1. *Let (H1)-(H3) hold. There exists a minimal speed $c_* > 0$ such that for each $c \ge c^* > 0$ the system* (1.1) *has a traveling wave solution* $(u(x,t),\ v(x,t),$

$w(x,t)) = (U(\xi),\ V(\xi), W(\xi))$ *where* $\xi = x - ct$, *that satisfies*

$$\begin{cases} 0 < U(\xi) \le \dfrac{1}{\beta} W(\xi) \le \dfrac{1}{\alpha} V < 1,\ \xi \in (-\infty, +\infty), \\ U'(\xi) < 0,\ V'(\xi) < 0,\ W'(\xi) < 0, \quad \xi \in (-\infty, +\infty), \\ (U, V, W)(-\infty) = (1, \alpha, \beta),\ (U, V, W)(+\infty) = (0, 0, 0). \end{cases}$$

The traveling wave solution is unique up to a translation in ξ. And for each $0 < c < c^*$ the system (1.1) has no traveling wave solution.

The rest of this article is organized in the following manner. In Section 2, we consider the transformed system of (1.1) and investigate the behavior of the trajectory in a neighborhood of the equilibrium points. In Section 3, we prove Theorem 1.1 by looking for the existence of heteroclinic orbits of the transformed system of (1.1) that connect the unstable equilibrium point to the stable equilibrium point.

2. Preliminaries

Traveling wave solutions of (1.1) with the moving coordinate $\xi := x - ct$ and the wave speed c of the form take the form $u(x,t) = U(\xi)$, $v(x,t) = V(\xi)$ and $w(x,t) = W(\xi)$ satisfying $(U(-\infty), V(-\infty), W(-\infty)) = (1, \alpha, \beta)$ and $(U(+\infty), V(+\infty), W(+\infty)) = (0, 0, 0)$. A direct computation shows that $(U(\xi), V(\xi), W(\xi))$ is a traveling wave solution of (1.1) if and only if $(U(\xi), V(\xi), W(\xi))$ is a solution of the system,

$$\begin{cases} -c\dfrac{dU}{d\xi} = \dfrac{d^2U}{d\xi^2} - \dfrac{d}{d\xi}(\chi_1(V)U\dfrac{dV}{d\xi}) - \dfrac{d}{d\xi}(\chi_2(W)U\dfrac{dW}{d\xi}) + rU(1-U), \\ -c\dfrac{dV}{d\xi} = V - \alpha U, \\ -c\dfrac{dW}{d\xi} = W - \beta U. \end{cases} \tag{2.1}$$

By introducing a new variable $Z = cU + \frac{dU}{d\xi} - \chi_1(V)U\frac{dV}{d\xi} - \chi_2(W)U\frac{dW}{d\xi}$, the resulting variables (U, V, W, Z) satisfy the four-dimensional ODEs

$$\begin{cases} \dot{U} = -cU + \dfrac{\chi_1(V)U(\alpha U - V)}{c} + \dfrac{\chi_2(W)U(\beta U - W)}{c} + Z, \\ \dot{V} = \dfrac{1}{c}(\alpha U - V), \\ \dot{W} = \dfrac{1}{c}(\beta U - W), \\ \dot{Z} = rU(U-1), \end{cases} \tag{2.2}$$

where the $\dot{X}$ represents differentiation with respect to the independent variable ξ. By straightforward calculation, we obtain that (2.2) has two equilibria $E_o = (0, 0, 0, 0)$ and $E_* = (1, \alpha, \beta, c)$.

To understand asymptotic behaviors of the trajectory $(U(\xi), V(\xi), W(\xi), Z(\xi))$ of system (2.2), we investigate the behavior of the trajectory in a neighborhood of the equilibrium points E_o and E_*.

At the equilibrium point E_o, the Jacobian matrix of (2.2) is

$$J_o = \begin{bmatrix} -c & 0 & 0 & 1 \\ \frac{\alpha}{c} & -\frac{1}{c} & 0 & 0 \\ \frac{\beta}{c} & 0 & -\frac{1}{c} & 0 \\ -r & 0 & 0 & 0 \end{bmatrix}.$$

By direct computations, $J(E_o)$ has four negative eigenvalues as follows:

$$\begin{aligned} \lambda_{o,1,2} &= -\frac{1}{c} < 0, \\ \lambda_{o,3} &= \frac{-c - \sqrt{c^2 - 4r}}{2}, \\ \lambda_{o,4} &= \frac{-c + \sqrt{c^2 - 4r}}{2}, \end{aligned}$$

where the corresponding eigenvectors to $\lambda_{o,i}$, $i = 1, 2, 3, 4$ are

$$\begin{aligned} \vec{l}_i &= [0, 1, 1, 0]^T, \ i = 1, 2, \\ \vec{l}_3 &= \left[1, \frac{\alpha}{c(\lambda_{o,3} + \frac{1}{c})}, \frac{\beta}{c(\lambda_{o,3} + \frac{1}{c})}, \frac{r}{\lambda_{o,3}}\right]^T, \\ \vec{l}_4 &= \left[1, \frac{\alpha}{c(\lambda_{o,4} + \frac{1}{c})}, -\frac{\beta}{c(\lambda_{o,4} + \frac{1}{c})}, -\frac{r}{\lambda_{o,4}}\right]^T. \end{aligned} \tag{2.3}$$

At the equilibrium point E_*, the Jacobian matrix of (2.2) is

$$J_* = \begin{bmatrix} -c & 0 & 0 & 1 \\ \frac{\alpha}{c} & -\frac{1}{c} & 0 & 0 \\ \frac{\beta}{c} & 0 & -\frac{1}{c} & 0 \\ r & 0 & 0 & 0 \end{bmatrix}.$$

By direct computations, $J(E_o)$ has three negative eigenvalues

$$\begin{aligned} \lambda_{1,2} &= -\frac{1}{c} < 0, \\ \lambda_3 &= \frac{-c - \sqrt{c^2 + 4r}}{2} < 0, \end{aligned}$$

and one positive eigenvalue

$$\lambda_4 = \frac{-c + \sqrt{c^2 + 4r}}{2} > 0,$$

where the corresponding eigenvector to λ_4 is

$$\vec{h} = \left[-1, -\frac{\alpha}{c(\lambda_4 + \frac{1}{c})}, -\frac{\beta}{c(\lambda_4 + \frac{1}{c})}, -\frac{r}{\lambda_4}\right]^T. \tag{2.4}$$

Let us summarize the above discussion in the following lemma.

Lemma 2.1. (I) *The equilibrium point E_o is stable node (including the degenerated stable node) of (2.2) if $c \geq c_*$. The local stable manifold $W^s_{loc}(E_o)$ of (2.2) is four-dimensional.*

(II) *The equilibrium point E_* is unstable. The local unstable manifold $W^u_{loc}(E_*)$ of (2.2) is one-dimensional.*

3. Existence and non-existence of traveling wave solutions

In this section, in order to prove the existence of traveling wave solutions of (1.1), we give two theorems for the existence of heteroclinic orbits of (2.2) connecting the equilibria E_o and E_*, which are a consequence of Theorem 1.1 as follows.

Theorem 3.1. *System (2.2) does not have a positive heteroclinic orbit connecting E_* and E_o for wave speed $0 < c < c_*$.*

Proof. If $0 < c < c_*$, then $\lambda_{o,3}$ and $\lambda_{o,4}$ are a pair of complex eigenvalues while the eigenvalues $\lambda_{o,1,2}$ correspond to the invariant set $\{U = Z = 0\}$ of system (2.2). This implies that any solution of (2.2) that converges to the origin E_o but not stay in the set $\{U = Z = 0\}$ must be oscillation around the origin E_o. $\square$

Theorem 3.2. *Let (H1)-(H3) hold and $c_* = 2\sqrt{r}$. For each $c \geq c^* > 0$, system (2.2) has a heteroclinic orbit $(u(x,t),\ v(x,t),\ w(x,t),\ z(x,t)) = (U(\xi),\ V(\xi),\ W(\xi),\ Z(\xi))$ where $\xi = x - ct$, that satisfies*

$$\begin{cases} 0 < U(\xi) \leq \dfrac{1}{\beta}W(\xi) \leq \dfrac{1}{\alpha}V(\xi) < 1, \quad 0 < Z(\xi) < c, \quad \xi \in (-\infty, +\infty), \\ U'(\xi) < 0,\ V'(\xi) < 0,\ W'(\xi) < 0,\ Z'(\xi) < 0, \quad \xi \in (-\infty, +\infty), \\ (U, V, W, Z)(-\infty) = (1, \alpha, \beta, c),\ (U, V, W, Z)(+\infty) = (0, 0, 0, 0), \end{cases}$$

and the heteroclinic orbit is unique up to a translation in ξ.

We prove the existence part of Theorem 3.2 by the following steps:

1. we construct a positively invariant set of (2.2) with the unstable equilibrium as its boundary point and the stable equilibrium,
2. we prove the existence of heteroclinic orbits of (2.2) connecting the equilibria E_o and E_* and investigate the monotonicity of the heteroclinic orbits for each $c \geq c_*$.

Step 1: Let σ be a constant defined by

$$\sigma = \frac{c + \sqrt{c^2 - 4r}}{2}.$$

It is clear that $0 < \sigma < c$ if $c \geq c_*$. Now, for $c \geq c_*$, we define a wedged like region $\Sigma \subset \mathbb{R}^4$ as follows:

$$\Sigma = \left\{ (U, V, W, Z) : 0 \leq U \leq \frac{1}{\beta}W \leq \frac{1}{\alpha}V \leq 1, \sigma U \leq Z \leq cU \right\}. \tag{3.1}$$

Then the boundary of Σ consists of surfaces $P_1 \sim P_6$ represented by

$$
\begin{aligned}
&P_1 = \{U = Z = 0\},\\
&P_2 = \{U = \frac{1}{\beta}W, \sigma U < Z < cU\},\\
&P_3 = \{W = \frac{\beta}{\alpha}V, \sigma U < Z < cU\},\\
&P_4 = \{V = \alpha, \sigma U < W < cU\},\\
&P_5 = \{0 < U < \frac{1}{\beta}W < \frac{1}{\alpha}V < 1, Z = \sigma U\},\\
&P_6 = \{0 < U < \frac{1}{\beta}W < \frac{1}{\alpha}V < 1, Z = cU\}.
\end{aligned}
$$

The vector field of (2.2) has a very simple property in the surface of Σ, which can be characterized by the following lemma.

Lemma 3.1. *Let assumptions (H1)-(H3) hold and $c \geq c_*$. Then the Σ is a positively invariant set of* (2.2)*, that is, any solution of* (2.2) *starting at a point in Σ cannot leave Σ at any positive time. Furthermore, the eigenvector $\vec{h}$ at E_* points to the interior point of Σ.*

Proof. Let $(U, V, W, Z) \in \partial\Sigma$. The main tool is to show that at any point $(U, V, W, Z) \in \partial\Sigma$, it satisfies

$$\vec{n} \cdot (U', V', W', Z') < 0,$$

where $\vec{n}$ denotes an outward normal vector at the point (see Theorem 4.2.2 in [16]).

On the face P_1, $\dot{U} = 0$ and $\dot{W} = 0$. It is obvious that the surface P_1 is an invariant set of (2.2).

On the face P_2, it is clear that the outward normal vector $\vec{n_1} = (\beta, 0, -1, 0)$ at the point (U, V, W, Z) . Then we have

$$
\begin{aligned}
\vec{n_1} \cdot (\dot{U}, \dot{V}, \dot{W}, \dot{Z})^T =& \beta\dot{U} - \dot{W}\\
=& \beta\left(-cU + \frac{\chi_1(V)U(\alpha U - V)}{c} + \frac{\chi_2(W)U(\beta \cdot \frac{W}{\beta} - W)}{c} + Z\right)\\
& - \frac{1}{c}\left(\beta \cdot \frac{W}{\beta} - W\right)\\
<& \beta\left(-cU + \frac{\chi_1(V)U(\alpha \cdot \frac{1}{\alpha}V - V)}{c} + cU\right)\\
=& 0.
\end{aligned}
$$

Thus, the vector field of equation (2.2) points towards the interior of Σ on the face.

On the face P_3, it is clear that the outward normal vector $\vec{n_2} = (0, -\frac{\beta}{\alpha}, 1, 0)$ at the point (U, V, W, Z) . Then we have

$$
\begin{aligned}
\vec{n_2} \cdot (\dot{U}, \dot{V}, \dot{W}, \dot{Z})^T &= \dot{W} - \frac{\beta}{\alpha}\dot{V}\\
&= \frac{\left(\beta U - \frac{\beta}{\alpha}V\right) - \frac{\beta}{\alpha}(\alpha U - V)}{C} = 0.
\end{aligned}
$$

It is obvious that the surface P_3 is an invariant set of (2.2).

On the face P_4, $\dot{V} = \frac{1}{c}(\alpha U - \alpha) = \frac{\alpha}{c}(U-1) \leq 0$. Hence, the vector field of (2.2) points interior of Σ on the face.

On the face P_5, it is clear that the outward normal vector $\vec{n_3} = (\sigma, 0, 0, -1)$ at the point (U, V, W, Z) . Then we have

$$
\begin{aligned}
\vec{n_3} \cdot (\dot{U}, \dot{V}, \dot{W}, \dot{Z})^T &= \sigma \dot{U} - \dot{Z} \\
&= \sigma \left(-cU + \frac{\chi_1(V)U(\alpha U - V)}{c} + \frac{\chi_2(W)U(\beta U - W)}{c} + \sigma U \right) \\
&+ rU(1-U) \\
&\leq \sigma\left(-cU + \sigma U\right) + rU \\
&= U\left(\sigma^2 - c\sigma + r\right) \\
&= 0.
\end{aligned}
$$

Thus, the vector field of equation (2.2) points towards the interior of Σ on the face.

On the face P_6, it is clear that the outward normal vector $\vec{n_4} = (-c, 0, 0, 1)$ at the point (U, V, W, Z) . By assumption (H1) we have

$$
\begin{aligned}
\vec{n_4} \cdot (\dot{U}, \dot{V}, \dot{W}, \dot{Z})^T &= -c\dot{U} + \dot{Z} \\
&= -\chi_1(V)U(\alpha U - V) - \chi_2(W)U(\beta U - W) + rU(U-1) \\
&\leq -\alpha\chi_1(V)U(U-1) - \beta\chi_2(W)U(U-1) + rU(U-1) \\
&= U(U-1)(r - \alpha\chi_1 - \beta\chi_2) \\
&< 0.
\end{aligned}
$$

Therefore, the vector field of (2.2) points interior of Σ on the face. Hence the Σ is a positively invariant set of (2.2). A direct verification shows that $\vec{h}$ defined in (2.4) is an eigenvector of J_* associated with λ_4. It follows from the signs of its components that $\vec{h}$ points to the interior of Σ. This completes the proof. □

Step 2: Now we state the lemma for the existence of heteroclinic orbits of (2.2) connecting the equilibria E_o and E_* as follows.

Lemma 3.2. *Let assumptions (H1)-(H3) hold and $c \geq c_*$. For each c, there is a unique heteroclinic orbit $(U(\xi), V(\xi), W(\xi), Z(\xi))$, where $\xi = x - ct$ in system* (2.2) *satisfying*

1. $(U, V, W, Z)(-\infty) = E_*$ *and* $(U, V, W, Z)(+\infty) = E_o$,
2. $(U, V, W, Z) \in Int(\Sigma)$ *for any* $\xi \in (-\infty, +\infty)$,
3. $U' < 0,\ V' < 0,\ W' < 0,\ Z' < 0$ *for any* $\xi \in (-\infty, +\infty)$.

Proof. By Lemma 3.1, $Int(\Sigma) \cap W^u_{loc}(E_*)$ is non-empty since the eigenvector $\vec{h}$ points to the interior of Σ at E_*. Define $\Phi_t(p)$ as a solution of (2.2) satisfying the initial condition $\Phi_0(p) = p \in \mathbb{R}^4$. Then, there exists a point $p_* \in W^u_{loc}(E_*)$ such that the solution $\Phi_t(p_*)$ of (2.2) initiated at p_* stays in $Int(\Sigma)$ for sufficiently negative p_* as proved in Lemma 3.1. Specifically, the solution must approach the equilibrium point E_o. By system (2.2) and the structure of Σ, we have $\dot{U},\ \dot{V},\ \dot{W}\ \dot{Z} < 0$. By Lemma 3.1, the heteroclinic orbit connecting E_* to E_o is unique up to a translation in ξ. □

As a consequence of Lemma 3.2, the proof of Theorem 3.2 is fulfilled.

4. Conclusions

In this paper, we investigate the existence and non-existence of traveling wave solutions for a chemotaxis model with two chemoattractants and analyze the monotonicity of traveling wave solutions. The above are the main research findings of this article. The follow-up work of this article can consider the following aspect: chemotactic diffusion is a type of directional diffusion, and another common directional motion is convective diffusion. We will further investigate the dynamic behavior of predator-prey systems with convective terms. By comparing the differences between chemotaxis and convection, we will characterize the impact of the combination of random diffusion and directional diffusion on predator-prey systems.

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