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The Upper Semicontinuity of Random Attractors for Non-Autonomous Stochastic Plate Equations with Multiplicative Noise and Nonlinear Damping*

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Abstract Based on the existence of pullback attractors for the non-autonomous stochastic plate equations with multiplicative noise and nonlinear damping defined in the entire space $\mathbb{R}^n$ by Xiaobin Yao in [15], in the paper, we further investigate the upper semicontinuity of pullback attractors for the problem.

Keywords Upper semicontinuity, attractors, plate equation, unbounded domains, multiplicative white noise

MSC(2010) 35B40, 35B41.

1. Introduction

Plate equations have been investigated for many years due to their importance in some physical areas such as vibration and elasticity theory of solid mechanics. The study of the long-time dynamics of plate equations has become an outstanding topic in the field of the infinite dimensional dynamical system [3–6, 9].

In this paper, we study the upper semicontinuity of pullback attractors for the following non-autonomous stochastic plate equation with multiplicative noise and nonlinear damping defined on the unbounded domain $\mathbb{R}^n$:

$$u_{tt} + \Delta^2 u + h(u_t) + \lambda u + f(x,u) = g(x,t) + \varepsilon u \circ \frac{dw}{dt} \tag{1.1}$$

with the initial value conditions

$$u(x,\tau) = u_0(x), \quad u_t(x,\tau) = u_1(x), \tag{1.2}$$

where $x \in \mathbb{R}^n$, $t > \tau$ with $\tau \in \mathbb{R}$, $\lambda > 0$ and ε are constants, $h(u_t)$ is a nonlinear damping term, f is a given interaction term, g is a given function satisfying $g \in L^2_{loc}(\mathbb{R}, H^1(\mathbb{R}^n))$, and w is a two-sided real-valued Wiener process on a probability space. The stochastic equation (1.1) is understood in the sense of Stratonovich's integration.

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The functions $h,\ f$ satisfy the following conditions.

(1) Let $F(x,u)=\int_0^u f(x,s)ds$ for $x\in\mathbb{R}^n$ and $u\in\mathbb{R}$. There exist positive constants $c_i(i=1,2,3,4)$, such that

$$|f(x,u)|\le c_1|u|^\gamma+\phi_1(x),\quad \phi_1\in L^2(\mathbb{R}^n), \tag{1.3}$$

$$f(x,u)u-c_2F(x,u)\ge\phi_2(x),\quad \phi_2\in L^1(\mathbb{R}^n), \tag{1.4}$$

$$F(x,u)\ge c_3|u|^{\gamma+1}-\phi_3(x),\quad \phi_3\in L^1(\mathbb{R}^n), \tag{1.5}$$

$$|\frac{\partial f}{\partial u}(x,u)|\le\beta,\quad |\frac{\partial f}{\partial x}(x,u)|\le\phi_4(x),\quad \phi_4\in L^2(\mathbb{R}^n), \tag{1.6}$$

where $\beta>0$ and $1\le\gamma\le\frac{n+4}{n-4}$.

(2) There exist two constants β_1,β_2 such that

$$h(0)=0,\qquad 0<\beta_1\le h'(v)\le\beta_2<\infty. \tag{1.7}$$

(3)

$$\delta>0 \text{ satisfies }\ \lambda+\delta^2-\beta_2\delta>0,\ \beta_1>\delta. \tag{1.8}$$

Just for problems (1.1)-(1.2) and the corresponding plate equations, on the unbounded domain, the authors investigated the asymptotic behavior for stochastic plate equation with different noise (see [12–15] for details). To the best of our knowledge, it has not been considered by any predecessors for the upper semicontinuity of pullback attractors for the stochastic plate equation with multiplicative noise on unbounded domain. It is well known that multiplicative noise makes the problem more complex and interesting even to the case of bounded domain. Based on the results in [15] as well as the theory and applications of B. Wang in [10,11], we decide to study the upper semicontinuity of pullback attractors for problems (1.1)-(1.2).

The rest of this paper is organized as follows. In the next section, we present some notations, definitions and a criteria concerning the upper semicontinuity of non-autonomous random attractors with respect to a parameter. In Section 3, we show the upper semi-continuity of random attractors.

Throughout the paper, we use $||\cdot||$ and $(\cdot,\cdot)$ to denote the norm and the inner product of $L^2(\mathbb{R}^n)$, respectively. The norms of $L^p(\mathbb{R}^n)$ and a Banach space X are generally written as $||\cdot||_p$ and $||\cdot||_X$, respectively. The letters c and c_i $(i=1,2,\ldots)$ are generic positive constants which may change their values from line to line or even in the same line and do not depend on ε.

2. Preliminaries

In this section, we first present some notations, then recall some definitions and known results regarding non-autonomous random dynamical systems from [1,2,7,8,11,16], which are useful to our problem.

Let $(\Omega,\mathcal{F},\mathcal{P})$ be the standard probability space, where $\Omega=\{\omega\in C(\mathbb{R},\mathbb{R}):\omega(0)=0\}$, $\mathcal{F}$ is the Borel σ-algebra induced by the compact open topology of Ω, and $\mathcal{P}$ is the Wiener measure on $(\Omega,\mathcal{F})$. There is a classical group $\{\theta_t\}_{t\in\mathbb{R}}$ acting on $(\Omega,\mathcal{F},\mathcal{P})$ which is defined by

$$\theta_t\omega(\cdot)=\omega(\cdot+t)-\omega(t),\quad \text{for all } \omega\in\Omega,\ t\in\mathbb{R},$$

then $(\Omega, \mathcal{F}, \mathcal{P}, \{\theta_t\}_{t\in\mathbb{R}})$ is a parametric dynamical system.

Let $-\Delta$ denote the Laplace operator in $\mathbb{R}^n$, $A = \Delta^2$ with the domain $D(A) = H^4(\mathbb{R}^n)$. We can also define the powers A^ν of A for $\nu \in \mathbb{R}$. The space $V_\nu = D(A^{\frac{\nu}{4}})$ is a Hilbert space with the following inner product and norm

$$(u, v)_\nu = (A^{\frac{\nu}{4}} u, A^{\frac{\nu}{4}} v), \qquad \|\cdot\|_\nu = \|A^{\frac{\nu}{4}} \cdot\|.$$

For brevity, the notation $(\cdot,\cdot)$ for L^2-inner product will also be used for the notation of duality pairing between dual spaces and $\|\cdot\|$ denotes the L^2-norm.

Let $E = H^2 \times L^2$, with the Sobolev norm

$$\|Y\|_E = (\|v\|^2 + (\lambda + \delta^2 - \beta_2\delta)\|u\|^2 + \|\Delta u\|^2)^{\frac{1}{2}}, \quad \text{for} \;\; Y = (u, v) \in E. \tag{2.1}$$

Definition 2.1. Let $\theta : \mathbb{R} \times \Omega \to \Omega$ be a $(\mathcal{B}(\mathbb{R}) \times \mathcal{F}, \mathcal{F})$-measurable mapping. We say $(\Omega, \mathcal{F}, \mathcal{P}, \theta)$ is a parametric dynamical system if $\theta(0, \cdot)$ is the identity on Ω, $\theta(s+t, \cdot) = \theta(t, \cdot) \circ \theta(s, \cdot)$ for all $t, s \in \mathbb{R}$, and $P\theta(t, \cdot) = P$ for all $t \in \mathbb{R}$.

Definition 2.2. Let $K : \mathbb{R} \times \Omega \to 2^X$ be a set-valued mapping with closed nonempty images. We say K is measurable with respect to $\mathcal{F}$ in Ω if the mapping $\omega \in \Omega \to d(x, K(\tau, \omega))$ is $(\mathcal{F}, \mathcal{B}(\mathbb{R}))$-measurable for every fixed $x \in X$ and $\tau \in \mathbb{R}$.

Definition 2.3. A mapping $\Phi : \mathbb{R}^+ \times \mathbb{R} \times \Omega \times X \to X$ is called a continuous cocycle on X over $\mathbb{R}$ and $(\Omega, \mathcal{F}, \mathcal{P}, \{\theta_t\}_{t\in\mathbb{R}})$ if for all $\tau \in \mathbb{R}$, $\omega \in \Omega$ and $t, s \in \mathbb{R}^+$, the following conditions (1)-(4) are satisfied:

(1) $\Phi(\cdot, \tau, \cdot, \cdot) : \mathbb{R}^+ \times \Omega \times X \to X$ is $(\mathcal{B}(\mathbb{R}^+) \times \mathcal{F} \times \mathcal{B}(X), \mathcal{B}(X))$-measurable;
(2) $\Phi(0, \tau, \omega, \cdot)$ is the identity on X;
(3) $\Phi(t+s, \tau, \omega, \cdot) = \Phi(t, \tau+s, \theta_s\omega, \cdot) \circ \Phi(s, \tau, \omega, \cdot)$;
(4) $\Phi(t, \tau, \omega, \cdot) : X \to X$ is continuous.

Hereafter, we assume that Φ is a continuous cocycle on X over $\mathbb{R}$ and $(\Omega, \mathcal{F}, \mathcal{P}, \{\theta_t\}_{t\in\mathbb{R}})$, and $\mathcal{D}$ is the collection of certain families of nonempty bounded subsets of X parameterized by $\tau \in \mathbb{R}$ and $\omega \in \Omega$:

$$\mathcal{D} = \{D = \{D(\tau, \omega) \subseteq X : D(\tau, \omega) \neq \emptyset, \tau \in \mathbb{R}, \omega \in \Omega\}\}.$$

Definition 2.4. Let $\mathcal{D}$ be a collection of certain families of nonempty subsets of X and $K = \{K(\tau, \omega) : \tau \in \mathbb{R}, \omega \in \Omega\} \in \mathcal{D}$. Then K is called a $\mathcal{D}$-pullback absorbing set for Φ if for all $\tau \in \mathbb{R}$ and $\omega \in \Omega$ and for every $B \in \mathcal{D}$, there exists $T = T(B, \tau, \omega) > 0$ such that

$$\Phi(t, \tau - t, \theta_{-t}\omega, B(\tau - t, \theta_{-t}\omega)) \subseteq K(\tau, \omega) \;\; \text{for all } t \geq T.$$

If, in addition, $K(\tau, \omega)$ is closed in X and is measurable in ω with respect to $\mathcal{F}$, then K is called a closed measurable $\mathcal{D}$-pullback absorbing set for Φ.

Definition 2.5. Let $\mathcal{D}$ be a collection of certain families of nonempty subsets of X and $\mathcal{A} = \{\mathcal{A}(\tau, \omega) : \tau \in \mathbb{R}, \omega \in \Omega\} \in \mathcal{D}$. Then $\mathcal{A}$ is called a $\mathcal{D}$-pullback attractor for Φ if the following conditions (1)-(3) are fulfilled: for all $t \in \mathbb{R}^+$, $\tau \in \mathbb{R}$ and $\omega \in \Omega$,

(1) $\mathcal{A}(\tau, \omega)$ is compact in X and is measurable in ω with respect to $\mathcal{F}$.
(2) $\mathcal{A}$ is invariant, that is,

$$\Phi(t, \tau, \omega, \mathcal{A}(\tau, \omega)) = \mathcal{A}(\tau + t, \theta_t\omega).$$

(3) For every $B=\{B(\tau,\omega):\tau\in\mathbb{R},\omega\in\Omega\}\in\mathcal{D}$,

$$\lim_{t\to\infty} d(\Phi(t,\tau-t,\theta_{-t}\omega,B(\tau-t,\theta_{-t}\omega)),\mathcal{A}(\tau,\omega))=0.$$

Finally, we present a criteria concerning the upper semicontinuity of non-autonomous random attractors with respect to a parameter.

Theorem 2.1. *Let $(X,\|\cdot\|_X)$ be a separable Banach space, Φ_ϵ be a continuous cocycle on X over $\mathbb{R}$ and $(\Omega,\mathcal{F},\mathcal{P},\{\theta_t\}_{t\in\mathbb{R}})$. Suppose that*
(i) Φ_ϵ has a closed measurable random absorbing set $K_\varepsilon=\{K_\varepsilon(\tau,\omega):\tau\in\mathbb{R},\omega\in\Omega\}$ in $\mathcal{D}(X)$ and a unique random attractor $\mathcal{A}_\epsilon=\{\mathcal{A}_\varepsilon(\tau,\omega):\tau\in\mathbb{R},\omega\in\Omega\}$ in $\mathcal{D}(X)$ and
(ii) there exists a map $\varsigma:\mathbb{R}\to\mathbb{R}$ such that for each $\tau\in\mathbb{R},\omega\in\Omega,K_0(\tau)=\{u\in X:\|u\|_X\le\varsigma(\tau)\}$ and

$$\limsup_{\varepsilon\to0}\|K_\varepsilon(\tau,\omega)\|_X=\limsup_{\varepsilon\to0}\limsup_{x\in K_\varepsilon(\tau,\omega)}\|x\|_X\le\varsigma(\tau). \tag{2.2}$$

(iii) There exists $\varepsilon_0>0$, such that for every $\tau\in\mathbb{R}$ and $\omega\in\Omega$,

$$\bigcup_{|\varepsilon|\le\epsilon_0}\mathcal{A}_\varepsilon(\tau,\omega)\ \textit{is precompact in } X. \tag{2.3}$$

(iv) For $t>0,\tau\in\mathbb{R},\omega\in\Omega,\varepsilon_n\to0$ when $n\to\infty$, and $x_n,x_0\in X$ with $x_n\to x_0$ when $n\to\infty$, it holds

$$\lim_{n\to\infty}\Phi_{\varepsilon_n}(t,\tau,\omega)x_n=\Phi_0(t,\tau)x_0. \tag{2.4}$$

Then for $\tau\in\mathbb{R},\omega\in\Omega$,

$$d_H(\mathcal{A}_\varepsilon(\tau,\omega),\mathcal{A}_0(\tau))=\sup_{u\in\mathcal{A}_\varepsilon(\tau,\omega)}\inf_{v\in\mathcal{A}_0(\tau)}\|u-v\|\to0,\quad \textit{as } \varepsilon\to0. \tag{2.5}$$

3. Upper semicontinuity of pullback attractors

In this section, we will consider the upper semicontinuity of pullback attractors for the stochastic plate equations (1.1)-(1.2) on $\mathbb{R}^n$.

Next, we will use Theorem 3.1 to prove the upper semicontinuity of random attractors $\mathcal{A}_\varepsilon(\tau,\omega)$ when $\varepsilon\to0$. To indicate the dependence of solutions on ε, we will write the solutions to problems (1.1)-(1.2) as $(u^{(\varepsilon)},\ v^{(\varepsilon)})$, that is, $\varphi^{(\varepsilon)}=(u^{(\varepsilon)},v^{(\varepsilon)})^T$ satisfies

$$\begin{cases}\frac{du^{(\varepsilon)}}{dt}+\delta u^{(\varepsilon)}-v^{(\varepsilon)}=\varepsilon u^{(\varepsilon)}z(\theta_t\omega),\\ \frac{dv^{(\varepsilon)}}{dt}-\delta v^{(\varepsilon)}+(\delta^2+\lambda+A)u^{(\varepsilon)}+f(x,u^{(\varepsilon)})=g(x,t)-h(v^{(\varepsilon)}\\ +\varepsilon u^{(\varepsilon)}z(\theta_t\omega)-\delta u^{(\varepsilon)})-\varepsilon(v^{(\varepsilon)}-3\delta u^{(\varepsilon)}+\varepsilon u^{(\varepsilon)}z(\theta_t\omega))z(\theta_t\omega),\\ u^{(\varepsilon)}(x,\tau,\tau)=u_0^{(\varepsilon)}(x),\qquad v^{(\varepsilon)}(x,\tau,\tau)=v_0^{(\varepsilon)}(x),\end{cases} \tag{3.1}$$

where $v(x,t)=u_t+\delta u-\varepsilon uz(\theta_t\omega)$(the definition of $z(\theta_t\omega)$, see [15]).

When $\varepsilon = 0$, the random problem (3.1) reduces to a deterministic dynamical system:

$$\begin{cases} \frac{du^{(0)}}{dt} + \delta u^{(0)} - v^{(0)} = 0, \\ \frac{dv^{(0)}}{dt} - \delta v^{(0)} + (\delta^2 + \lambda + A)u^{(0)} + f(x, u^{(0)}) = g(x,t) - h(v^{(0)} - \delta u^{(0)}), \\ u^{(0)}(x,\tau,\tau) = u_0^{(0)}(x), \qquad v^{(0)}(x,\tau,\tau) = v_0^{(0)}(x), \end{cases} \tag{3.2}$$

Accordingly, by virtue of Theorem 5.1 in [15], the deterministic non-autonomous system Φ_0 generated by (3.2) is readily verified to admit a unique $\mathcal{D}_0(E(\mathbb{R}^n))$-pullback attractor $\mathcal{A}_0(\tau)$ if $g(x,\cdot) \in L^2_{loc}(\mathbb{R}, H^1(\mathbb{R}^n))$.

Theorem 3.1. *Assume that* (1.3)-(1.8) *hold. Then the cocycle* Φ_ε *generated by* (3.1) *has a unique* $\mathcal{D}$*-pullback attractor* $\{\mathcal{A}_\varepsilon(\tau,\omega)\}_{\omega\in\Omega}$ *in* $\mathcal{H}(\mathbb{R}^n)$*. Moreover, the family of random attractors* $\{\mathcal{A}_\varepsilon\}_{\varepsilon>0}$ *is upper semicontinuous.*

Proof. (i) From Lemma 4.1 and Theorem 5.1 in [15], we know that Φ_ϵ has a closed measurable random absorbing set $E_\varepsilon = \{E_\varepsilon(\tau,\omega) : \tau \in \mathbb{R}, \omega \in \Omega\} \in \mathcal{D}\big(E(\mathbb{R}^n)\big)$, where $E_\varepsilon(\tau,\omega) = \big\{\varphi^{(\varepsilon)} \in E(\mathbb{R}^n) : \|\varphi^{(\varepsilon)}\|^2_{E(\mathbb{R}^n)} \le R(\varepsilon,\tau,\omega)\big\}$, and a unique random attractor $\mathcal{A}_\varepsilon = \{\mathcal{A}_\varepsilon(\tau,\omega) : \tau \in \mathbb{R}, \omega \in \Omega\} \in \mathcal{D}(E(\mathbb{R}^n))$, for each $\tau \in \mathbb{R}, \omega \in \Omega$, $\mathcal{A}_\varepsilon(\tau,\omega) \subseteq E_\varepsilon(\tau,\omega)$.

(ii) Given $\varepsilon \le 1$, by (4.2) in [15], we have

$$R(\varepsilon,\tau,\omega) \le R(1,\tau,\omega) < \infty,$$

and

$$\limsup_{\varepsilon\to 0} R(\varepsilon,\tau,\omega) \le R(1,\tau,\omega).$$

So, for every $\tau \in \mathbb{R}$ and $\omega \in \Omega$,

$$\limsup_{\varepsilon\to 0} \|E_\varepsilon(\tau,\omega)\| = \limsup_{\varepsilon\to 0} \sup_{x\in E_\varepsilon(\tau,\omega)} \|x\|_{E(\mathbb{R}^n)} \le R^{\frac{1}{2}}(1,\tau,\omega). \tag{3.3}$$

Letting $E_1(\tau,\omega) = \{\varphi^{(\varepsilon)} \in E(\mathbb{R}^n) : \|\varphi^{(\varepsilon)}\|^2_{E(\mathbb{R}^n)} \le R(1,\tau,\omega)\}$, then

$$\bigcup_{\varepsilon\le 1} \mathcal{A}_\varepsilon(\tau,\omega) \subseteq \bigcup_{\varepsilon\le 1} E_\varepsilon(\tau,\omega) \subseteq E_1(\tau,\omega). \tag{3.4}$$

(iii) Given $\varepsilon \le 1$. Let us prove the precompactness of $\bigcup\limits_{\varepsilon\le 1} \mathcal{A}_\varepsilon(\tau,\omega)$ for every $\tau \in \mathbb{R}$ and $\omega \in \Omega$. For one thing, by (3.4), Lemma 4.3 in [15] and the invariance of $\mathcal{A}_\varepsilon(\tau,\omega)$, for every $\eta > 0, \varepsilon > 0, \tau \in \mathbb{R}$ and $\omega \in \Omega$, there exist $T = T(\tau,\omega,E_1,\varepsilon,\eta) > 0$ and $K = K(\tau,\omega,\varepsilon,\eta) \ge 1$, such that for all $t \ge T,\ k \ge K$, the solution $\varphi^{(\varepsilon)}$ to (3.1) satisfies

$$\sup_{\varphi^{(\varepsilon)} \in \bigcup\limits_{\varepsilon\le 1} \mathcal{A}_\varepsilon(\tau,\omega)} \big\|\varphi^{(\varepsilon)}\big(\tau, \tau - t, \theta_{-\tau}\omega, \varphi_0^{(\varepsilon)}\big)\big\|^2_{E(\mathbb{R}^n\setminus\mathbb{B}_k)} \le \eta.$$

For another thing, by (3.4) we find that the set $\bigcup\limits_{\varepsilon\le 1} \mathcal{A}_\varepsilon(\tau,\omega)$ is precompact in $E(\mathbb{B}_k)$ and hence $\bigcup\limits_{\varepsilon\le 1} \mathcal{A}_\varepsilon(\tau,\omega)$ is precompact in $E(\mathbb{R}^n)$.

(iv) Let $\varphi^{(0)}=(u^{(0)},v^{(0)})$ be a solution to (3.2) with the initial data $\varphi_0^{(0)}=(u_0^{(0)},v_0^{(0)})$, and $U=u^{(\varepsilon)}-u^{(0)}$, $V=v^{(\varepsilon)}-v^{(0)}$. It follows from (3.1) and (3.2) that

$$\begin{cases}\frac{dU}{dt}+\delta U-V=\varepsilon z(\theta_t\omega)U+\varepsilon z(\theta_t\omega)u^{(0)},\\ \frac{dV}{dt}-\delta V+(\lambda+\delta^2+A)U+f\big(x,u^{(\varepsilon)}\big)-f\big(x,u^{(0)}\big)\\ =h\big(v^{(0)}-\delta u^{(0)}\big)-h\big(v^{(\varepsilon)}+\varepsilon u^{(\varepsilon)}z(\theta_t\omega)-\delta u^{(\varepsilon)}\big)-\varepsilon z(\theta_t\omega)V\\ -\varepsilon z(\theta_t\omega)v^{(0)}+\varepsilon(3\delta-\varepsilon z(\theta_t\omega))z(\theta_t\omega)U+\varepsilon(3\delta-\varepsilon z(\theta_t\omega))z(\theta_t\omega)u^{(0)},\\ U(x,\tau,\tau)=U_0(x),\quad V(x,\tau,\tau)=V_0(x).\end{cases}\tag{3.5}$$

Taking the inner product of the second equation of (3.5) with V in $L^2(\mathbb{R}^n)$, and then using the first equation of (3.5) to simplify the resulting equality, we obtain

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|V\|^2-\delta\|V\|^2+(\lambda+\delta^2)(U,V)+(AU,V)+\Big(f(x,u^{(\varepsilon)})-f(x,u^{(0)}),V\Big)\\ =&-\Big(h\big(v^{(\varepsilon)}+\varepsilon u^{(\varepsilon)}z(\theta_t\omega)-\delta u^{(\varepsilon)}\big)-h\big(v^{(0)}-\delta u^{(0)}\big),V\Big)\Big(\varepsilon z(\theta_t\omega)V\\ &-\varepsilon z(\theta_t\omega)v^{(0)}+\varepsilon(3\delta-\varepsilon z(\theta_t\omega))z(\theta_t\omega)U+\varepsilon(3\delta-\varepsilon z(\theta_t\omega))z(\theta_t\omega)u^{(0)},V\Big).\end{aligned}\tag{3.6}$$

Similar to Lemma 4.1 in [15], we now estimate the terms in (3.6) as follows:

$$\begin{aligned}&-\Big(h\big(v^{(\varepsilon)}+\varepsilon u^{(\varepsilon)}z(\theta_t\omega)-\delta u^{(\varepsilon)}\big)-h\big(v^{(0)}-\delta u^{(0)}\big),V\Big)\\ \le&-\beta_1\|V\|^2+h'(\vartheta)\delta(U,V)-h'(\vartheta)\varepsilon z(\theta_t\omega)(U,V)-h'(\vartheta)\varepsilon z(\theta_t\omega)(u^{(0)},V),\end{aligned}\tag{3.7}$$

$$\begin{aligned}h'(\vartheta)\delta(U,V)\le&\frac{\beta_2\delta}{2}\frac{d}{dt}\|U\|^2+\beta_2\delta^2\|U\|^2-h'(\vartheta)\delta\varepsilon z(\theta_t\omega)\|U\|^2\\ &+h'(\vartheta)\delta\varepsilon z(\theta_t\omega)(U,u^{(0)}),\end{aligned}\tag{3.8}$$

$$\begin{aligned}(\lambda+\delta^2)(U,V)=&\frac{\lambda+\delta^2}{2}\frac{d}{dt}\|U\|^2+\delta(\lambda+\delta^2)\|U\|^2-(\lambda+\delta^2)\varepsilon z(\theta_t\omega)\|U\|^2\\ &+(\lambda+\delta^2)\varepsilon z(\theta_t\omega)(U,u^{(0)}),\end{aligned}\tag{3.9}$$

$$(AU,V)=\frac{1}{2}\frac{d}{dt}\|\Delta U\|^2+\delta\|\Delta U\|^2-\varepsilon z(\theta_t\omega)\|\Delta U\|^2+\varepsilon z(\theta_t\omega)(U,\Delta^2u^{(0)}),\tag{3.10}$$

$$\begin{aligned}&-\Big(f(x,u^{(\varepsilon)})-f(x,u^{(0)}),V\Big)\\ &\le\beta\|U\|\|V\|\le c\big(\|V\|^2+(\lambda+\delta^2-\beta_2\delta)\|U\|^2\big).\end{aligned}\tag{3.11}$$

Substitute (3.7)-(3.11) into (3.6) to obtain

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\big(\|V\|^2+(\lambda+\delta^2-\beta_2\delta)\|U\|^2+\|\Delta U\|^2\big)\\ &+(\beta_1-\delta)\|V\|^2+\delta(\lambda+\delta^2-\beta_2\delta)\|U\|^2+\delta\|\Delta U\|^2\\ \le& c\big(\|V\|^2+(\lambda+\delta^2-\beta_2\delta)\|U\|^2+\|\Delta U\|^2\big)+(\lambda+\delta^2)\varepsilon z(\theta_t\omega)\|U\|^2\\ &-(\lambda+\delta^2)\varepsilon z(\theta_t\omega)(U,u^{(0)})+\varepsilon z(\theta_t\omega)\|\Delta U\|^2-\varepsilon z(\theta_t\omega)(U,\Delta^2u^{(0)})\end{aligned}$$

$$
\begin{aligned}
&- h'(\vartheta)\delta\varepsilon z(\theta_t\omega)\|U\|^2 + h'(\vartheta)\delta\varepsilon z(\theta_t\omega)(U,u^{(0)}) - h'(\vartheta)\varepsilon z(\theta_t\omega)(U,V)\\
&- h'(\vartheta)\varepsilon z(\theta_t\omega)(u^{(0)},V) - \Big(\varepsilon z(\theta_t\omega)V - \varepsilon z(\theta_t\omega)v^{(0)}\\
&+ \varepsilon(3\delta - \varepsilon z(\theta_t\omega))z(\theta_t\omega)U + \varepsilon(3\delta - \varepsilon z(\theta_t\omega))z(\theta_t\omega)u^{(0)}, V\Big). \qquad (3.12)
\end{aligned}
$$

Thanks to Young's inequality, we find that from the second term to the last term on the right hand side of (3.12) are controlled by $|\varepsilon|c(1+|z(\theta_t\omega)|^2)(\|U\|^2_{H^2(\mathbb{R}^n)} + \|V\|^2 + \|u^{(0)}\|^2_{H^2(\mathbb{R}^n)} + \|v^{(0)}\|^2)$, which along with (3.12) implies

$$
\begin{aligned}
&\frac{d}{dt}\big(\|V\|^2 + (\lambda+\delta^2-\beta_2\delta)\|U\|^2 + \|\Delta U\|^2\big)\\
\le& c\big(\|V\|^2 + (\lambda+\delta^2-\beta_2\delta)\|U\|^2 + \|\Delta U\|^2\big)\\
&+ |\varepsilon|c(1+|z(\theta_t\omega)|^2)(\|U\|^2_{H^2(\mathbb{R}^n)} + \|V\|^2 + \|u^{(0)}\|^2_{H^2(\mathbb{R}^n)} + \|v^{(0)}\|^2). \qquad (3.13)
\end{aligned}
$$

Applying Lemma 4.1 in [15], there exists a constant $c_0 = c_0(\tau,\omega,R,T) > 0$ such that for all $t \ge T$,

$$
\|u^{(0)}\|^2_{H^2(\mathbb{R}^n)} + \|v^{(0)}\|^2 \le c_0. \qquad (3.14)
$$

Together with (3.13) and (3.14) we get

$$
\begin{aligned}
&\frac{d}{dt}\big(\|V\|^2 + (\lambda+\delta^2-\beta_2\delta)\|U\|^2 + \|\Delta U\|^2\big)\\
\le& c\big(\|V\|^2 + (\lambda+\delta^2-\beta_2\delta)\|U\|^2 + \|\Delta U\|^2\big) + |\varepsilon|c(1+|z(\theta_t\omega)|^2). \qquad (3.15)
\end{aligned}
$$

Therefore, applying the Gronwall inequality to (3.15) over (τ,t), we have

$$
\begin{aligned}
&\|u^{(\varepsilon)}(t,\tau,\omega,u_0^{(\varepsilon)}) - u^{(0)}(t,\tau,u_0^{(0)})\|^2_{H^2(\mathbb{R}^n)} + \|v^{(\varepsilon)}(t,\tau,\omega,v_0^{(\varepsilon)}) - v^{(0)}(t,\tau,v_0^{(0)})\|^2_{L^2(\mathbb{R}^n)}\\
\le& ce^{c(t-\tau)}(\|u_0^{(\varepsilon)} - u_0^{(0)}\|^2_{H^2(\mathbb{R}^n)} + \|v_0^{(\varepsilon)} - v_0^{(0)}\|^2_{L^2(\mathbb{R}^n)})\\
&+ \varepsilon c\int_\tau^t e^{c(t-s)}(1+|z(\theta_s\omega)|^2)ds,
\end{aligned}
$$

which along with (i),(ii), (iii) and Theorem 2.1 complete the proof. □

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