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Dynamics of a Tick-borne Disease Model with Birth Pulse and Pesticide Pulse at Different Moments*

Shuyu Yan[1] and Xue Zhang[1,†]

Abstract Tick-borne diseases pose a potential risk to public health, which is influenced by the stage structure and seasonal reproduction of tick populations. In this paper, a model that explains the transmission dynamics of pathogens among ticks and hosts is formulated and analyzed, considering birth pulse and pesticide pulse on tick population at different moments. Using the stroboscopic mapping for the disease-free system, we prove a globally asymptotically stable positive periodic solution exists when the pulsed pesticide spraying intensity is less than a critical threshold. Applying the comparison theorem for the impulsive differential system, the conditions for global attraction of the disease-free periodic solution to the investigated system are given. Moreover, we demonstrate the persistence of the studied system and give numerical simulations to verify it. Ultimately, we discuss the case with multiple pesticide sprays and conclude that fewer sprays are more favorable for disease extinction.

Keywords Tick-borne disease, stage structure, double pulse, stability, permanence

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1. Introduction

Tick-borne diseases, like tick-borne encephalitis(TBE) [1], tick-borne relapsing fever [2], Lyme disease [3], have become a major problem for woodland populations living in many parts of Europe, the former Soviet Union and North America. This problem has become increasingly serious over the last 20 years as people spend more time outdoors [4]. Lyme disease is an important infectious disease in the United States, with more than 40,000 cases reported each year, but eight to ten times as many people are actually infected [5]. Lyme disease has a high incidence among forest and field workers [6]. The clinical manifestations include meningitis, encephalitis, neuritis, motor and sensory neuritis and other neurological damage [7]. During the initial phase of the disease, chronic erythema migrans affect the skin, while in

†the corresponding author.
Email address: zhangxue@mail.neu.edu.cn (X. Zhang), shuyuyancc@163.com (S. Yan)
1College of Sciences, Northeastern University, Shenyang, Liaoning 110819, China
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the intermediate stage, it leads to lesions in the nerves, heart, or joints, and the treatment for nervous system damage has not been effective thus far.

Tick-borne diseases are spread through ticks bites, which are parasitic arthropods that commonly attach themselves to various mammals, including humans. Ticks act as carriers or vectors for several diseases affecting humans and animals [8]. The typical life cycle of ticks spans a duration of 2 to 3 years in their natural habitat, during which they undergo four distinct stages of development: the egg, larva, nymph, and adult phases [6]. Except for the egg stage, each other stage contains three sub-stages: searching for a host, feeding blood and engorgement [9]. The larvae and nymphs are normally parasitic on small rodents (like birds and mice) while the adults mainly feed on medium and large mammals (like sheep and deer) [10].

We can control the prevalence of tick-borne diseases in several ways, such as vaccination [11], biological control [12] and chemical control (pesticide spraying). However, there is currently only one vaccine for TBE, which is not permanent. People need three intramuscular vaccinations (primary series) on day 0, months 1-3 and months 5-12, with a booster shot after 3 years and every 5 years thereafter [13]. Irregular vaccination schedules will result in slightly less effective vaccines [14]. Since most countries do not include tick-borne vaccines in health insurance, the vaccine coverage is low. For biological control, it is impractical to introduce a large number of natural enemies to prey on ticks because of the small number of species of ticks, and it is easy to destroy the original biological chain. Therefore, chemical control i.e., spraying pesticides on animals, grasses and forests to kill ticks, is the most important means of tick control.

At present, a number of modeling efforts have been conducted by many scholars to research tick population dynamics and the spread of tick-borne diseases. Tosato et al. [15] modeled disease transmission in ticks and rodents, and studied the effects of two means of control: insect repellents and acaricides. Analysis suggests that host control with chemical insecticides in areas with rapidly growing tick populations may prolong the duration of the disease and even allow the disease to spread in disease-free areas. Rosà and Pugliese [4] discussed the influence of host population density and structure on the spread of tick-borne diseases, concluding that low host population densities do not sustain cycles of infection, while high host population densities may dilute pathogen transmission. Egyed et al. [16] analyzed the effect of season on Ixodes ricinus activity and the prevalence of infection with major tick-borne disease pathogens in Hungary. Lou and Wu [17] gave basic regeneration numbers of complex disease systems for twelve developmental stages of tick populations.

Recently, a number of researchers have developed epidemiological and population dynamics models with impulsive differential equations [18,19]. Pulsed differential equations exist in almost every field of modern science and have been applied in numerous studies. Wang et al. [20] introduced the impulse control approach to pest biological control by considering the global nature of epidemiological models and bifurcating models with periodic impulse effects. Li et al. [21] constructed a generalized predator-prey system containing a nonlinear pulse to study the effects of pulse-released predators on prey population outbreaks. Tang and Chen [22] presented a stage-structured population system with different fertility function pulses, comparing the dynamic behavior of Ricker functional and Beverton-Holt functional fertility pulses. Sisodiya et al. [23] proposed a model of mosquito-borne disease that considers three means of impulse control at the same moment, namely vaccination, pesticides, and adulticides. However, tick-borne diseases have not been seen in articles with pulse situations. Considering the effects of tick birth and pesticide spraying are controlled both in pulsatile manners, it is necessary to propose a double-pulse tick-borne disease model.

The paper is structured as follows: In section 2, a model is introduced and the meaning of each parameter is given. In section 3, definitions and lemmas are given to provide the basis for later proofs. In section 4, a positive periodic solution exists and conditions for its global asymptotic stability are given, using the stroboscopic mapping of the disease-free subsystem. In section 5, conditions for the global attraction of the investigated model are given, based on the comparison theorem of the impulse differential equation. In section 6, the persistence of the investigated model is presented. Finally, we give numerical simulations to conclude and direct for future work.

2. The model formulation

A model is developed, which explains the transmission dynamics of pathogens among ticks and hosts, considering birth pulse and pesticide pulse on tick population at different moments.

$$\begin{cases} \left.\begin{aligned} &(L_t)' = -\beta^L H_t L_t - \mu^T L_t, \\ &(N_t^S)' = \eta^L \beta^L [H_t^S + (1-p^N) H_t^I] L_t - \beta^N H_t N_t^S - \mu^T N_t^S, \\ &(N_t^I)' = \eta^L \beta^L p^N H_t^I L_t - \beta^N H_t N_t^I - \mu^T N_t^I, \\ &(H_t^S)' = \Lambda - \beta^N p^H N_t^I H_t^S - \mu^H H_t^S, \\ &(H_t^I)' = \beta^N p^H N_t^I H_t^S - \mu^H H_t^I - \alpha^H H_t^I, \end{aligned}\right\} \begin{aligned} &t \neq (n+l)T, \\ &t \neq (n+1)T, \end{aligned} \\ \left.\begin{aligned} &L_{t^+} = (1-q) L_t, \\ &N_{t^+}^S = (1-q) N_t^S, \\ &N_{t^+}^I = (1-q) N_t^I, \end{aligned}\right\} t = (n+l)T,\ n = 1, 2\cdots, \\ \left.\begin{aligned} &L_{t^+} = L_t + \frac{r\eta^N \beta^N H_t N_t}{1 + w\eta^N \beta^N H_t N_t}, \\ &N_{t^+}^S = N_t^S, \\ &N_{t^+}^I = N_t^I, \end{aligned}\right\} t = (n+1)T,\ n = 1, 2\cdots. \end{cases} \tag{2.1}$$

Consider L_t, N_t, H_t as variables representing the population numbers of larvae, nymphs and hosts at time t, respectively. The nymphs N_t can be further categorized into two groups: susceptible individuals, denoted as N_t^S, and infected individuals, denoted as N_t^I. Thus,

$$N_t = N_t^S + N_t^I.$$

The host population H_t can be further categorized into two groups: the susceptible hosts H_t^S and the infected hosts H_t^I. Thus,

$$H_t = H_t^S + H_t^I.$$

T is the period of birth pulse and pesticide pulse. $0 < q < 1$ denotes the death rate due to pesticide in ticks at $t = (n+l)T$, $0 < l < 1$. $\frac{r\eta^N \beta^N H_t N_t}{1+w\eta^N \beta^N H_t N_t}$ denotes the pulse recruitment effect of larvae at $t = (n+1)T$, where r is the number of eggs per adult female tick surviving to reach larval stage, and η^N is the conversion rate from nymphs to adults. Define $F_{t^+} = \lim\limits_{x\to t^+} F_x$.

All parameters in model (2.1) are positive constants. Detailed explanations and specific ranges of parameters are listed in Table 1.

Table 1. Model parameters description

Parameter	Description	Value range	Source
β^L	Biting rate of larvae	[0.025, 0.105]	[24]
β^N	Biting rate of nymphs	[0.025, 0.105]	[24]
η^L	Conversion rate from larvae to nymphs	[0.35, 0.5]	[25]
η^N	Conversion rate from nymphs to adults	[0.35, 0.5]	[25]
μ^T	Natural mortality rate of ticks	[0.007, 0.087]	[25]
μ^H	Natural mortality rate of hosts	[0, 1]	Assumed
Λ	Recruitment rate due to birth in hosts	[0.5, 0.7]	[26]
α^H	Mortality rate due to disease in infected hosts	[0.0083, 0.33]	[27]
q	Mortality rate due to pesticide in ticks	[0, 1]	Assumed
r	Number of eggs per adult female tick surviving to reach larval stage	200	[28]
w	Density dependent rate of adults	[0, 200]	Assumed
p^N	Infection probability from infectious hosts to suscpectible nymphs	[0, 0.9]	[25]
p^H	Infection probability from infectious nymphs to suscpectible hosts	[0, 0.9]	[25]

3. Preliminaries

Some definitions and lemmas are introduced for convenience.

From system (2.1), we obtain

$$(H_t)' = \Lambda - \mu^H H_t - \alpha^H H_t^I,$$

then

$$\Lambda - (\mu^H + \alpha^H) H_t \le (H_t)' \le \Lambda - \mu^H H_t.$$

We derive that

$$\underline{H} \triangleq \frac{\Lambda}{\mu^H + \alpha^H} \le \lim_{t\to\infty} Inf\, H_t \le \lim_{t\to\infty} Sup\, H_t \le \frac{\Lambda}{\mu^H} \triangleq \overline{H}. \tag{3.1}$$

Lemma 3.1. *When t is large enough, there exists a constant $M = \frac{r}{w\cdot(1-e^{-\mu^T \eta^L T})} + \zeta$, such that $L_t \le \frac{M}{\eta^L}$, $N_t^S \le M$ and $N_t^I \le M$, where $\zeta > 0$ is sufficiently small.*

Proof. By defining $R_t = \eta^L L_t + N_t^S + N_t^I$, we get

$$\begin{aligned}(R_t)' &= -\eta^L \mu^T L_t - (\beta^N H_t + \mu^T) N_t \\ &\le -\eta^L \mu^T L_t - \mu^T N_t \\ &\le -\eta^L \mu^T R_t.\end{aligned}$$

It can be seen that when $t = (n+l)T$,

$$R_{t^+} = (1-q)R_t \le R_t.$$

When $t = (n+1)T$,

$$\begin{aligned}R_{t^+} &= R_t + \frac{r\eta^N \beta^N N_t H_t}{1 + w\eta^N \beta^N N_t H_t} \\ &= R_t + \frac{r}{w}\left(1 - \frac{1}{1 + w\eta^N \beta^N N_t H_t}\right) \\ &\le R_t + \frac{r}{w}.\end{aligned}$$

For the comparison system

$$\begin{cases} (X_{1_t})' = -\mu^T \eta^L X_{1_t}, & t \ne nT, \\ X_{1_{t^+}} = X_{1_t} + \dfrac{r}{w}, & t = nT, \end{cases}$$

there exists a global asymptotically stable periodic solution

$$\widetilde{X_{1_t}} = \frac{r}{w} \cdot \frac{e^{-\eta^L \eta^L_{(t-nT)}}}{1 - e^{-\mu^T \eta^L T}},\ nT < t \le (n+1)T.$$

For sufficiently small $\zeta > 0$ and large enough t, it holds that

$$R_t \le X_{1_t} < \widetilde{X_{1_t}} + \zeta < \frac{r}{w} \cdot \frac{1}{1 - e^{-\mu^T \eta^L T}} + \zeta \triangleq M.$$

Based on the definition of R_t, we derive

$$L_t \le \frac{M}{\eta^L}, \quad N_t^S \le M, \quad N_t^I \le M,$$

when t is large enough. □

4. Disease-free subsystem

If $N_t^I = H_t^I = 0$, the disease-free subsystem of system (2.1) is defined as:

$$\begin{cases} \left.\begin{aligned} &(L_t)' = -\beta^L H_t^S L_t - \mu^T L_t, \\ &(N_t^S)' = \eta^L \beta^L H_t^S L_t - \beta^N H_t^S N_t^S - \mu^T N_t^S, \\ &(H_t^S)' = \Lambda - \mu^H H_t^S, \end{aligned}\right\} \begin{aligned} &t \ne (n+l)T, \\ &t \ne (n+1)T, \end{aligned} \\ \left.\begin{aligned} &L_{t^+} = (1-q)L_t, \\ &N_{t^+}^S = (1-q)N_t^S, \end{aligned}\right\} t = (n+l)T, \\ \left.\begin{aligned} &L_{t^+} = L_t + \frac{r\eta^N \beta^N H_t^S N_t^S}{1 + w\eta^N \beta^N H_t^S N_t^S}, \\ &N_{t^+}^S = N_t^S, \end{aligned}\right\} t = (n+1)T. \end{cases} \tag{4.1}$$

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Since $(H_t^S)' = \Lambda - \mu^H H_t^S$, then

$$H_t^S = \frac{\Lambda}{\mu^H} + (H_{0+}^S - \frac{\Lambda}{\mu^H})e^{-\mu^H t} \to \frac{\Lambda}{\mu^H}, \quad as \quad t \to \infty.$$

In terms of $N_t^S = N_t$ and $H_t^S = H_t$, system (4.1) and the following system are equivalent when we consider their dynamical properties.

$$\begin{cases} \left.\begin{aligned} (L_t)' &= -aL_t, \\ (N_t)' &= bL_t - cN_t, \end{aligned}\right\} t \neq (n+l)T, \ t \neq (n+1)T, \\ \left.\begin{aligned} L_{t^+} &= (1-q)L_t, \\ N_{t^+} &= (1-q)N_t, \end{aligned}\right\} t = (n+l)T, \\ \left.\begin{aligned} L_{t^+} &= L_t + \frac{\overline{r}N_t}{1+\overline{w}N_t}, \\ N_{t^+} &= N_t, \end{aligned}\right\} t = (n+1)T, \end{cases} \tag{4.2}$$

where $a = \beta^L\overline{H} + \mu^T$, $b = \eta^L\beta^L\overline{H}$, $c = \beta^N\overline{H} + \mu^T$, $\overline{r} = r\eta^N\beta^N\overline{H}$, and $\overline{w} = w\eta^N\beta^N\overline{H}$.

System (4.2) has the following analytic solution between pluses:

$$L_t = \begin{cases} e^{-a(t-nT)}L_{nT^+}, & nT < t \le (n+l)T, \\ e^{-a(t-(n+l)T)}L_{(n+l)T^+}, & (n+l)T < t \le (n+1)T. \end{cases}$$

$$N_t = \begin{cases} \frac{b}{c-a}\left(e^{-a(t-nT)} - e^{-c(t-nT)}\right)L_{nT^+} + e^{-c(t-nT)}N_{nT^+}, \\ \qquad nT < t \le (n+l)T, \\ \frac{b}{c-a}\left(e^{-a(t-(n+l)T)} - e^{-c(t-(n+l)T)}\right)L_{(n+l)T^+} + e^{-c(t-(n+l)T)}N_{(n+l)T^+}, \\ \qquad (n+l)T < t \le (n+1)T. \end{cases}$$

The stroboscopic map of system(4.2) is as follows:

$$\begin{cases} L_{(n+1)T^+} = CL_{nT^+} + \frac{\overline{r}(AL_{nT^+} + BN_{nT^+})}{1+\overline{w}(AL_{nT^+} + BN_{nT^+})}, \\ N_{(n+1)T^+} = AL_{nT^+} + BN_{nT^+}, \end{cases} \tag{4.3}$$

where $\xi = \frac{e^{-aT}-e^{-cT}}{c-a} > 0$, $A = (1-q)b\frac{e^{-aT}-e^{-cT}}{c-a} = (1-q)b\xi$, $B = (1-q)e^{-cT}$, and $C = (1-q)e^{-aT}$.

Make a notation as

$$\begin{aligned} q^* =& \frac{1}{2}e^{(a+c)T}\Big[-(e^{-aT} + e^{-cT} + \overline{r}b\xi - 2e^{-(a+c)T}) \\ &+ \sqrt{(e^{-aT} + e^{-cT} + \overline{r}b\xi)^2 - 4e^{-(a+c)T}}\Big]. \end{aligned}$$

It is clear to see the following equivalent relation

$$q < q^* \iff \overline{r}A - (1-B)(1-C) > 0,$$

which indicates that the two fixed points of stroboscopic map (4.3) are obtained as $Q_1(0,\ 0)$ and $Q_2(L^*,\ N^*)$, where

$$\left.\begin{aligned} L^* &= \left(\frac{\overline{r}}{1-C} - \frac{1-B}{A}\right)\frac{1}{\overline{w}},\\ N^* &= \left(\frac{\overline{r}A}{(1-B)(1-C)} - 1\right)\frac{1}{\overline{w}}, \end{aligned}\right\} q < q^*. \tag{4.4}$$

Theorem 4.1. *(i) If $q > q^*$, $Q_1(0,\ 0)$ is globally asymptotically stable;*
(ii) If $q < q^*$, $Q_2(L^*,\ N^*)$ is globally asymptotically stable.

Proof. We represent the linear form of (4.3) as follows:

$$\begin{pmatrix} L_{(n+1)T^+} \\ N_{(n+1)T^+} \end{pmatrix} = G \begin{pmatrix} L_{nT^+} \\ N_{nT^+} \end{pmatrix}. \tag{4.5}$$

Linear system(4.5) determines the dynamical properties of $Q_1(0,\ 0)$ and $Q_2(L^*,\ N^*)$. The stabilities of $Q_1(0,\ 0)$ and $Q_2(L^*,\ N^*)$ depend on the relationship between the magnitude of G eigenvalues and 1. If G meets the Jury criterion [29] , then the eigenvalues of G are less than 1.
(i) If $q > q^*$, namely $(1-B)(1-C) - A\overline{r} > 0$, system(4.3) has a unique fixed point $Q_1(0,\ 0)$,

$$G = \begin{pmatrix} C + A\overline{r} & B\overline{r} \\ A & B \end{pmatrix},$$

and

$$\begin{aligned} 1 - \mathrm{tr}G + \det G &= 1 - (C + A\overline{r} + B) + (BC + AB\overline{r} - AB\overline{r}) \\ &= (1-B)(1-C) - A\overline{r} > 0. \end{aligned}$$

Thus, $Q_1(0,\ 0)$ is locally stable. Since (4.2) is a linear differential system, $Q_1(0,\ 0)$ is globally asymptotically stable.
(ii) If $q < q^*$, namely $A\overline{r} - (1-B)(1-C) > 0$, then $Q_1(0,\ 0)$ is unstable. For $Ar' - (1-B)(1-C) > 0$, $Q_2(L^*,\ N^*)$ exists, and

$$G = \begin{pmatrix} C + \dfrac{A\overline{r}}{[1+\overline{w}(AL^* + BN^*)]^2} & \dfrac{B\overline{r}}{[1+\overline{w}(AL^* + BN^*)]^2} \\ A & B \end{pmatrix},$$

$$\begin{aligned} &1 - \mathrm{tr}G + \det G \\ =&1 - [C + \frac{A\overline{r}}{[1+\overline{w}(AL^* + BN^*)]^2} + B] + BC \\ &+ \frac{AB\overline{r}}{[1+\overline{w}(AL^* + BN^*)]^2} - \frac{AB\overline{r}}{[1+\overline{w}(AL^* + BN^*)]^2} \\ =&(1 - B - C + BC) - \frac{A\overline{r}}{[1+\overline{w}(AL^* + BN^*)]^2} \end{aligned}$$

$$=(1-B)(1-C)-\frac{A\overline{r}}{[1+(\frac{\overline{r}}{1-C}-\frac{1-B}{A})A+(\frac{\overline{r}A}{(1-B)(1-C)}-1)B]^2}$$
$$=\frac{(1-B)(1-C)\left[A\overline{r}-(1-B)(1-C)\right]}{A\overline{r}}>0.$$

Similarly, $Q_2(L^*,\ N^*)$ is locally stable. Since (4.2) is a linear differential system, $Q_2(L^*,\ N^*)$ is globally asymptotically stable. □

Theorem 4.2. *(i) If $q > q^*$, there exists a globally asymptotically stable trivial periodic solution $(0,\ 0)$ for system (4.2);*
(ii) If $q < q^*$, there exists a globally asymptotically stable nontrivial periodic solution $(\widetilde{L_t},\ \widetilde{N_t})$ for system (4.2), where

$$\widetilde{L_t}=\begin{cases} e^{-a(t-nT)}L^*, & nT<t\le(n+l)T,\\ (1-q)e^{-a(t-nT)}L^*, & (n+l)T<t\le(n+1)T,\end{cases}$$
$$\widetilde{N_t}=\begin{cases} \frac{b}{a-c}\left(e^{-c(t-nT)}-e^{-a(t-nT)}\right)L^*+e^{-c(t-nT)}N^*, \\ \qquad nT<t\le(n+l)T,\\ (1-q)\left[\frac{b}{a-c}\left(e^{-c(t-nT)}-e^{-a(t-nT)}\right)L^*+e^{-c(t-nT)}N^*\right], \\ \qquad (n+l)T<t\le(n+1)T,\end{cases} \tag{4.6}$$

where L^* and N^* are determined as in (4.4).

We apply numerical simulation to examine the global asymptotic stability of the periodic solution of system (4.2). We fix the following parameters: $T=1,\ l=0.5,\ r=200,\ w=100,\ \beta^L=0.045,\ u^T=0.2,\ \eta^L=0.35,\ \Lambda=0.5,\ u^H=0.1,\ \beta^N=0.04,\ \eta^N=0.4$, then $q=0.5>q*=0.4822$, the larvae and nymphs go to extinction. Its dynamical behaviors are depicted in Figure $1(a)$ and Figure $1(b)$. If $q=0.3(<q*)$, the population of ticks is permanent. Its dynamics behaviours are depicted in Figure $1(c)$ and Figure $1(d)$.

5. Global attractivity of disease-free periodic solution

It is clear from the above discussion that system (2.1) has a disease-free periodic solution $(\widetilde{L_t},\ \widetilde{N_t},\ 0,\ \overline{H},\ 0)$. Next, we will present that $(\widetilde{L_t},\ \widetilde{N_t},\ 0,\ \overline{H},\ 0)$ is globally attractive.

Theorem 5.1. *There exists a globally attractive periodic solution $(\widetilde{L_t},\ \widetilde{N_t},\ 0,\ \overline{H},\ 0)$ of system (2.1), if $q<q^*$ and*

$$\max\left\{\eta^L\beta^Lp^N(\frac{r}{w}\cdot\frac{1}{1-e^{-(\beta^L\underline{H}+\mu^T)T}}+\varepsilon_0),\ \beta^Np^H\overline{H}\right\}$$
$$<\min\left\{\mu^T+\beta^N\underline{H},\ \mu^H+\alpha^H\right\},$$

where $\varepsilon_0>0$ is sufficiently small.

Proof. It holds from system (2.1) that

$$\begin{cases} (L_t)' \leq -(\beta^L \underline{H} + \mu^T) L_t, & t \neq (n+l)T, \ t \neq (n+1)T, \\ L_{t^+} \leq L_t, & t = (n+l)T, \\ L_{t^+} \leq L_t + \dfrac{r}{w}, & t = (n+1)T. \end{cases}$$

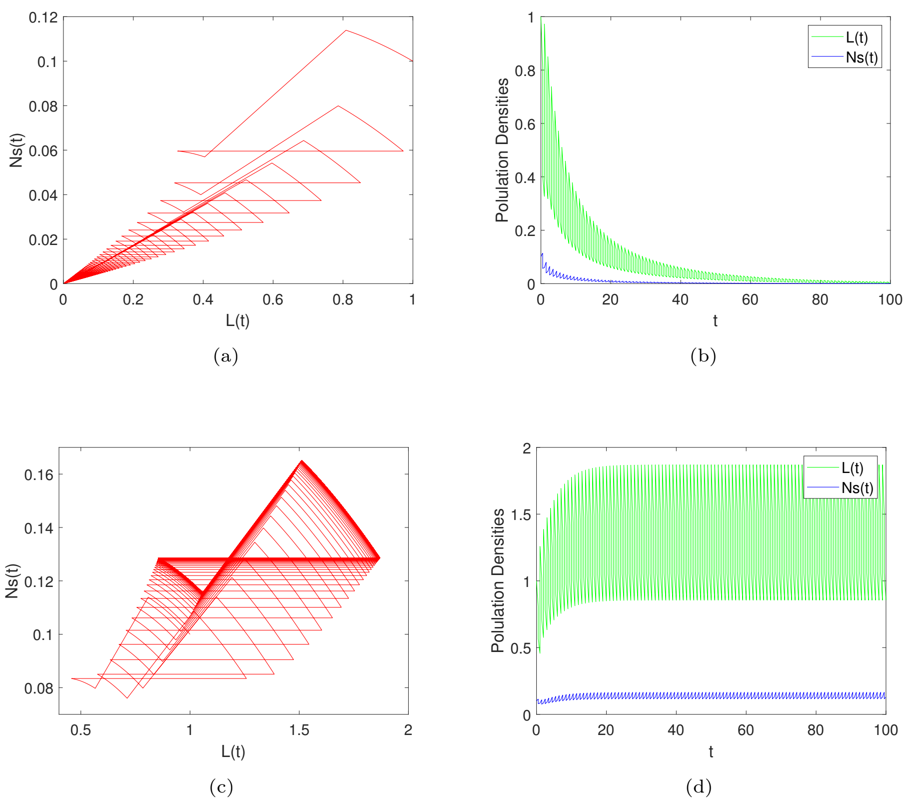

Figure 1. The phase portrait(a) and time-series diagram(b) of trivial periodic solution $(0,0)$ of system (4.2) with initial conditions $L_0 = 1, N_0^S = 0.1$ and $q = 0.5$; the phase portrait(c) and time-series diagram(d) of nontrivial periodic solution (L^*, N^*) of system (4.2) with $q = 0.3$.

The comparison system is given by:

$$\begin{cases} (X_{2_t})' = -(\beta^L \underline{H} + \mu^T) X_{2_t}, & t \neq nT, \\ X_{2_{t^+}} = X_{2_t} + \dfrac{r}{w}, & t = nT. \end{cases} \tag{5.1}$$

We get the periodic solution to (5.1)

$$\widetilde{X_{2_t}} = \frac{r}{w} \cdot \frac{e^{-(\beta^L \underline{H} + \mu^T)(t - nT)}}{1 - e^{-(\beta^L \underline{H} + \mu^T)T}}, \quad nT < T \leq (n+1)T,$$

which is globally asymptotically stable, with an initial value

$$\widetilde{X_{2_{0^+}}} = \frac{r}{w} \cdot \frac{1}{1 - e^{-(\beta^L \underline{H} + \mu^T)T}} \ .$$

Hence for $\varepsilon_0 > 0$ sufficiently small, we get

$$L_t \leq X_{2_t} < \widetilde{X_{2_t}} + \varepsilon_0.$$

Thus, we derive

$$L_t < \widetilde{X_{2_t}} + \varepsilon_0 \leq \frac{r}{w} \cdot \frac{1}{1 - e^{-(\beta^L \underline{H} + \mu^T)T}} + \varepsilon_0 \triangleq M^L,$$

for all t large enough.

Now considering $V_t = N_t^I + H_t^I$, it holds that

$$\begin{aligned}(V_t)' &= (N_t^I)' + (H_t^I)' \\ &= [\eta^L \beta^L p^N H_t^I L_t - (\mu^T + \beta^N H_t) N_t^I] + [\beta^N p^H N_t^I H_t^S - (\mu^H + \alpha^H) H_t^I] \\ &\leq \eta^L \beta^L p^N M^L H_t^I - (\mu^T + \beta^N \underline{H}) N_t^I + \beta^N p^H \overline{H} N_t^I - (\mu^H + \alpha^H) H_t^I \\ &\leq (\theta - \varphi) V_t,\end{aligned}$$

where

$$\theta = \max\left\{\eta^L \beta^L p^N M^L,\ \beta^N p^H \overline{H}\right\}, \quad \varphi = \min\left\{\mu^T + \beta^N \underline{H},\ \mu^H + \alpha^H\right\}.$$

When $t = (n+l)T$,

$$V_{t^+} = N_{t^+}^I + H_{t^+}^I = (1-q) N_t^I + H_t^I \leq V_t.$$

Therefore, we derive

$$(V_t)' \leq (\theta - \varphi) V_t, \tag{5.2}$$

for t sufficiently large.

Integrating (5.2) on $(nT,\ (n+1)T]$ yields

$$V_{(n+1)T} \leq V_{nT^+} e^{(\theta - \varphi)T}.$$

We obtain $V_{nT} \leq V_{0^+} e^{(\theta-\varphi)nT}$, hence $V_{nT} \to 0$ as $t \to \infty$. From (5.2), we can derive that $0 < V_t \leq V_{nT}\ e^{(\theta-\varphi)(t-nT)}$ for $nT < t \leq (n+1)T$, thus $V_t \to 0$ as $t \to \infty$.

Considering the positivity of N_t^I and H_t^I, we obtain

$$\lim_{t \to \infty} V_t = 0\ ,$$

which is equivalent to

$$\lim_{t \to \infty} N_t^I = 0,\ \lim_{t \to \infty} H_t^I = 0\ .$$

Therefore, when t is large enough, for sufficiently small $\zeta_1,\ \zeta_2 > 0$, it holds that $0 < N_t^I < \zeta_1$, and $0 < H_t^I < \zeta_2$.

From system (2.1), we get

$$(H_t^S)' \geq \Lambda - (\beta^N p^H \zeta_1 + \mu^H) H_t^S. \tag{5.3}$$

Considering its comparison system

$$(X_{3_t})' = \Lambda - (\beta^N p^H \zeta_1 + \mu^H) X_{3_t},$$

we derive

$$X_{3_t} = \left(X_{3_{0+}} - \frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} \right) e^{-(\beta^N p^H \zeta_1 + \mu^H)t} + \frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} \ ,$$

then $X_{3_t} \to \frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H}$ as $t \to \infty$. Thus, for sufficiently small $\varepsilon_1 > 0$, it holds that

$$H_t^S \geq \frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} - \varepsilon_1. \tag{5.4}$$

According to (2.1), we obtain

$$H_t^S \leq \Lambda - \mu^H H_t^S.$$

Similar to (5.3), for sufficiently small $\varepsilon_2 > 0$, we derive

$$H_t^S \leq \frac{\Lambda}{\mu^H} + \varepsilon_2. \tag{5.5}$$

Combining (5.4) and (5.5), it holds that

$$\frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} - \varepsilon_1 \leq H_t^S \leq \frac{\Lambda}{\mu^H} + \varepsilon_2$$

when t is large enough. Let $\zeta_1, \varepsilon_1, \varepsilon_2 \to 0$. We get $H_t^S \to \frac{\Lambda}{\mu^H}$ as $t \to \infty$.

From system (2.1), we obtain

$$\begin{cases} \left.\begin{array}{l} (L_t)' \geq -(\beta^L \overline{H} + \mu^T) L_t, \\ (N_t^S)' \geq \eta^L \beta^L (\frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} - \varepsilon_1) L_t - (\beta^N \overline{H} + \mu^T) N_t^S, \end{array}\right\} \begin{array}{l} t \neq (n+l)T, \\ t \neq (n+1)T, \end{array} \\ \left.\begin{array}{l} L_{t^+} = (1-q) L_t, \\ N_{t^+}^S = (1-q) N_t^S, \end{array}\right\} t = (n+l)T, \\ \left.\begin{array}{l} L_{t^+} \geq L_t + \dfrac{r\eta^N \beta^N (\frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} - \varepsilon_1) N_t^S}{1 + w\eta^N \beta^N \overline{H} (N_t^S + \zeta_1)}, \\ N_{t^+}^S = N_t^S. \end{array}\right\} t = (n+1)T. \end{cases}$$

Comparison impulsive differential system is given by:

$$\begin{cases} \left.\begin{array}{l} (L_{1_t})' = -a_1 L_{1_t}, \\ (N_{1_t})' = b_1 L_{1_t} - c_1 N_{1_t}, \end{array}\right\} \begin{array}{l} t \neq (n+l)T, \\ t \neq (n+1)T, \end{array} \\ \left.\begin{array}{l} L_{1_{t^+}} = (1-q) L_{1_t}, \\ N_{1_{t^+}} = (1-q) N_{1_t}, \end{array}\right\} t = (n+l)T, \\ \left.\begin{array}{l} L_{1_{t^+}} = L_{1_t} + \dfrac{r_1 N_{1_t}}{1 + w_1 (N_{1_t} + \zeta_1)}, \\ N_{1_{t^+}} = N_{1_t}, \end{array}\right\} t = (n+1)T, \end{cases} \tag{5.6}$$

where $a_1 = \beta^L\overline{H} + \mu^T$, $b_1 = \eta^L\beta^L(\frac{\Lambda}{\beta^N p^H\zeta_1+\mu^H} - \varepsilon_1)$, $c_1 = \beta^N\overline{H} + \mu^T$, $r_1 = r\eta^N\beta^N(\frac{\Lambda}{\beta^N p^H\zeta_1+\mu^H} - \varepsilon_1)$, and $w_1 = w\eta^N\beta^N\overline{H}$.
Similar to (4.2), we obtain the periodic solution to (5.6) as follows:

$$\widetilde{L_{1_t}} = \begin{cases} e^{-a_1(t-nT)}L_1^*, & nT < t \le (n+l)T, \\ (1-q)e^{-a_1(t-nT)}L_1^*, & (n+l)T < t \le (n+1)T, \end{cases}$$
$$\widetilde{N_{1_t}} = \begin{cases} \frac{b_1}{a_1-c_1}\left(e^{-c_1(t-nT)} - e^{-a_1(t-nT)}\right)L_1^* + e^{-c_1(t-nT)}N_1^*, \\ \qquad nT < t \le (n+l)T, \\ (1-q)\left[\frac{b_1}{a_1-c_1}\left(e^{-c_1(t-nT)} - e^{-a_1(t-nT)}\right)L_1^* + e^{-c_1(t-nT)}N_1^*\right], \\ \qquad (n+l)T < t \le (n+1)T, \end{cases}$$

where

$$\left.\begin{aligned} L_1^* &= \frac{1}{w_1}\left(\frac{r_1}{1-C_1} - \frac{1-B_1}{A_1}\right) - \zeta_1, \\ N_1^* &= \frac{1}{w_1}\left(\frac{r_1A_1}{(1-B_1)(1-C_1)} - 1\right) - \frac{1-B_1}{A_1}\zeta_1, \end{aligned}\right\} q < q_1^*,$$

and

$$\xi_1 = \frac{e^{-a_1T} - e^{-c_1T}}{c_1 - a_1} > 0,\ A_1 = (1-q)b_1\xi_1,\ B_1 = (1-q)e^{-c_1T},\ C_1 = (1-q)e^{-a_1T},$$

$$\begin{aligned} q_1^* =& \frac{1}{2}e^{(a_1+c_1)T}\Big[-(e^{-a_1T} + e^{-c_1T} + r_1b_1\xi_1 - 2e^{-(a_1+c_1)T}) \\ &+ \sqrt{(e^{-a_1T} + e^{-c_1T} + r_1b_1\xi_1)^2 - 4e^{-(a_1+c_1)T}}\Big]. \end{aligned}$$

Therefore, for $\varepsilon_3, \varepsilon_4 > 0$ sufficiently small, we have

$$\begin{cases} \widetilde{L_{1_t}} - \varepsilon_3 < L_{1_t} \le L_t, \\ \widetilde{N_{1_t}} - \varepsilon_4 < N_{1_t} \le N_t^S, \end{cases} \tag{5.7}$$

when t is large enough.

From system (2.1), it holds that

$$\begin{cases} \left.\begin{aligned} &(L_t)' \le -[\beta^L(\frac{\Lambda}{\beta^N p^H\zeta_1+\mu^H} - \varepsilon_1) + \mu^T]L_t, \\ &(N_t^S)' \le \eta^L\beta^L\overline{H}L_t - [\beta^N(\frac{\Lambda}{\beta^N p^H\zeta_1+\mu^H} - \varepsilon_1) + \mu^T]N_t^S, \end{aligned}\right\} \begin{aligned} &t \ne (n+l)T, \\ &t \ne (n+1)T, \end{aligned} \\ \left.\begin{aligned} &L_{t^+} = (1-q)L_t, \\ &N_{t^+}^S = (1-q)N_t^S, \end{aligned}\right\} t = (n+l)T, \\ \left.\begin{aligned} &L_{t^+} \le L_t + \frac{r\eta^N\beta^N\overline{H}(N_t^S + \zeta_1)}{1 + w\eta^N\beta^N(\frac{\Lambda}{\beta^N p^H\zeta_1+\mu^H} - \varepsilon_1)N_t^S}, \\ &N_{t^+}^S = N_t^S. \end{aligned}\right\} t = (n+1)T. \end{cases} \tag{5.8}$$

The comparison system is given by:

$$\begin{cases} \left.\begin{aligned} (L_{2_t})' &= -a_2 L_{2_t}, \\ (N_{2_t})' &= b_2 L_{2_t} - c_2 N_{2_t}, \end{aligned}\right\} t \neq (n+l)T, t \neq (n+1)T, \\ \left.\begin{aligned} L_{2_{t^+}} &= (1-q)L_{2_t}, \\ N_{2_{t^+}} &= (1-q)N_{2_t}, \end{aligned}\right\} t = (n+l)T, \\ \left.\begin{aligned} L_{2_{t^+}} &= L_{2_t} + \frac{r_2(N_{2_t}+\zeta_1)}{1+w_2 N_{2_t}}, \\ N_{2_{t^+}} &= N_{2_t}, \end{aligned}\right\} t = (n+1)T, \end{cases} \tag{5.9}$$

where $a_2 = \beta^L(\frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} - \varepsilon_1) + \mu^T$, $b_2 = \eta^L \beta^L \overline{H}$, $c_2 = \beta^N(\frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} - \varepsilon_1) + \mu^T$, $r_2 = r\eta^N \beta^N \overline{H}$, and $w_2 = w\eta^N \beta^N(\frac{\Lambda}{\beta^N p^H \zeta_1 + \mu^H} - \varepsilon_1)$.

System (5.9) has the following stroboscopic map:

$$\begin{cases} L_{2_{(n+1)T^+}} = C_2 L_{2_{nT^+}} + \dfrac{r_2[A_2 L_{2_{nT^+}} + B_2 N_{2_{nT^+}} + \zeta_1]}{1 + w_2[A_2 L_{2_{nT^+}} + B_2 N_{2_{nT^+}}]}, \\ N_{2_{(n+1)T^+}} = A_2 L_{2_{nT^+}} + B_2 N_{2_{nT^+}}, \end{cases} \tag{5.10}$$

where
$\xi_2 = \frac{e^{-a_2 T} - e^{-c_2 T}}{c_2 - a_2} > 0$, $A_2 = (1-q)b_2\xi_2$, $B_2 = (1-q)e^{-c_2 T}$, $C_2 = (1-q)e^{-a_2 T}$.

The positive fixed point of (5.10) is as follows:

$$\left.\begin{aligned} L_2^* &= \frac{1}{2}\left[\frac{1}{w_2}\left(\frac{r_2 A_2}{(1-B_2)(1-C_2)} - 1\right) + \sqrt{\Delta}\right], \\ N_2^* &= \frac{1}{2}\left[\frac{1}{w_2}\left(\frac{r_2}{1-C_2} - \frac{1-B_2}{A_2}\right) + \frac{1-B_2}{A_2}\sqrt{\Delta}\right], \end{aligned}\right\} q < q_2^*,$$

where

$$\Delta = \left[\frac{1}{w_2}\left(\frac{r_2 A_2}{(1-B_2)(1-C_2)} - 1\right)\right]^2 + \frac{4 r_2 A \zeta_1}{w_2(1-B)(1-C)} > 0,$$

and

$$q_2^* = \frac{1}{2}e^{(a_2+c_2)T}[-(e^{-a_2 T} + e^{-c_2 T} + r_2 b_2 \xi_2 - 2e^{-(a_2+c_2)T}) \\ + \sqrt{(e^{-a_2 T} + e^{-c_2 T} + r_2 b_2 \xi_2)^2 - 4e^{-(a_2+c_2)T}}\ ].$$

System (5.9) has the following periodic solution:

$$\widetilde{L_{2_t}} = \begin{cases} e^{-a_2(t-nT)}L_2^*, & nT < t \leq (n+l)T, \\ (1-q)e^{-a_2(t-nT)}L_2^*, & (n+l)T < t \leq (n+1)T, \end{cases}$$

$$\widetilde{N_{2_t}} = \begin{cases} \dfrac{b_2}{a_2-c_2}\left(e^{-c_2(t-nT)} - e^{-a_2(t-nT)}\right)L_2^* + e^{-c_2(t-nT)}N_2^*, \\ \qquad nT < t \leq (n+l)T, \\ (1-q)\left[\dfrac{b_2}{a_2-c_2}\left(e^{-c_2(t-nT)} - e^{-a_2(t-nT)}\right)L_2^* + e^{-c_2(t-nT)}N_2^*\right], \\ \qquad (n+l)T < t \leq (n+1)T. \end{cases}$$

Therefore, for $\varepsilon_5, \varepsilon_6 > 0$ sufficiently small, we have

$$\begin{cases} L_t \leq L_{2_t} < \widetilde{L_{2_t}} + \varepsilon_5, \\ N_t^S \leq N_{2_t} < \widetilde{N_{2_t}} + \varepsilon_6, \end{cases} \tag{5.11}$$

for t large enough.
Combining (5.7) and (5.11), we obtain

$$\begin{cases} \widetilde{L_{1_t}} - \varepsilon_3 < L_t < \widetilde{L_{2_t}} + \varepsilon_5, \\ \widetilde{N_{1_t}} - \varepsilon_4 < N_t^S < \widetilde{N_{2_t}} + \varepsilon_6, \end{cases} \tag{5.12}$$

when t is sufficiently large.

Let $\zeta_1,\ \varepsilon_3,\ \varepsilon_4,\ \varepsilon_5,\ \varepsilon_6 \to 0$. We get $\lim_{t\to\infty} \widetilde{L_{1_t}} = \widetilde{L_t}$, $\lim_{t\to\infty} \widetilde{L_{2_t}} = \widetilde{L_t}$ and $\lim_{t\to\infty} \widetilde{N_{1_t}} = \widetilde{N_t}$, $\lim_{t\to\infty} \widetilde{N_{2_t}} = \widetilde{N_t}$. Hence $\lim_{t\to\infty} L_t = \widetilde{L_t}$ and $\lim_{t\to\infty} N_t = \widetilde{N_t}$. Due to $q_1^*,\ q_2^* \to q^*$ as $\zeta_1 \to 0$, we can find that sufficiently small ζ_1 makes $q < \min\{\mathrm{q}^*,\ \mathrm{q}_1^*,\ \mathrm{q}_2^*\}$. Therefore, system (2.1) has a globally attractive disease-free periodic solution $(\widetilde{L_t},\ \widetilde{N_t},\ 0,\ \overline{H},\ 0)$. □

We use numerical simulations to verify the global attractiveness of the disease-free periodic solution $(\widetilde{L_t},\ \widetilde{N_t},\ 0,\ \overline{H},\ 0)$ of system (2.1). We choose $T = 1$, $l = 0.5$, $r = 200$, $w = 50$, $\beta^L = 0.045$, $\eta^L = 0.4$, $\eta^N = 0.5$, $\beta^N = 0.05$, $p^N = 0.4$, $\mu^T = 0.05$, $\Lambda = 0.55$, $p^H = 0.6$, $\mu^H = 0.1$, and $\alpha^H = 0.1$, then we can compute $q = 0.32 < q^* = 0.6967$ and $\theta = 0.1806 < \varphi = 0.1875$. Hence, the disease-free periodic solution $(\widetilde{L_t},\ \widetilde{N_t},\ 0,\ \overline{H},\ 0)$ is globally attractive. Its dynamical behaviors are depicted in Figure 2.

6. Permanence

We will demonstrate in this section that system (2.1) is uniformly persistent. Based on the above discussion, we can find a constant $\overline{M}$ such that, for all solutions $(L_t,\ N_t^S,\ N_t^I,\ H_t^S,\ H_t^I)$ of system (2.1), $\overline{M}$ is an upper bound when t is sufficiently large. Therefore, we just need to find a positive constant $\underline{m}$ that is a lower bound for every solution $(L_t,\ N_t^S,\ N_t^I,\ H_t^S,\ H_t^I)$ of system (2.1) when t is sufficiently large.

From system (2.1) and Lemma 1, we get

$$(H_t^S)' \geq \Lambda - (\beta^N p^H M + \mu^H)H_t^S$$

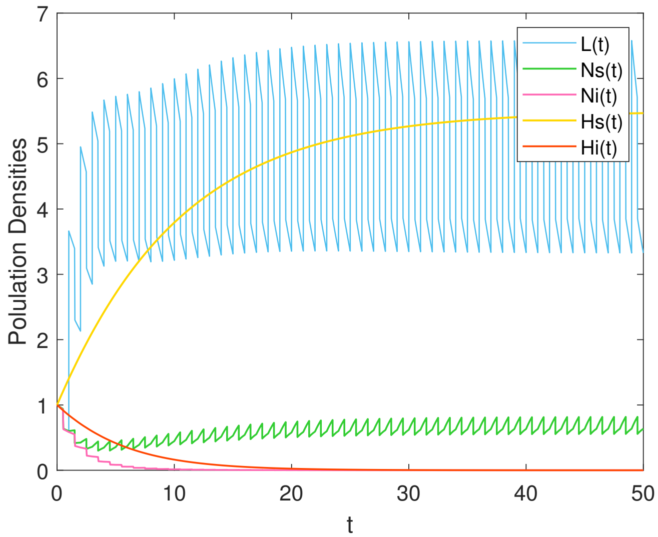

Figure 2. The time-series diagram of globally attractive disease-free periodic solution $(\widetilde{L_t},\ \widetilde{N_t},\ 0,\ \overline{H},\ 0)$ to system (2.1) with initial conditions $(1,1,1,1,1)$ and the following parameter values $T=1,\ l=0.5,\ q=0.32,\ r=200,\ w=50,\ \beta^L=0.045,\ \eta^L=0.4,\ \eta^N=0.5,\ \beta^N=0.05,\ p^N=0.4,\ \mu^T=0.05,\ \Lambda=0.55,\ p^H=0.6,\ \mu^H=0.1,\ \alpha^H=0.1.$

when t is sufficiently large.

The comparison system is as follows:

$$(X_{4_t})^{'}=\Lambda-(\beta^N p^H M+\mu^H)X_{4_t}.$$

We can easily obtain $X_{4_t}\to\frac{\Lambda}{\beta^N p^H M+\mu^H}$ as $t\to\infty$. Thus, for $\varepsilon_7>0$ sufficiently small, we get

$$H_t^S>\frac{\Lambda}{\beta^N p^H M+\mu^H}-\varepsilon_7\triangleq m^H.$$

It holds from system (2.1) that

$$\begin{cases}\left.\begin{aligned}&(L_t)^{'}\geq-(\beta^L\overline{H}+\mu^T)L_t,\\&(N_t^S)'\geq\eta^L\beta^L(\frac{\Lambda}{\beta^N p^H M+\mu^H}-\varepsilon_7)L_t-(\beta^N\overline{H}+\mu^T)N_t^S,\end{aligned}\right\}\begin{aligned}&t\neq(n+l)T,\\&t\neq(n+1)T,\end{aligned}\\\left.\begin{aligned}&L_{t^+}=(1-q)L_t,\\&N_{t^+}^S=(1-q)N_t^S,\end{aligned}\right\}t=(n+l)T,\\\left.\begin{aligned}&L_{t^+}\geq L_t+\frac{r\eta^N\beta^N\underline{H}N_t^S}{1+w\eta^N\beta^N\overline{H}(N_t^S+M)},\\&N_{t^+}^S=N_t^S.\end{aligned}\right\}t=(n+1)T.\end{cases}$$

Consider the following comparison system

$$\begin{cases} \left.\begin{array}{l} (L_{3_t})' = -a_3 L_{3_t}, \\ (N_{3_t})' = b_3 L_{3_t} - c_3 N_{3_t}, \end{array}\right\} & \begin{array}{l} t \neq (n+l)T, \\ t \neq (n+1)T, \end{array} \\ \left.\begin{array}{l} L_{3_{t^+}} = (1-q) L_{3_t}, \\ N_{3_{t^+}} = (1-q) N_{3_t}, \end{array}\right\} t = (n+l)T, \\ \left.\begin{array}{l} L_{3_{t^+}} = L_{3_t} + \dfrac{r_3 N_{3_t}}{1 + w_3 (N_{3_t} + M)}, \\ N_{3_{t^+}} = N_{3_t}, \end{array}\right\} t = (n+1)T, \end{cases} \tag{6.1}$$

where $a_3 = \beta^L \overline{H} + \mu^T$, $b_3 = \eta^L \beta^L (\frac{\Lambda}{\beta^N p^H M + \mu^H} - \varepsilon_7)$, $c_3 = \beta^N \overline{H} + \mu^T$, $r_3 = r\eta^N \beta^N \underline{H}$, and $w_3 = w\eta^N \beta^N \overline{H}$.

Similar to (5.6), system (6.1) has a positive periodic solution

$$\widetilde{L_{3_t}} = \begin{cases} e^{-a_3(t-nT)} L_3^*, & nT < t \leq (n+l)T, \\ (1-q) e^{-a_3(t-nT)} L_3^*, & (n+l)T < t \leq (n+1)T, \end{cases}$$

$$\widetilde{N_{3_t}} = \begin{cases} \dfrac{b_3}{a_3 - c_3} \left(e^{-c_3(t-nT)} - e^{-a_3(t-nT)} \right) L_3^* + e^{-c_3(t-nT)} N_3^*, \\ \qquad nT < t \leq (n+l)T, \\ (1-q) \left[\dfrac{b_3}{a_3 - c_3} \left(e^{-c_3(t-nT)} - e^{-a_3(t-nT)} \right) L_3^* + e^{-c_3(t-nT)} N_3^* \right], \\ \qquad (n+l)T < t \leq (n+1)T, \end{cases}$$

where

$$\left.\begin{array}{l} L_3^* = \left(\dfrac{r_3}{1 - C_3} - \dfrac{1 - B_3}{A_3} \right) \dfrac{1}{w_3} - M, \\ N_3^* = \left(\dfrac{r_3 A_3}{(1 - B_3)(1 - C_3)} - 1 \right) \dfrac{1}{w_3} - \dfrac{1 - B_3}{A_3} M, \end{array}\right\} q < q_3^*,$$

and
$\xi_3 = \dfrac{e^{-a_3 T} - e^{-c_3 T}}{c_3 - a_3} > 0$, $A_3 = (1-q) b_3 \xi_3 > 0$, $B_3 = (1-q) e^{-c_3 T}$, $C_3 = (1-q) e^{-a_3 T}$,

$$\begin{aligned} q_3^* = & \frac{1}{2} e^{(a_3+c_3)T} \Big[-(e^{-a_3 T} + e^{-c_3 T} + r_3 b_3 \xi_3 - 2e^{-(a_3+c_3)T}) \\ & + \sqrt{(e^{-a_3 T} + e^{-c_3 T} + r_3 b_3 \xi_3)^2 - 4e^{-(a_3+c_3)T}} \ \Big]. \end{aligned}$$

Therefore, for sufficiently small $\varepsilon_8, \varepsilon_9 > 0$, it holds that

$$\begin{cases} \widetilde{L_{3_t}} - \varepsilon_8 \leq L_{3_t} < L_t, \\ \widetilde{N_{3_t}} - \varepsilon_9 \leq N_{3_t} < N_t^S, \end{cases} \tag{6.2}$$

when t is large enough. Thus,

$$\begin{aligned} & L_t > (1-q) L_3^* e^{-a_3 T} - \varepsilon_8 \triangleq m^L, \\ & N_t^S > (1-q) \left[\frac{b_3}{c_3 - a_3} (e^{(c_3 - a_3)T} - 1) L_3^* + N_3^* \right] e^{-c_3 T} - \varepsilon_9 \triangleq m^N. \end{aligned} \tag{6.3}$$

In summary, we have shown that $H_t^S > m^H > 0,\ L_t > m^L > 0,\ N_t^S > m^N > 0$ when t is sufficiently large. Next, we will show that there exists a $\rho > 0$ satisfying $N_t^I + H_t^I \geq \rho$ when t is sufficiently large.

Theorem 6.1. *There exists a $\rho > 0$, such that when t is large enough, each solution $(L_t,\ N_t^S,\ N_t^I,\ H_t^S,\ H_t^I)$ of system (2.1) meets $N_t^I + H_t^I \geq \rho$. This holds true under the conditions that $q > q_4^*$ and $(1-q)e^{(\sigma-\tau)T} > 1$, where*

$$\sigma = \min\{\eta^{\mathrm{L}}\beta^{\mathrm{L}}\mathrm{p}^{\mathrm{N}}\delta_1,\ \beta^{\mathrm{N}}\mathrm{p}^{\mathrm{H}}\delta_2\},\ \tau = \max\{\mu^{\mathrm{T}} + \beta^{\mathrm{N}}\overline{\mathrm{H}},\ \mu^{\mathrm{H}} + \alpha^{\mathrm{H}}\}.$$

Proof. We will prove it in two steps.
Step I :
We claim that $N_t^I + H_t^I < \rho$ cannot be true for all $t \geq 0$. Otherwise,

$$N_t^I < \rho, H_t^I < \rho, \quad t \geq 0. \tag{6.4}$$

Therefore, from system (2.1), we derive

$$(H_t^S)' \geq \Lambda - (\beta^N p^H \rho + \mu^H) H_t^S.$$

Similar to (5.3), for $\varepsilon_{10} > 0$ sufficiently small, we obtain

$$H_t^S > \frac{\Lambda}{\beta^N p^H \rho + \mu^H} - \varepsilon_{10} \triangleq \delta_2, \tag{6.5}$$

when t is large enough.

From (6.4) and system (2.1), we get

$$\begin{cases} \left.\begin{aligned} &(L_t)' \geq -(\beta^L\overline{H} + \mu^T)L_t, \\ &(N_t^S)' \geq \eta^L\beta^L\delta_2 L_t - (\beta^N\overline{H} + \mu^T)N_t^S, \end{aligned}\right\} \begin{aligned} &t \neq (n+l)T, \\ &t \neq (n+1)T, \end{aligned} \\ \left.\begin{aligned} &L_{t^+} = (1-q)L_t, \\ &N_{t^+}^S = (1-q)N_t^S, \end{aligned}\right\} t = (n+l)T, \\ \left.\begin{aligned} &L_{t^+} \geq L_t + \frac{r\eta^N\beta^N\delta_2 N_t^S}{1 + w\eta^N\beta^N\overline{H}(N_t^S + \rho)}, \\ &N_{t^+}^S = N_t^S. \end{aligned}\right\} t = (n+1)T. \end{cases}$$

Consider the following comparison system

$$\begin{cases} \left.\begin{aligned} &(L_{4_t})' = -a_4 L_{4_t}, \\ &(N_{4_t})' = b_4 L_{4_t} - c_4 N_{4_t}, \end{aligned}\right\} \begin{aligned} &t \neq (n+l)T, \\ &t \neq (n+1)T, \end{aligned} \\ \left.\begin{aligned} &L_{4_{t^+}} = (1-q)L_{4_t}, \\ &N_{4_{t^+}} = (1-q)N_{4_t}, \end{aligned}\right\} t = (n+l)T, \\ \left.\begin{aligned} &L_{4_{t^+}} = L_{4_t} + \frac{r_4 N_{4_t}}{1 + w_4(N_{4_t} + \rho)}, \\ &N_{4_{t^+}} = N_{4_t}, \end{aligned}\right\} t = (n+1)T, \end{cases} \tag{6.6}$$

where $a_4 = \beta^L\overline{H} + \mu^T$, $b_4 = \eta^L\beta^L\delta_2$, $c_4 = \beta^N\overline{H} + \mu^T$, $r_4 = r\eta^N\beta^N\delta_2$, and $w_4 = w\eta^N\beta^N\overline{H}$.

The fixed points of (6.6) are

$$\left.\begin{aligned} L_4^* &= \frac{1}{w_4}\left(\frac{r_4}{1-C_4} - \frac{1-B_4}{A_4}\right) - \rho, \\ N_4^* &= \frac{1}{w_4}\left(\frac{r_4A_4}{(1-B_4)(1-C_4)} - 1\right) - \frac{(1-B_4)\rho}{A_4}, \end{aligned}\right\} q < q_4^*,$$

where $\xi_4 = \frac{e^{-a_4T} - e^{-c_4T}}{c_4 - a_4} > 0$, $A_4 = b_4(1-q)\xi_4$, $B_4 = (1-q)e^{-c_4T}$, $C_4 = (1-q)e^{-a_4T}$, and

$$\begin{aligned} q_4^* =& \frac{1}{2}e^{(a_4+c_4)T}\Big[-(e^{-a_4T} + e^{-c_4T} + r_4b_4\xi_4 - 2e^{-(a_4+c_4)T}) \\ & + \sqrt{(e^{-a_4T} + e^{-c_4T} + r_4b_4\xi_4)^2 - 4e^{-(a_4+c_4)T}}\ \Big]. \end{aligned}$$

Smiliar to (6.3), for $\varepsilon_{11} > 0$ sufficiently small, we have

$$L_t > (1-q)L_4^*e^{-a_4T} - \varepsilon_{11} \triangleq \delta_1, \tag{6.7}$$

for t large enough.

Combining (6.5) and (6.7), we obtain

$$\begin{aligned} (V_t)' &= (N_t^I)' + (H_t^I)' \\ &= [\eta^L\beta^Lp^NL_tH_t^I - (\mu^T + \beta^NH_t)N_t^I] + [\beta^Np^HH_t^SN_t^I - (\mu^H + \alpha^H)H_t^I] \\ &\geq \eta^L\beta^Lp^N\delta_1H_t^I - (\mu^T + \beta^N\overline{H})N_t^I + \beta^Np^H\delta_2N_t^I - (\mu^H + \alpha^H)H_t^I, \\ &\geq (\sigma - \tau)V_t, \end{aligned}$$

where

$$\sigma = \min\{\eta^L\beta^Lp^N\delta_1,\ \beta^Np^H\delta_2\},\ \tau = \max\{\mu^T + \beta^N\overline{H},\ \mu^H + \alpha^H\}.$$

When $t = (n+l)T$,

$$V_{t^+} = (1-q)N_t^I + H_t^I \geq (1-q)V_t.$$

Thus, we obtain

$$\begin{cases} V_t \geq (\sigma - \tau)V_t, & t \neq (n+l)T, \\ V_{t^+} \geq (1-q)V_t, & t = (n+l)T. \end{cases} \tag{6.8}$$

Integrating (6.8) on $((n+l-1)T, (n+l)T]$, we obtain

$$V_{(n+l)T^+} \geq V_{(n+l-1)T^+}(1-q)e^{(\sigma-\tau)T},$$

thus

$$V_{(n+l+k)T^+} \geq V_{(n+l)T^+}[(1-q)e^{(\sigma-\tau)T}]^k,\ k \in N_+.$$

When $(1-q)e^{(\sigma-\tau)T} > 1$, we can derive $\lim\limits_{k\to+\infty} V_{((n+l+k)T^+)} = +\infty$, which contradicts the boundedness of V_t. Hence we can find a $t_1 > 0$ satisfying $V_{t_1} = N_{t_1}^I + H_{t_1}^I \geq$

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ρ.

Step II :
If $V_t \geq \rho$ for all $t \geq t_1$, then we have achieved our goal. Otherwise $V_t < \rho$ for some $t \geq t_1$. Defining $\hat{t_1} = inf_{t \geq t_1}\{V_t < \rho\}$, $\hat{t_1}$ has two cases as follows.
Case i: If $\hat{t_1} = (k_1 + l - 1)T$, $k_1 \in N_+$, then $V_t \geq \rho$ for $t \in [t_1,\ \hat{t_1})$ and $V_{\hat{t_1}} = \rho$, and $V_{\hat{t}_1^+} = (1-q)N^I_{\hat{t_1}} + H^I_{\hat{t_1}} \leq V^I_{\hat{t_1}} = \rho$. Let $T' = k_2T + k_3T$ and select $k_2, k_3 \in N$, such that

$$
\begin{aligned}
&(k_2 - 1)T > -\frac{ln(\frac{\varepsilon'}{E_1})}{\beta^L \underline{H} + \mu^T}\ , \\
&(1-q)^{k_2+k_3} e^{(\sigma-\tau)k_3T} e^{-\tau k_2 T} > 1\ .
\end{aligned}
$$

We declare that there must be a $t_2 \in [\hat{t_1},\ \hat{t_1} + T']$ satisfying $V_{t_2} > \rho$. Otherwise, $V(t) \leq \rho$ holds for all $t \in [\hat{t_1},\ \hat{t_1} + T']$. Considering (5.1) with $X_{2_{k_1T^+}} = L_{k_1T^+}$, we have

$$
X_{2_t} = \widetilde{X_{2_t}} + e^{-(\beta^L \underline{H} + \mu^T)(t - k_1T)}[X_{2_{k_1T^+}} -\ \widetilde{X_{2_{0+}}}],\quad (n-1)T < t \leq nT, \tag{6.9}
$$

where $k_1 + 1 \leq n \leq k_1 + k_2 + k_3$. Then

$$
|X_{2_t} - \widetilde{X_{2_t}}| \leq E_1 e^{-(\beta^L \underline{H} + \mu^T)(t - k_1T)} < \varepsilon'\ ,
$$

where $E_1 = |X_{2_{k_1T^+}} - \widetilde{X_{2_{0+}}}|$ and $L_t \leq X_{2_t} \leq \widetilde{X_{2_t}} + \varepsilon'$ for $(k_1 + k_2 - 1)T \leq t \leq \hat{t_1} + T'$. Integrating system (6.8) on $[\hat{t_1} + k_2T,\ \hat{t_1} + T']$, we derive

$$
V_{(\hat{t_1}+T')} \geq V_{(\hat{t_1}+k_2T)}(1-q)^{k_3} e^{(\sigma-\tau)k_3T}. \tag{6.10}
$$

It holds from system (2.1) that

$$
\begin{cases}
V_t \geq -\tau V_t, & t \neq (n+l)T, \\
V_{t^+} \geq (1-q)V_t, & t = (n+l)T.
\end{cases} \tag{6.11}
$$

Integrating (6.11) on $[\hat{t_1},\ \hat{t_1} + k_2T]$ yields

$$
V_{(\hat{t_1}+k_2T)} \geq \rho(1-q)^{k_2} e^{-\tau k_2 T}. \tag{6.12}
$$

Combining (6.10) and (6.12), we derive

$$
V_{(\hat{t_1}+T')} \geq \rho(1-q)^{k_2+k_3} e^{-\tau k_2 T} e^{(\sigma-\tau)k_3T} > \rho,
$$

which contradicts that $V_t \leq \rho$ for all $t \in [\hat{t_1},\ \hat{t_1}+T']$. So we can find a $t_2 \in [\hat{t_1},\ \hat{t_1}+T']$ satisfying $V_{t_2} > \rho$.

Let $\hat{t_2} = inf_{t \geq \hat{t_1}}\{V_t > \rho\}$, then $V_t \leq \rho$ for $t \in (\hat{t_1},\ \hat{t_2})$, and $V_{\hat{t_2}} = \rho$. Suppose that $t \in (\hat{t_1} + (g_1 - 1)T,\ \hat{t_1} + KT] \subset (\hat{t_1},\ \hat{t_2}]$, g_1 is a positive integer and $g_1 \leq k_2 + k_3$.

According to system (6.11), we obtain

$$
\begin{aligned}
V_t &\geq V_{(\hat{t_1}+(g_1-1)T)} e^{-\tau(t - \hat{t_1} - (g_1-1)T)} \\
&\geq V_{\hat{t}_1^+} e^{-\tau(g_1-1)T}(1-q)^{g_1-1} e^{-\tau T}
\end{aligned}
$$

$$\geq (1-q)^{g_1}\rho e^{-\tau g_1 T}$$
$$\geq (1-q)^{g_1}\rho e^{-\tau(k_2+k_3)T} \triangleq \rho_1,$$

for $t > \hat{t_2}$. Due to $V_{\hat{t_2}} \geq \rho$, the same conclusions can be drawn. Thus $V_t \geq \rho_1$ holds for all $t \geq t_1$.

Case ii: If $\hat{t_1} \neq (n+l-1)T$, then $V_t \geq \rho$ for $t \in [t_1,\ \hat{t_1})$ and $V_{\hat{t_1}} = \rho$. Assume that $\hat{t_1} \in ((k_4+l-1)T,\ (k_4+l)T)$. There are two subcases for $t \in (\hat{t_1},\ (k_4+l)T)$.

Case a: $V_t \leq \rho$ for all $t \in (\hat{t_1},\ (k_4+l)T)$. Similar to Case i, we demonstrate that there exists a $t_3 \in [(k_4+l)T,\ (k_4+l)T+T']$ satisfying $V_{t_3} > \rho$. Here we omit it.

Let $\hat{t_3} = inf_{t>\hat{t_1}}\{V_t > \rho\}$. Then $V_t \leq \rho$ for $t \in (\hat{t_1},\ \hat{t_3})$ and $V_{\hat{t_3}} = \rho$. Choosing $t \in ((k_4+l-1)T+(g_2-1)T,\ (k_4+l-1)T+g_2T] \subset (\hat{t_1},\ \hat{t_3})$, $g_2 \in N_+$ and $g_2 < 1+k_2+k_3$, it holds that

$$\begin{aligned} V_t &\geq V_{((k_4+l-1)T+(g_2-1)T)}e^{-\tau[t-((k_4+l-1)T+(g_2-1)T)]}, \\ &\geq (1-q)^{g_2-1}V_{\hat{t_1}}e^{-\tau(t-\hat{t_1})}, \\ &\geq (1-q)^{k_2+k_3}\rho e^{-\tau(k_2+k_3+1)T} \triangleq \rho_2, \end{aligned}$$

hence $V_t \geq \rho_2$ for $t \in (\hat{t_1},\ \hat{t_3})$. When $t \geq \hat{t_3}$, the same conclusions can be drawn due to $V_{\hat{t_3}} \geq \rho$.

Case b: If there exists a $t_3 \in (\hat{t_1},\ (k_4+l)T)$ satisfying $V_{t_3} \geq \rho$. Let $\hat{t_4} = inf_{t>\hat{t_1}}\{V_t > \rho\}$. Then $V_t \leq \rho$ for $t \in (\hat{t_1},\ \hat{t_4})$ and $V_{\hat{t_4}} = \rho$. Thus (6.11) holds for $t \in (\hat{t_1},\ \hat{t_4})$. Integrating (6.11) on $(\hat{t_1},\ \hat{t_4})$, we derive

$$V_t \geq V_{\hat{t_1}}e^{-\tau(t-\hat{t_1})} \geq \rho e^{-\tau T} \triangleq \rho_3.$$

Since $V_{\hat{t_4}} \geq \rho$ for $t \geq \hat{t_4}$, the same conclusions can be drawn. Therefore, $V_t \geq \rho$ for $t \geq t_1$. □

We use numerical simulations to check the persistence of system (2.1). Fix $T = 1,\ l = 0.5,\ r = 200,\ w = 50,\ \beta^L = 0.045,\ \eta^L = 0.8,\ \eta^N = 0.7,\ \beta^N = 0.04,\ p^N = 0.6,\ \mu^T = 0.05,\ \Lambda = 0.55,\ p^H = 0.8,\ \mu^H = 0.05,\ \alpha^H = 0.05$. Then we obtain $q = 0.02 < q_4^* = 0.2231$, $\sigma = 0.5331 > \tau = 0.49$ and $(1-q)e^{(\sigma-\tau)T} > 1$. Therefore, system (2.1) is permanent. Its dynamical behaviors are demonstrated in Figure 3.

7. Discussions

Considering that tick eggs, laid by adult females after blood feeding in the fall, will hatch into larvae in spring, we developed a novel tick-borne disease transmission model with double pulses, including a birth pulse of larval ticks and an insecticide pulse. We give conditions for the global asymptotic stability of the periodic solution to the disease-free subsystem. We also demonstrate the global attractiveness of disease-free periodic solutions and the persistence of the studied system.

In theoretical analysis, we showed the dynamics of the tick-borne pathogen model with one pesticide pulse. However, in the natural world, the CDC may spray pesticides several times in wild forests and woods each year. We further explore and compare the effects between multiple pesticide pulses and single pulse numerically. We investigate the relationship between the minimum intensity of the

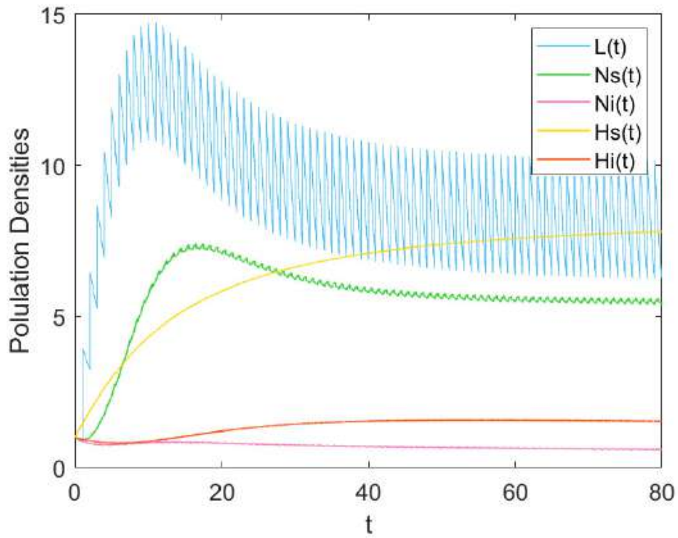
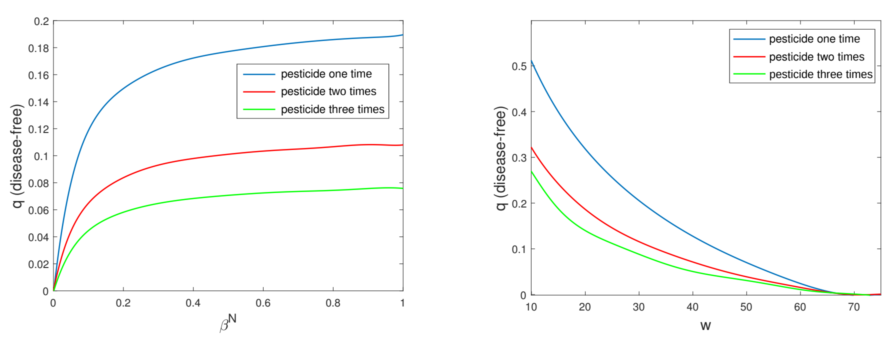


Figure 3. The time-series diagram of disease persistent periodic solution to system (2.1) with initial conditions $(1, 1, 1, 1, 1)$ and the following parameter values $T = 1,\ l = 0.5,\ q = 0.02,\ r = 200,\ w = 50,\ \beta^L = 0.045,\ \eta^L = 0.8,\ \eta^N = 0.7,\ \beta^N = 0.04,\ p^N = 0.6,\ \mu^T = 0.05,\ \Lambda = 0.55,\ p^H = 0.8,\ \mu^H = 0.05,\ \alpha^H = 0.05$.



(a) (b)

Figure 4. The minimum intensity of the sprayed pesticide that makes the disease disappear with respect to different parameters: (a) nymphal biting rate β^N and (b) adult density-dependent rate w.

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sprayed pesticide that makes the disease disappear and nymphal biting rate β^N (density dependent rate of adult ticks w). Figure 4(a) (Figure 4(b)) shows the minimum intensity of the sprayed pesticide is increasing (decreasing) with respect to β^N (w): if pesticide is sprayed once, i.e., at the moment $t = (n + 0.5)T$, the blue curve depicts the minimum intensity; if pesticides are applied twice ($l = 0.5,\ 0.8$) and three times ($l = 0.5,\ 0.65,\ 0.8$), the corresponding curves are in red and green, respectively. From Figure 4(a) and Figure 4(b), we can observe that the total intensity of pesticide in one year: total intensity q for spraying once $<$ total intensity q for spraying twice $<$ total intensity q for spraying three times. Therefore, when the total intensity of pesticide is fixed, the effect of spraying once is better than multiple sprays. The reason is that ticks can develop rapidly during pulse intervals of pesticide spray, which leads to an increase in total pesticide intensity.

Currently, there are relatively few studies on tick-borne disease transmission with pulses and stage structures. However, there are still many theoretical issues that deserve to be studied and solved. Beyond temporal pesticide spray, state impulsive control is also a commonly used strategy by forestry administration, i.e. pesticides are sprayed when the population of ticks grows to a certain threshold. In addition, it is well-known that co-feeding transmission is a significant spread mode of tick-borne pathogens even though pathogens have not been established within reservoir hosts. The state impulsive control of tick-born diseases with co-feeding transmission will be our future work.

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