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ON CONVERGENCE OF GENERAL GAMMA TYPE OPERATORS

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Abstract. The present paper deals with the new type of Gamma operators, here we estimate the rate of pointwise convergence of these new Gamma type operators $M_{n,k}$ for functions of bounded variation, by using some techniques of probability theory.

Key words: *rate of convergence, gamma operator, bounded variation, total variation*

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1 Introduction

Let $BV_\gamma[0,\infty)$, $(\gamma \geq 0)$ be the class of all functions defined on $[0,\infty)$, being bounded variation on every finite subinterval of $[0,\infty)$ and satisfying the growth condition $|f(t)| \leq Mt^\gamma$ for every $t > 0$ and some constant $M > 0$.

For a measurable complex valued locally bounded function f defined on $[0,\infty)$, Lupas and Müller[1] introduced and investigated some approximation properties of the sequence of linear positive operators $\{G_n\}$ defined by

$$G_n(f;x) = \int_0^\infty g_n(x,u) f\left(\frac{n}{u}\right) \mathrm{d}u,$$

which is called Gamma operator, where $g_n(x,u) = \dfrac{x^{n+1}}{n!} e^{-xu} u^n$, $x > 0$. Some approximation properties of these operators were studied by Chen and Guo[2] for functions in $BV[0,\infty)$ and recently for functions in $BV[0,\infty)$ and $DBV[0,\infty)$ by Zeng[3].

In [4], Mazhar defined and studied some approximation properties of the following sequence of linear positive operators

$$\begin{aligned} F_n(f;x) &= \int_0^\infty g_n(x,u)\mathrm{d}u \int_0^\infty g_{n-1}(u,t)f(t)\mathrm{d}t \\ &= \frac{(2n)!x^{n+1}}{n!(n-1)!}\int_0^\infty \frac{t^{n-1}}{(x+t)^{2n+1}}f(t)\mathrm{d}t, \quad n>1, \quad x>0, \end{aligned}$$

where $g_n(x,u)$ is the same function, which has been used by Lupas and Müller in paper [1].

Recently, by using the techniques due to Mazhar, Izgi and Buyukyazici[5] and independently Karsli[6] considered the following Gamma type linear and positive operators

$$\begin{aligned} L_n(f;x) &= \int_0^\infty g_{n+2}(x,u)\mathrm{d}u \int_0^\infty g_n(u,t)f(t)\mathrm{d}t \\ &= \frac{(2n+3)!x^{n+3}}{n!(n+2)!}\int_0^\infty \frac{t^n}{(x+t)^{2n+4}}f(t)\mathrm{d}t, \qquad x>0. \end{aligned}$$

For a very recent results on the local and global approximation results on $L_n(f;x)$ see Karsli and Ozarslan[15].

In 2007 Mao[14] defined the following gamma type operators

$$\begin{aligned} (M_{n,k}f)(x) &= \int_0^\infty g_n(x,u)\mathrm{d}u \int_0^\infty g_{n-k}(u,t)f(t)\mathrm{d}t \\ &= \frac{(2n-k+1)!x^{n+1}}{n!(n-k)!}\int_0^\infty \frac{t^{n-k}}{(x+t)^{2n-k+2}}f(t)\mathrm{d}t, \qquad x>0, \end{aligned}$$

whose special cases are:

If $k=1$, then $(M_{n,1}f)(x)=F_n(f;x)$,

If $k=2$, then $(M_{n,2}f)(x)=L_{n-2}(f;x)$.

In addition, if f is right-side continuous at $x=0$, we define

$$(M_{n,k}f)(0) := f(0), \qquad n,k\in\mathbf{N}.$$

For the convenience we can rewrite the operators $(M_{n,k}f)(x)$ as

$$(M_{n,k}f)(x) = \int_0^\infty K_{n,k}(x,t)f(t)\mathrm{d}t, \tag{1}$$

where

$$K_{n,k}(x,t) := \frac{(2n-k+1)!x^{n+1}}{n!(n-k)!}\frac{t^{n-k}}{(x+t)^{2n-k+2}}, \qquad x,t\in(0,\infty).$$

The rate of approximation for functions of bounded variation is an interesting topic in approximation theory, several researchers have studied on these subjects for three decades. We mention the work of Bojanic-Vuilleumier and Cheng (see [9,10]) who estimated the rate of convergence of bounded variation for Fourier- Legendre series and Bernstein polynomials by using