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ON THE LOCATION OF ZEROS OF A POLYNOMIAL

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Abstract. In this paper we extend Enestrom -Kakeya theorem to a large class of polynomials with complex coefficients by putting less restrictions on the coefficients. Our results generalise and extend many known results in this direction.

Key words: *polynomial, zero, bound*

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1 Introduction and Statement of Results

Let $P(z)$ be a polynomial of degree n. A classical result due to Enestrom and Kakeya[8] concerning the bound for the moduli of the zeros of polynomials having positive coefficients is often stated as in the following theorem(see [8]) :

Theorem A (Enestrom-Kakeya). *Let* $P(z)=\sum_{j=0}^{n} a_j z^j$ *be a polynomial of degree n whose coefficients satisfy*

$$0 \le a_1 \le a_2 \le \ldots \le a_n.$$

Then P(z) has all its zeros in the closed unit disk $|z| \le 1$.

In the literature there exist several generalisations of this result (see [1], [3], [4], [7], [8]). Recently Aziz and Zargar[2] relaxed the hypothesis in several ways and proved:

Theorem B. *Let* $P(z)=\sum_{j=0}^{n} a_j z^j$ *be a polynomial of degree n such that for some* $k \ge 1$

$$ka_n \ge a_{n-1} \ge \ldots \ge a_0.$$

Then all the zeros of $P(z)$ *lie in*

$$|z+k-1| \le \frac{ka_n + |a_0| - a_0}{|a_n|}.$$

For ploynomials, whose coeffiencents are not necessarily real, Govil and Rehman[6] proved the following generalisation of Theorem A:

Theorem C. *Let* $P(z)=\sum_{j=0}^{n} a_j z^j$ *be a polynomial of degree n with* $\mathrm{Re}(a_j)=\alpha_j$ *and* $\mathrm{Im}(a_j)=\beta_j,\ j=0,\ 1,\ \cdots,\ n$, *such that*

$$\alpha_n \geq \alpha_{n-1} \geq \cdots \geq \alpha_0 \geq 0,$$

where $\alpha_n>0$, *then* $P(z)$ *has all its zeros in*

$$|z| \leq 1+\left(\frac{2}{\alpha_n}\right)\left(\sum_{j=0}^{n}|\beta_j|\right).$$

More recently, Govil and Mc-tume[5] proved the follwoing generalisations of Theorems B and C:

Theorem D. *Let* $P(z)=\sum_{j=0}^{n} a_j z^j$ *be a polynomial of degree n with* $\mathrm{Re}(a_j)=\alpha_j$ *and* $\mathrm{Im}(a_j)=\beta_j,\ j=0,\ 1,\ \cdots,\ n$. *If for some* $k\geq 1$,

$$k\alpha_n \geq \alpha_{n-1} \geq \cdots \geq \alpha_0,$$

then $P(z)$ *has all its zeros in*

$$|z+k-1| \leq \frac{k\alpha_n-\alpha_0+|\alpha_0|+2\sum_{j=0}^{n}|\beta_j|}{|\alpha_n|}.$$

Theorem E. *Let* $P(z)=\sum_{j=0}^{n} a_j z^j$ *be a polynomial of degree n with* $\mathrm{Re}(a_j)=\alpha_j$ *and* $\mathrm{Im}(a_j)=\beta_j,\ j=0,\ 1,\ \cdots,\ n$. *If for some* $k\geq 1$,

$$k\beta_n \geq \beta_{n-1} \geq \cdots \geq \beta_0,$$

then $P(z)$ *has all its zeros in*

$$|z+k-1| \leq \frac{k\beta_n-\beta_0+|\beta_0|+2\sum_{j=0}^{n}|\alpha_j|}{|\beta_n|}.$$

In this paper we shall present some interesting generalizations of Theorems D and E and consequently of Enestrom-Kakeya Theorem. Our first result in this direction is the following: