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WEIGHTED ESTIMATES FOR MULTIVARIATE HAUSDORFF OPERATORS

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Abstract. In this paper, some weighted estimates for the multivariate Hausdorff operators are obtained. It is proved that the multivariate Hausdorff operators are bounded on L^p spaces with power weights, which is based on the boundedness of multivariate Hausdorff operators on Herz spaces, and are bounded on weighted L^p spaces with the weights satisfying the homogeneity of degree zero.

Key words: *multivariate Hausdorff operator, weighted L^p space, Herz space*

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1 Introduction

The notion of the Hausdorff operator with respect to a positive measure on the unit interval $[0,1]$ is introduced by Hardy in [1]. The operator with respect to a complex measure in the real line $\mathbf{R}$ is defined and studied by Brown and Móricz in [2]. Following that, the multivariate Hausdorff operator with respect to complex Borel measures on $\mathbf{R}^n$ is introduced in a more general framework in [3].

Let μ be a $\sigma-$finite complex Borel measure on $\mathbf{R}^n$ and c be a Borel measurable function on $\mathbf{R}^n$, which is nonzero μ-a.e. Assume that $\mathcal{A} := [a_{jk}]$ is an $n\times n$ matrix whose entries $a_{jk} : \mathbf{R}^n \to \mathbf{C}$ are all Borel measurable functions and such that $\mathcal{A}$ is nonsingular μ-a.e. For a measurable complex valued function f on $\mathbf{R}^n$, the multivariate Hausdorff operator $\mathcal{H} = \mathcal{H}(\mu,c,\mathcal{A})$ is defined

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by:

$$\mathcal{H}f(x) := \int_{\mathbf{R}^n} c(s) f(\mathcal{A}(s)x) \mathrm{d}\mu(s). \tag{1.1}$$

The operator $\mathcal{H}^*$ adjoint to $\mathcal{H}$ is given by

$$\mathcal{H}^* f(x) := \int_{\mathbf{R}^n} c(s) |\det \mathcal{A}^{-1}(s)| f(\mathcal{A}^{-1}(s)x) \mathrm{d}\mu(s). \tag{1.2}$$

Both the above two integrals on the right hand side exist as Lebesgue-Stieltjes integrals [4,5]. It is obvious that $\mathcal{H}^*$ is also a Hausdorff operator corresponding to the triple $\mu(s), c(s)|\det \mathcal{A}^{-1}(s)|$, $\mathcal{A}^{-1}(s)$, that is

$$\mathcal{H}^* = \mathcal{H}(\mu, c|\det \mathcal{A}^{-1}|, \mathcal{A}^{-1}). \tag{1.3}$$

In [3], Brown and Móricz obtained the boundedness of the multivariate Hausdorff operator on $L^p(\mathbf{R}^n)$:

Theorem A. *If μ is a complex measure on $\mathbf{R}^n$ and*

$$k_p := \int_{\mathbf{R}^n} |c(s)||\det \mathcal{A}^{-1}(s)|^{\frac{1}{p}} \mathrm{d}|\mu|(s) < \infty \tag{1.4}$$

for some $1 \le p \le \infty$, then the Hausdorff operator $\mathcal{H} = \mathcal{H}(\mu, c, \mathcal{A})$ defined in (1.1) *is bounded on $L^p(\mathbf{R}^n)$:*

$$\|\mathcal{H}f\|_p \le k_p \|f\|_p, \tag{1.5}$$

where $|\mu|$ is the total variation of μ.

In [6], Móricz proved that the multivariate Hausdorff operator is bounded on the real Hardy space $H^1(\mathbf{R}^n)$ and BMO($\mathbf{R}^n$).

In this paper, we will generalize some results in [3] to the weighted L^p space and obtain some useful estimates for multivariate Hausdorff operators.

Note that the Herz space is a natural generalization of the L^p space with power weights (see [7]). We will firstly consider the boundedness of the multivariate Hausdorff operator on the Herz space. As a direct corollary of it, we can obtain the estimates for the operator on the L^p space with power weights. Next, we will estimate the multivariate Hausdorff operator on the weighted L^p space, where the weight functions are homogeneous of degree zero.

2 Main Results

Assume $1 \le p \le \infty$ and denote the exponent conjugate to p by p^*, that is, let $\dfrac{1}{p} + \dfrac{1}{p^*} = 1$ with the agreement that $\dfrac{1}{\infty} = 0$. Let $k \in \mathbf{Z}$, $B_k = \{x \in \mathbf{R}^n : |x| \le 2^k\}$, $D_k = B_k \backslash B_{k-1}$, and $\chi_k = \chi_{D_k}$ is the characteristic function of D_k.