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Toeplitz Operator Related to Singular Integral with Non-Smooth Kernel on Weighted Morrey Space

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Abstract. Let T_1 be a singular integral with non-smooth kernel or $\pm I$, let T_2 and T_4 be the linear operators and let $T_3=\pm I$. Denote the Toeplitz type operator by

$$T^b=T_1M^bI_\alpha T_2+T_3I_\alpha M^bT_4,$$

where $M^bf=bf$, and I_α is the fractional integral operator. In this paper, we investigate the boundedness of the operator T^b on the weighted Morrey space when b belongs to the weighted BMO space.

Key Words: Toeplitz operator, non-smooth kernel, weighted BMO, fractional integral, weighted Morrey space.

AMS Subject Classifications: 42B20, 42B35

1 Introduction

Let b be a locally integrable function on $\mathbb{R}^n$. The Toeplitz operator related to singular integral operator T and fractional integral operator I_α is defined by

$$T^b=T_1M^bI_\alpha T_2+T_3I_\alpha M^bT_4, \tag{1.1}$$

where $T_1=T$ or $\pm I$ (the identity operator), T_2 and T_4 are the linear operators, $T_3=\pm I$, and $M^bf=bf$.

Note that the commutators $[b,I_\alpha](f)=bI_\alpha(f)-I_\alpha(bf)$ are the particular cases of the Toeplitz operators T^b. The Toeplitz operators T^b are the non-trivial generalization of these commutators.

Lin and Lu in [1] obtained the boundedness of Toeplitz operator as (1.1) on $L^p(\mathbb{R}^n)$ when T is a strongly singular Calderón-Zygmund operator and b belongs to the Lipschitz function space Λ_β; In [2], Lu and Mo proved that the Toeplitz operator is bounded

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on $L^p(\mathbb{R}^n)$ when T is a singular integral with non-smooth kernel and b is a Lipschitz function. More results about Toeplitz operator can be found in [3,4]. In this paper, we investigate the boundedness of the operator T^b on weighted Morrey space when T is the singular integral with non-smooth kernel and b belongs to the weighted BMO space.

Definition 1.1 (see [5]). Let $1\le p<\infty$, $0<\kappa<1$ and ω be a weight function. Then the weighted Morrey space is defined by

$$L^{p,\kappa}(\omega)=\Big\{f\in L^p_{loc}(\omega):\|f\|_{L^{p,\kappa}(\omega)}<\infty\Big\},$$

where

$$\|f\|_{L^{p,\kappa}(\omega)}=\sup_B\left(\frac{1}{\omega(B)^\kappa}\int_B|f(x)|^p\omega(x)dx\right)^{1/p},$$

and the supremum is taken over all balls $B\subset\mathbb{R}^n$.

In order to deal with the fractional order case, we need to consider the weighted Morrey space with two weights.

Definition 1.2 (see [5]). Let $1\le p<\infty$ and $0<\kappa<1$. Then for two weights μ and ν, the weighted Morrey space is defined by

$$L^{p,\kappa}(\mu,\nu)=\Big\{f\in L^p_{loc}(\mu):\|f\|_{L^{p,\kappa}(\mu,\nu)}<\infty\Big\},$$

where

$$\|f\|_{L^{p,\kappa}(\mu,\nu)}=\sup_B\left(\frac{1}{\nu(B)^\kappa}\int_B|f(x)|^p\mu(x)dx\right)^{1/p},$$

and the supremum is taken over all balls $B\subset\mathbb{R}^n$.

We introduce the definition of the Hardy-Littlewood maximal operator and several variants.

Definition 1.3. The Hardy-Littlewood maximal operator Mf is defined by

$$M(f)(x)=\sup_{x\in B}\frac{1}{|B|}\int_B|f(y)|dy.$$

For $0\le\alpha<n$, $r\ge1$, we define the fractional maximal operator $M_{\alpha,r}f$ by

$$M_{\alpha,r}(f)(x)=\sup_{x\in B}\left(\frac{1}{|B|^{1-\alpha r/n}}\int_B|f(y)|^rdy\right)^{1/r},$$