Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2024-0192

Vol. **39**, No. 5, pp. 1512-1535
May 2026

Kinetic Representation of the Unified Gas-Kinetic Wave-Particle Method and Beyond

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Received 12 August 2024; Accepted (in revised version) 8 January 2025

Abstract. The unified gas-kinetic wave-particle (UGKWP) method is a hybrid method for multiscale flow simulations, in which the contributions to the whole gas evolution from deterministic hydrodynamic wave and stochastic particle transport are combined simultaneously. Originally, the UGKWP method was developed as a direct modeling approach at discrete level. In this work, we revisit the time evolution of each part of the involved simulation particles and wave molecules in UGKWP, and present the corresponding kinetic equations. The resultant kinetic system can be viewed as a collision decomposition of the original kinetic equation, which can serve as a basis for developing other kinetic methods for flows in all flow regimes.

AMS subject classifications: 76-10, 76M28, 76N15, 76P05

Key words: Collision decomposition, multiscale method, UGKWP, kinetic equation.

1 Introduction

It is a challenging task to simulate gas flows involving different flow regimes in which the Knudsen number ϵ can vary over a wide range. Classical computational fluid dynamics (CFD) techniques based on Euler or Navier–Stokes equations are limited to continuum flows, while stochastic particle methods, such as the Direct Simulation Monte Carlo (DSMC) method, are mainly suitable for high-speed rarefied flows but usually encounter difficulties for continuum flows.

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In recent years, a variety of deterministic discrete-velocity methods (DVMs) based on gas kinetic theory have been developed for simulating multiscale flows [1, 2]. Particularly, kinetic schemes with asymptotic preserving (AP) properties have received much attention due to their capability in capturing the hydrodynamic behaviors in the limit of $\epsilon \to 0$ without resolving kinetic scales [3, 4]. A unified preserving (UP) concept was further proposed recently [5], which can be used to assess the order of asymptotics of a kinetic scheme for finite ϵ. The unified gas-kinetic scheme (UGKS) [6] and discrete unified gas-kinetic scheme (DUGKS) [7] are two typical kinetic methods with UP properties, which have found wide applications in multiscale flow simulations [8, 9].

Although the deterministic DVMs with AP or UP properties can avoid limitations on mesh size and time step by kinetic scales, a large number of discrete velocities are required for flows involving multiple flow regimes (particularly for high Mach number flows), leading to rather expensive memory and computational costs. Recently, a hybrid approach, named as unified gas-kinetic wave-particle (UGKWP) method, was developed to combine the advantages of stochastic particle method and the UGKS [10]. In UGKWP, the gas system is represented as a two-phase system, i.e., continuum hydrodynamic wave phase composed of gas molecules and discrete particle phase composed of simulation particles. The hydrodynamic wave reflects the collective dynamics of molecules undergoing intensive collisions, and describes the evolution of the near-equilibrium state; on the other hand, the discrete particle phase is composed of a number of simulation particles representing a cluster of gas molecules, which models the evolution of non-equilibrium states. The interaction between the two phases is realized by absorbing collided particles into wave and generating free transport particles from wave.

In the UGKWP method, particle transport is tracked directly such that no discrete particle velocity space is required, which significantly reduces the memory and computational costs. On the other hand, the hydrodynamic wave is captured from the equilibrium state deterministically. The contributions to the gas evolution from the wave and particle phases depend on the local cell Knudsen number, such that the transport of discrete particles dominates the solution in rarefied regime, while the hydrodynamic wave dominates the solution in continuum regime. Additionally, the number of particles changes adaptively with the local cell Knudsen number. This suggests that the particle phase and wave phase become negligible in the continuum and free molecular limits, respectively. Consequently, the UGKWP can serve as an efficient method for multiscale flow simulations.

The evolution of the entire gas system in UGKWP is described by the dynamics of simulation particles and hydrodynamic wave molecules, during which the two phases interact with each other. It should be noted, however, that UGKWP is designed at discrete level as a direct modeling method [8]. Here the concept of "direct modeling" means the gas evolution is modeled directly at a discrete numerical (mesh size and time step) scale, emphasizing that the transport process is implemented directly to construct a multi-scale scheme. This concept is different from that in the Molecular Dynamics (MD) and Direct Simulation Monte Carlo (DSMC) methods, in which real physics is modeled by simula-