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Adaptive Reconstruction Method Using Discontinuity Feedback for High-Order Accuracy Schemes

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Abstract. This paper presents efficient adaptive stencil extension reconstruction methods using a discontinuity feedback factor to address weak robustness and high computational costs in high-order schemes (7th-order and above). The method features two innovations: accuracy order adaptively increases from the lowest level based on local stencil smoothness, unlike WENO and MUSCL limiters that reduce order from highest level; and the Discontinuity Feedback Factor detects sub-cell discontinuity strength while serving as a local smoothness measure. This eliminates expensive smoothness indicators in very high-order schemes and generalizes to arbitrary high orders. Rigorous tests, including a Mach 20000 jet, demonstrate exceptional robustness.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

Key words: Computational efficiency, discontinuity feedback factor, adaptive stencil extension, high-order schemes.

1 Introduction

Contemporary CFD research emphasizes developing high-order schemes for turbulent flow simulation, building on Harten et al.'s foundational work [12]. Key methodological advances include Essential Non-Oscillatory (ENO) schemes [12, 30], Weighted Essential

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Non-Oscillatory (WENO) schemes [19,26], and Discontinuous Galerkin (DG) methods [6, 7], which have enhanced capabilities for handling complex flow problems with improved accuracy and efficiency.

High-order numerical methods face two significant challenges. First, algorithm robustness decreases rapidly with increasing order when calculating discontinuity problems. Recent improvements include new stencil selection strategies, combining nonlinear and linear weights [10, 27], and TENO schemes that adapt to local flow characteristics [19,30]. Limiters provide another approach, though prior limiters (van Leer [32], van Albada [32]) are parameter-sensitive and post limiters like MOOD [5] are computationally intensive. Second, these schemes are computationally expensive. While one-stage high-order methods [24] reduce temporal advancement costs compared to classical Runge-Kutta approaches, spatial reconstruction remains challenging. WENO schemes require complex smoothness indicators, especially at higher orders. Current solutions include using lower-order stencil combinations [14, 20, 21] and weighted-least-squares methods [13, 25], but these either lack arbitrary high-order applicability or suffer from weak robustness and non-unified global smoothness indicators.

In order to deal with the above challenges, the Adaptive Stencil Extension reconstruction methods based on Discontinuity Feedback factor (ASE-DF) are constructed. As presented in Section 3.1, DF is to define the discontinuity strength associated with the stencil-based reconstruction. Without imposing the continuous flow distribution assumption inside each cell, the use of DF makes it more flexible in developing high-order FVM and its transition to be first-order scheme for preserving positivity property. The use of DF not only improves the robustness of the algorithm, but also replaces the smoothness indicators for each stencil, which significantly reduces the computational cost while achieving arbitrary high-order schemes. In the DF-based schemes presented in this paper, two flux functions are considered. The first one is the Lax-Friedrichs (L-F) with the SSP-RK [11] temporal discretization, which has the positivity stability preserving, and the other is the 2nd-order gas-kinetic scheme (GKS) [34,35] with two-stages fourth-order (S2O4) temporal discretization.

This paper is organized as follows: Section 2 provides a brief overview of the high-order finite volume scheme. Section 3 introduces the ASP-DF reconstruction methods. In Section 4, some numerical results of the viscous and inviscid flow problems will be presented. The last section is the conclusion.

2 High-order finite volume scheme

2.1 Finite volume framework

The Finite Volume Method (FVM) involves dividing the computational domain into a finite amount of control volumes. Within each control volume, the physical quantities