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Stability Analysis and Structure Preserving Schemes for the Reactive Euler Equations with a New Equation of State

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Abstract. This paper is concerned with the multi-dimensional reactive Euler equations under the new equation of state (EoS) recently proposed in [45]. We show that under this EoS the classical thermodynamic entropy is a strictly convex entropy function in general dimension. Based on this we further prove that the reactive Euler equations satisfy the stability conditions for hyperbolic relaxation systems, which guarantee the existence of zero relaxation limit. The eigen-decompositions of the Jacobian matrices in two and three dimensions are also provided. Moreover, we develop a positivity preserving and oscillation-free entropy stable discontinuous Galerkin scheme by adapting that in [46] for the EoS of ideal gas to the newly proposed one. A key step in doing so is to prove that the HLL (Harten-Lax-van Leer) flux is entropy stable, which is established by tactfully using a natural assumption on a function in the EoS. The high convergence orders stable entropy, no oscillation and positivity of the scheme are demonstrated with numerical examples.

AMS subject classifications: 76M25, 65M60, 65M12

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1 Introduction

In this work, we are interested in inviscid compressible reactive Euler equations in D dimensions [9,10]:

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$$\partial_t \begin{pmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \vdots \\ \rho u_D \\ \rho E \\ \rho Y \end{pmatrix} + \partial_{x_1} \begin{pmatrix} \rho u_1 \\ \rho {u_1}^2 + p \\ \rho u_1 u_2 \\ \vdots \\ \rho u_1 u_D \\ \rho E u_1 + p u_1 \\ \rho u_1 Y \end{pmatrix} + \partial_{x_2} \begin{pmatrix} \rho u_2 \\ \rho u_1 u_2 \\ \rho {u_2}^2 + p \\ \vdots \\ \rho u_2 u_D \\ \rho E u_2 + p u_2 \\ \rho u_2 Y \end{pmatrix} + \cdots + \partial_{x_D} \begin{pmatrix} \rho u_D \\ \rho u_1 u_D \\ \rho u_2 u_D \\ \vdots \\ \rho {u_D}^2 + p \\ \rho E u_D + p u_D \\ \rho u_D Y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \omega \end{pmatrix}, \tag{1.1}$$

where ρ is the fluid density, u_i is the velocity along the direction x_i with $i = 1,2,\cdots,D$, p is the pressure, E is the total energy per unit mass and $Y \in [0,1]$ is the reactant mass fraction. The non-positive source term ω represents the chemical reaction and we adopt the widely used Arrhenius form

$$\omega = -\rho Y \tilde{K} e^{-\tilde{T}/T}, \tag{1.2}$$

where $\tilde{K} > 0$ is a constant rate coefficient, $T = p/\rho$ is the temperature and $\tilde{T} > 0$ is the activation constant temperature. The total energy is related to the internal energy ρe (e is the internal energy per unit mass) as

$$\rho E = \frac{1}{2}\rho \sum_{i=1}^{D} u_i^2 + \rho e + q\rho Y, \tag{1.3}$$

where $q > 0$ is the heat release of reaction. To close the system, we adopt the equation of state (EoS) proposed in [45]:

$$p = (\gamma - 1)(\rho e + q\rho Y - \rho\zeta), \tag{1.4}$$

where γ is the specific heat ratio, $\zeta := \zeta(Y)$ is a smooth function of Y. When $\zeta = 0$, (1.4) degenerates to the EoS for the ideal gas.

For the original Euler equations with chemical reactions of N species, there are N species conservation equations. Unlike this, the reactive Euler equations (1.1) are a simplified system under the assumption that there is only one-step irreversible chemical reaction converting the unreacted species to the reacted species [7,9,10]. Though relatively simple in form, (1.1) can give complicated stable and unstable wave patterns, which have been observed in experiments [2,7]. Due to their practical importance, the reactive Euler equations have been widely investigated both theoretically and numerically in the past decades, see *e.g.* [1,2,4,6,7,11,13,15,16,23,24,35].

Eqs. (1.1) are a system of hyperbolic balance laws and degenerate to the Euler equations when Y = 0. It is well known that hyperbolic balance laws can generate shock discontinuities in a finite time, and thus the solutions are considered in a weak sense and are