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The Implicit Unified Gas Kinetic Scheme for Neutron Transport with Arbitrary Anisotropic Scattering Model

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Abstract. In previous work, we developed the unified gas dynamics scheme (UGKS) and its corresponding time-implicit variant (IUGKS) for neutron transport. The multi-scale characteristics of these schemes are particularly well-suited for transport equations, which show good performances in large-scale conditions. However, only the isotropic scattering model equation was considered in these studies, which does not meet the application requirements in complex engineering cases. In this paper, we extend our studies of isotropic neutron IUGKS to encompass the transport equation with arbitrary anisotropic scattering. We first clarify the construction principle of IUGKS-PN scheme according to the general form of anisotropic scattering models. However, the construction of the scheme is complex and the versatility is not strong. To address these challenges, a more concise scheme IUGKS-AM is proposed based on the isotropic IUGKS framework, in which the higher-order moments of the neutron distribution function are estimated. We theoretically analyze the asymptotic property of IUGKS-AM, and verify the numerical performances in anisotropic model by numerical experiments. Finally, several complex anisotropic scattering models are computed by using the numerical scheme, demonstrating its general applicability. This study significantly enhances the practical utility of multi-scale neutron transport UGKS in engineering applications.

AMS subject classifications: 82D75, 76P05, 65M08
Key words: 3D neutron transport, UGKS, anisotropic scattering, time implicit.

1 Introduction

In previous works, we developed the unified gas kinetic scheme (UGKS) and the corresponding time-implicit scheme (IUGKS) for neutron transport simulations [1, 2]. The

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results show that IUGKS achieves ideal accuracy and efficiency in the calculation of three-dimensional isotropic neutron transport equation, including the multi-group model calculations. In this paper, we further enhance the computational capabilities of IUGKS by extending it to accommodate anisotropic scattering models for practical applications. The transport equation form with general collision term is,

$$
\begin{aligned}
&\frac{1}{v(E)}\frac{\partial\phi(\mathbf{x},t,\mathbf{\Omega},E)}{\partial t}+\mathbf{\Omega}\cdot\nabla\phi(\mathbf{x},t,\mathbf{\Omega},E)+\Sigma(\mathbf{x},E)\phi(\mathbf{x},t,\mathbf{\Omega},E)\\
=&\int\int\Sigma(\mathbf{x},E')f(E\to E',\mathbf{\Omega}\cdot\mathbf{\Omega}')\phi(\mathbf{x},t,\mathbf{\Omega}',E')\mathrm{d}E'\mathrm{d}\mathbf{\Omega}'+q(\mathbf{x},t,\mathbf{\Omega},E).
\end{aligned}
$$

Here the scattering phase function $f(\mathbf{x},\mathbf{\Omega}',E'\to\mathbf{\Omega},E)$ relys on the angle between $\mathbf{\Omega}$ and $\mathbf{\Omega}'$.

For single-scale SN methods, researches have been conducted on the computation of anisotropic equations [3]. The SN methods employ the splitting approach to deal with the convection and collision terms, and updates the transport equation by the source iteration method. By introducing the anisotropic scattering function directly into the collision term, the corresponding anisotropic numerical scheme can be constructed. The anisotropic collision term does not affect the calculation of the convection term, which is usually the core of numerical scheme construction. Despite the simple constructions, the SN schemes usually suffer from numerical stiffness issues. Therefore, the acceleration methods are needed to improve the convergence speed of the scheme. The common used acceleration methods include high-order/low-order (HOLO) [4], diffusion synthesis acceleration (DSA)/transport synthesis acceleration (TSA) [5,6], quasi-diffusion (QD), multi-grid method [7], etc. These methods have shown good performances in isotropic scattering simulations. But for the anisotropic models, due to that the scattering term has a significant effect on the spectral radius of the iterative equation, the acceleration method needs to be modified to maintain effective convergence. In practical three-dimensional high-order scattering models, these acceleration methods still present certain complexities and difficulties in the algorithms and code.

The mathematical properties of the neutron transport equation vary with the strength of the collision term. In the free transport region with weak collision, the equation is hyperbolic, while in the diffusion region with strong collision, the equation is parabolic. Therefore, transport is a typical multi-scale process. The corresponding numerical scheme should have asymptotic preserving (AP) properties. When the grid size or time step is fixed, and the Knudsen number approaches zero, a numerical scheme with AP properties can automatically recover the solution of the diffusion equation. This ensures the validity and accuracy of the scheme in a wide range of conditions. For isotropic scattering transport equation, the AP schemes have been widely studied [8–11]. Meanwhile, it is challenging to develop numerical schemes with AP properties for anisotropic scattering [12,13].

In recent years, UGKS has been widely studied in solving multi-scale equations. In UGKS, the cell-interface quantities and numerical fluxes are constructed with the time