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A Diffusion Model Based Iterative Convolution Thresholding Method for Structural Topological Optimization

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Abstract. In this study, we introduce the Diffusion Model with Iterative Convolution Thresholding Method (DICTM), a novel hybrid approach designed to address the minimum compliance problem in topology optimization. DICTM synergistically combines the robustness of diffusion models with the precision of threshold dynamics to tackle the complexities inherent in linear elasticity problems, while substantially enhancing computational efficiency. Our approach facilitates the generation of initial configurations via the diffusion model, which dramatically improves the efficiency of the subsequent threshold dynamics process, reducing the iteration count to about one-tenth of that required by traditional methods. This significant reduction in computational effort also enables more effective hyperparameter tuning without added cost. The integration of deep-generative models with a rigorous threshold dynamics framework positions DICTM as a powerful tool in topology optimization, producing designs not only with low compliance, but also in a computationally efficient way.

AMS subject classifications: 74B05, 90C26

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1 Introduction

Topology optimization is a powerful method to determine the material distribution within a given design domain to achieve the best possible structural performance under

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specified constraints. Traditional approaches, such as the Solid Isotropic Material with Penalization (SIMP) method [4], level set methods [1, 22], phase field methods [13, 23], threshold dynamics methods [6, 7], have been widely used to solve topology optimization problems. However, these methods often require careful tuning of parameters and can be computationally intensive, especially for complex structures.

With the advent of data-driven approaches, deep learning techniques have been integrated into topology optimization to accelerate the design process. Generative models have shown promise in achieving near-optimal compliance and shape similarity while substantially reducing computational demands [24]. Variational Autoencoders (VAE) [10, 25] and Generative Adversarial Networks (GAN) [17, 18] have been used to generate designs conditioned on boundary conditions and loads. Despite these advances, there remain challenges to establish reliable and generalizable mappings from boundary conditions to optimal structures. GANs, in particular, face issues with training stability and generalization due to their adversarial nature.

Diffusion models have recently emerged as a stable and effective alternative for generative modeling. First introduced by Sohl-Dickstein et al. [19] and later improved by Ho et al. [11] and Song et al. [21], diffusion models have outperformed GANs in generating high-quality images [9] and have found applications in computer vision [2, 14], natural language processing [3, 12], and medical imaging [5, 8].

However, deep learning methods often struggle to strictly adhere to the mathematical requirements inherent in optimization problems. Generated structures may lack necessary connectivity and smoothness, leading to suboptimal or infeasible solutions. Traditional numerical methods, while mathematically rigorous, can be computationally expensive and sensitive to initial guesses and parameter settings.

In [6], an iterative convolution thresholding method (ICTM) was introduced for topology optimization in linear elasticity. We propose a simple energy decaying algorithm to solve the two-phase minimum compliance problem. The material domain is implicitly represented by its characteristic function, and the problem is formulated as a minimization-minimization problem by the principle of minimum complementary energy. We show that the energy is decreasing in each iteration and a local minimum solution is quickly approached. Two effective continuation schemes are proposed to avoid being trapped in a bad local minimum.

In this study, we propose a novel approach that integrates diffusion models with the iterative convolution thresholding method (ICTM) for topology optimization in linear elasticity. Using the diffusion model to provide a high-quality initial guess, we significantly accelerate the convergence of the threshold dynamics method. This hybrid approach, termed the DICTM (Diffusion model with Iterative Convolution Thresholding Method), leverages the predictive capabilities of deep-generative models and the mathematical rigor of numerical methods.

Our contributions can be summarized as follows.

- We develop a hybrid topology optimization method that combines diffusion mod-