Commun. Comput. Phys.
doi: 10.4208/cicp.140911.050412a

Vol. **13**, No. 4, pp. 985-1012
April 2013

Wave Propagation Across Acoustic/Biot's Media: A Finite-Difference Method

Guillaume Chiavassa[1,*] and Bruno Lombard[2]

[1] *Centrale Marseille, Laboratoire de Mécanique Modélisation et Procédés Propres, UMR 7340 CNRS, Technopôle de Chateau-Gombert, 38 rue Frédéric Joliot-Curie, 13451 Marseille, France.*

[2] *Laboratoire de Mécanique et d'Acoustique, UPR 7051 CNRS, 31 chemin Joseph Aiguier, 13402 Marseille, France.*

Received 14 September 2011; Accepted (in revised version) 5 April 2012

Communicated by Wei Cai

Available online 21 September 2012

Abstract. Numerical methods are developed to simulate the wave propagation in heterogeneous 2D fluid/poroelastic media. Wave propagation is described by the usual acoustics equations (in the fluid medium) and by the low-frequency Biot's equations (in the porous medium). Interface conditions are introduced to model various hydraulic contacts between the two media: open pores, sealed pores, and imperfect pores. Well-posedness of the initial-boundary value problem is proven. Cartesian grid numerical methods previously developed in porous heterogeneous media are adapted to the present context: a fourth-order ADER scheme with Strang splitting for time-marching; a space-time mesh-refinement to capture the slow compressional wave predicted by Biot's theory; and an immersed interface method to discretize the interface conditions and to introduce a subcell resolution. Numerical experiments and comparisons with exact solutions are proposed for the three types of interface conditions, demonstrating the accuracy of the approach.

AMS subject classifications: 35L05, 35L50, 65N06, 65N85, 74F10

Key words: Biot's model, poroelastic waves, jump conditions, imperfect hydraulic contact, high-order finite differences, immersed interface method.

---

1 Introduction

The theory developed by Biot in 1956 [3, 4] is largely used to describe the wave propagation in poroelastic media. Three kinds of waves are predicted: the usual shear wave and

*Corresponding author. *Email addresses:* `guillaume.chiavassa@centrale-marseille.fr` (G. Chiavassa), `lombard@lma.cnrs-mrs.fr` (B. Lombard)

"fast" compressional wave (as in elastodynamics), and an additional "slow" compressional wave observed experimentally in 1981 [33]. This slow wave is a static mode below a critical frequency, depending on the viscosity of the saturating fluid. In the current study, we will focus on this low-frequency range.

The coupling between acoustic and poroelastic media is of high interest in many applications: sea bottom in underwater acoustics [38], borehole logging in civil engineering [36], and bones in biomechanics [21]. Many theoretical efforts have dealt with the acoustic/porous wave propagation. Various boundary conditions have been proposed to describe the hydraulic contacts: open pores, sealed pores, and imperfect pores involving the hydraulic permeability of the interface [6, 15, 36]. Reflection and transmission coefficients of plane waves have been derived [39]. The influence of the interface conditions on the existence of surface waves has been investigated in the case of inviscid [15] and viscous saturating fluids [13, 17] in the porous material. The time-domain Green's function has been computed by the Cagniard-de Hoop's method [12, 16]. Experimental works have shown the crucial importance of hydraulic contact on the generation of slow compressional wave [35].

The literature dedicated to numerical methods for porous wave propagation is large: see [23], [9] for a review, and the introduction of [11] for a list of time-domain methods. Coupled fluid/porous configurations have been addressed by an integral method [18], a spectral-element method [32], and a pseudospectral method [37], to cite a few. To simulate efficiently wave propagation in fluid/porous media, numerical methods must overcome the following difficulties:

- In the low-frequency range, the slow compressional wave is a diffusive-like solution, and the evolution equations become stiff [34]. It drastically restricts the stability condition of any explicit method;
- The diffusive slow compressional wave remains localized near the interfaces. Capturing this wave - that plays a key role on the balance equations - requires a very fine spatial mesh;
- An accurate description of arbitrary-shaped geometries with various interface conditions is crucial. These properties are badly discretized by finite-difference methods on Cartesian grids. Alternatively, unstructured meshes provide accurate descriptions, but the computational effort greatly increases;
- An accurate modeling of the hydraulic contact at the interface is also required. In particular, as far as we know, imperfect pore conditions still have not been addressed in numerical models.

To overcome these difficulties, we adapt a methodology previously developed in porous/porous media [11] and fluid/viscoelastic media [28]. Three Cartesian grid numerical methods are put together. A fourth-order ADER scheme with Strang splitting is used to integrate the evolution equations, ensuring an optimal CFL condition of stability. Specific solvers are used in the fluid medium and in the porous medium. Their