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Runge-Kutta Discontinuous Galerkin Method with a Simple and Compact Hermite WENO Limiter

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Abstract. In this paper, we propose a new type of weighted essentially non-oscillatory (WENO) limiter, which belongs to the class of Hermite WENO (HWENO) limiters, for the Runge-Kutta discontinuous Galerkin (RKDG) methods solving hyperbolic conservation laws. This new HWENO limiter is a modification of the simple WENO limiter proposed recently by Zhong and Shu [29]. Both limiters use information of the DG solutions only from the target cell and its immediate neighboring cells, thus maintaining the original compactness of the DG scheme. The goal of both limiters is to obtain high order accuracy and non-oscillatory properties simultaneously. The main novelty of the new HWENO limiter in this paper is to reconstruct the polynomial on the target cell in a least square fashion [8] while the simple WENO limiter [29] is to use the entire polynomial of the original DG solutions in the neighboring cells with an addition of a constant for conservation. The modification in this paper improves the robustness in the computation of problems with strong shocks or contact discontinuities, without changing the compact stencil of the DG scheme. Numerical results for both one and two dimensional equations including Euler equations of compressible gas dynamics are provided to illustrate the viability of this modified limiter.

AMS subject classifications: 65M60, 35L65

Key words: Runge-Kutta discontinuous Galerkin method, HWENO limiter, conservation law.

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1 Introduction

In this paper, we are interested in solving the hyperbolic conservation law

$$\begin{cases} u_t+f(u)_x=0, \\ u(x,0)=u_0(x), \end{cases} \tag{1.1}$$

and its two-dimensional version

$$\begin{cases} u_t+f(u)_x+g(u)_y=0, \\ u(x,y,0)=u_0(x,y), \end{cases} \tag{1.2}$$

using the Runge-Kutta discontinuous Galerkin (RKDG) methods [4–7], where u, $f(u)$ and $g(u)$ can be either scalars or vectors.

It is not an easy task to solve (1.1) and (1.2) since solutions may contain discontinuities even if the initial conditions are smooth. Discontinuous Galerkin (DG) methods can capture weak shocks and other discontinuities without further modification. However, for problems with strong discontinuous solutions, the numerical solution has significant oscillations near discontinuities, especially for high order methods. A common strategy to control these spurious oscillations is to apply a nonlinear limiter. One type of limiters is based on the slope methodology, such as the *minmod* type limiters [4–7], the moment based limiter [2] and an improved moment limiter [3]. These limiters do control the oscillations well, however they may degrade accuracy when mistakenly used in smooth regions of the solution. Another type of limiters is based on the weighted essentially non-oscillatory (WENO) methodology [11–13,17], which can achieve both high order accuracy and non-oscillatory properties. The WENO limiters introduced in [18–20,22,30] and the Hermite WENO limiters in [19,22] belong to this type. These limiters are designed based on the WENO finite volume methodology which require a wider stencil for higher order schemes. Therefore, it is difficult to implement them for multi-dimensional problems, especially on unstructured meshes. An alternative family of DG limiters which serves at the same time as a new PDE-based limiter, as well as a troubled cells indicator, was introduced by Dumbser et al. [10].

More recently, a particularly simple and compact WENO limiter, which utilizes fully the advantage of DG schemes in that a complete polynomial is available in each cell without the need of reconstruction, is designed for RKDG schemes in [29]. The two major advantages of this simple WENO limiter are the compactness of its stencil, which contains only immediate neighboring cells, and the simplicity in implementation, especially for unstructured meshes [31]. However, it was observed in [29] that the limiter might not be robust enough for problems containing very strong shocks or low pressure problem, especially for higher order polynomials, for example the blast wave problems [23,28] and the double rarefaction wave problem [16], making it necessary to apply additional positivity-preserving limiters [27] in such situation. In order to overcome this difficulty, without compromising the advantages of compact stencil and simplicity of linear weights, we present a modification of the limiter in the step of preprocessing the