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Pseudo-Arclength Continuation Algorithms for Binary Rydberg-Dressed Bose-Einstein Condensates

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Abstract. We study pseudo-arclength continuation methods for both Rydberg-dressed Bose-Einstein condensates (BEC), and binary Rydberg-dressed BEC which are governed by the Gross-Pitaevskii equations (GPEs). A divide-and-conquer technique is proposed for rescaling the range/ranges of nonlocal nonlinear term/terms, which gives enough information for choosing a proper stepsize. This guarantees that the solution curve we wish to trace can be precisely approximated. In addition, the ground state solution would successfully evolve from one peak to vortices when the affect of the rotating term is imposed. Moreover, parameter variables with different number of components are exploited in curve-tracing. The proposed methods have the advantage of tracing the ground state solution curve once to compute the contours for various values of the coefficients of the nonlocal nonlinear term/terms. Our numerical results are consistent with those published in the literatures.

AMS subject classifications: 65N35, 35P30, 35Q55

Key words: Ground state solution, solution branch, spectral collocation method, divide-and-conquer..

1 Introduction

In this paper, we are concerned with pseudo-arclength continuation algorithms for computing numerical solutions of nonlinear eigenvalue problems of the following form

$$\mathbf{F}(\mathbf{x},\Lambda)=0, \tag{1.1}$$

where $\mathbf{F}:\mathbf{B_1}\times\mathbf{R}^m\to\mathbf{B_2}$ is a smooth mapping with $\mathbf{x}\in\mathbf{B_1}$, $\Lambda=(\lambda_1,\cdots,\lambda_m)\in\mathbf{R}^m$, $\mathbf{x}$ is the state variable, Λ is the parameter variable, and $\mathbf{B_1}$ and $\mathbf{B_2}$ are two Banach spaces. A

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typical example of Eq. (1.1) is the Gross-Pitaevskii equation (GPE), where the parameters $\lambda_i, 1\le i\le m$, have specific physical meaning.

Recently, it has been observed that a quantum system of interacting particles can exhibit both crystalline structure and superfluid property [1, 2]. Henkel *et al.* [3] showed that the particles of Bose-Einstein condensates (BEC) interacting through an isotropically repulsive van der Waals interaction with a softened core might support a density modulation. They observed that spontaneously crystalline ground states, called quantum crystals, could exist on trapped Rydberg-dressed BEC. Based on the mean-field theory, we consider two-component (rotating) Rydberg-dressed BEC which is governed by the coupled GPEs of the following form:

$$\begin{aligned}\mathbf{i}\partial_t\Psi_1(\mathbf{r},t)=&\Big[-\frac{1}{2}\nabla^2+V(\mathbf{r})+\gamma_{11}|\Psi_1(\mathbf{r},t)|^2+\gamma_{12}|\Psi_2(\mathbf{r},t)|^2+\mu_{11}\int\frac{|\Psi_1(\mathbf{r}',t)|^2}{1+|\mathbf{r}-\mathbf{r}'|^6}d\mathbf{r}'\\&+\mu_{12}\int\frac{|\Psi_2(\mathbf{r}',t)|^2}{1+|\mathbf{r}-\mathbf{r}'|^6}d\mathbf{r}'-\omega L_z\Big]\Psi_1(\mathbf{r},t),\quad t>0,\\\mathbf{i}\partial_t\Psi_2(\mathbf{r},t)=&\Big[-\frac{1}{2}\nabla^2+V(\mathbf{r})+\gamma_{22}|\Psi_2(\mathbf{r},t)|^2+\gamma_{21}|\Psi_1(\mathbf{r},t)|^2+\mu_{22}\int\frac{|\Psi_2(\mathbf{r}',t)|^2}{1+|\mathbf{r}-\mathbf{r}'|^6}d\mathbf{r}'\\&+\mu_{21}\int\frac{|\Psi_1(\mathbf{r}',t)|^2}{1+|\mathbf{r}-\mathbf{r}'|^6}d\mathbf{r}'-\omega L_z\Big]\Psi_2(\mathbf{r},t),\quad t>0,\end{aligned}\tag{1.2}$$

where (Ψ_1,Ψ_2) is the vector of two-component complex wave functions, $V(\mathbf{r})=\eta^2\mathbf{r}^2/2$ is the harmonic trapping potential with η the radius frequency and $\mathbf{r}\in\mathbf{R}^n, n=2,3$, γ_{jj} $(j=1,2)$ and γ_{jl} $(j,l=1,2)$ are the short-range intra-component interactions and inter-component interactions in the two-component BECs, respectively, μ_{jj} $(j=1,2)$ and μ_{jl} $(j,l=1,2)$ are the long-range intra-component interactions and inter-component interactions in the two-component BECs, or the coefficients of the nonlocal nonlinear terms, respectively, ω is an angular velocity, and $L_z=xp_y-yp_x=-\mathbf{i}(x\partial y-y\partial x)$ is the z-component of the angular momentum $\mathbf{L}=\mathbf{r}\times\mathbf{P}$ with the momentum operator $\mathbf{P}=-\mathbf{i}\nabla=(p_x,p_y,p_z)^T$. A special case of Eq. (1.2) is the one-component Rydberg-dressed BEC which is governed by the GPE of the following form:

$$\mathbf{i}\partial_t\Psi(\mathbf{r},t)=\Big[-\frac{1}{2}\nabla^2+V(\mathbf{r})+\gamma|\Psi(\mathbf{r},t)|^2+\mu\int\frac{|\Psi(\mathbf{r}',t)|^2}{1+|\mathbf{r}-\mathbf{r}'|^6}d\mathbf{r}'-\omega L_z\Big]\Psi(\mathbf{r},t),\tag{1.3}$$

where the parameters used in Eq. (1.3) have the same physical meaning as those used in Eq. (1.2). Substituting the formula

$$\Psi_j(\mathbf{r},t)=e^{-\mathbf{i}\lambda_j t}\psi_j(\mathbf{r}),\quad j=1,2$$

into Eq. (1.2), we obtain the coupled stationary state nonlinear eigenvalue problem

$$\begin{aligned}F_1(\psi_1,\psi_2,\lambda_1,\lambda_2)=&\Big[-\frac{1}{2}\nabla^2+V(\mathbf{r})+\gamma_{11}|\psi_1(\mathbf{r})|^2+\gamma_{12}|\psi_2(\mathbf{r})|^2+\mu_{11}\int\frac{|\psi_1(\mathbf{r}')|^2}{1+|\mathbf{r}-\mathbf{r}'|^6}d\mathbf{r}'\\&+\mu_{12}\int\frac{|\psi_2(\mathbf{r}')|^2}{1+|\mathbf{r}-\mathbf{r}'|^6}d\mathbf{r}'-\omega L_z-\lambda_1\Big]\psi_1(\mathbf{r})=0,\end{aligned}$$