Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2017-0007

Vol. **22**, No. 3, pp. 863-888
September 2017

# Equilibrium Configurations of Classical Polytropic Stars with a Multi-Parametric Differential Rotation Law: A Numerical Analysis

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Received 13 January 2017; Accepted (in revised version) 27 February 2017

**Abstract.** In this paper we analyze in detail the equilibrium configurations of classical polytropic stars with a multi-parametric differential rotation law of the literature using the standard numerical method introduced by Eriguchi and Mueller. Specifically we numerically investigate the parameters' space associated with the velocity field characterizing both equilibrium and non-equilibrium configurations for which the stability condition is violated or the mass-shedding criterion is verified.

**PACS**: 04.40.-b, 02.30.Jr, 02.60.-x

**Key words**: Free boundary problems, self-gravitating systems, numerical methods for partial differential equations.

## 1 Introduction

The problem of equilibrium of rotating self-gravitating systems, dating back to Newton's Principia Mathematica studies on the Earth's shape, still represents a very actual topic in

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the field of astrophysics. Its main target is to reconstruct the structure of rotating stars considered to be, in a first approximation, in hydrostatic equilibrium although more complicated hydrodynamical effects can be taken into account by using modern tools of numerical analysis. Historically the studies on spherical non rotating self-gravitating bodies (well summarized in the classical Chandrasekhar's monograph on stellar structure [1]) and on uniformly rotating ones in the case of incompressible fluids (deeply analysed too in the companion Chandrasekhar's monograph on ellipsoidal figures of equilibrium [2], as well as, for instance, in [4–12]) preceded the study of compressible uniformly rotating polytropic stars [3]. All of these studies were completed by a series of refined numerical integrations of the complicated field equations governing the problem, performed by the Japanese school, which specifically investigated the problem of self-gravitating fluid's shape bifurcations [13–20]. The next step then has been the inclusion of differential rotation laws in the treatment, for instance in [21, 22], where rotation profiles were considered admitting an exact integral relation leading to an analytical expression of the centrifugal potential term in the hydrostatic equilibrium equation. In the literature it is known that differential rotation plays an important role in modelling the rotating stars' structure, in particular for both initial and ending phases of the stars' life. Most of the aforementioned works dealt with barotropic stars, i.e. configurations in which isopycnic (constant density) and isobaric (constant pressure) surfaces coincide, although it has been recently stressed the importance to consider also more general situations, like the baroclinic one (in which isopycnic surfaces are inclined over isobaric ones) in order to obtain more realistic configurations [23]. We have to point out also that although many recent papers dealt with relativistic figures of equilibrium (see for instance e.g. [24] and references therein) in relation to the problem of modelling possible sources of gravitational waves, the initial step to investigate the effects of pure rotation is to consider the problem of classical figures of equilibrium first. In the present paper, we will analyse in detail i) a polytropic classical self-gravitating fluid, ii) with axial and equatorial symmetry and with iii) a multi-parametric differential rotation law, which was proposed in [25] without a systematic analysis of the possible configurations belonging to such a velocity profile. The main feature of this rotation profile is that, with respect to the study in [21], this one can be considered as a generalization because it does not admit an analytical expression for the integral for centrifugal potential term. In addition, the presence of different free-parameters allows a more detailed study of the way in which the star rotates. By using the general method given in [21] in order to perform an analysis of the free-parameters' space, we identify the presence of possible bifurcation points in the configurations' sequences. The article is organized as follows. In Section 2, the numerical method by Eriguchi and Mueller [21] is briefly reviewed, the multi-parametric differential rotation profile taken by [25] is discussed and an analysis of possible instabilities which may be reached is performed. In Section 3, we show results locating stable configurations within the free-parameters' space and focusing on how different values of parameters in the rotation law could lead to different shaped configurations. The correctness of results is checked and already known results of [21] are recovered. In Section 4 we summarize and