Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2016-0171

Vol. **23**, No. 1, pp. 264-295
January 2018

Second-Kind Boundary Integral Equations for Scattering at Composite Partly Impenetrable Objects

Xavier Claeys[1], Ralf Hiptmair[2,*] and Elke Spindler[2]

[1] *Sorbonne Universités, UPMC Univ Paris 06, CNRS, INRIA, UMR 7598, Laboratoire Jacques-Louis Lions, équipe Alpines, F-75005, Paris, France.*
[2] *Seminar for Applied Mathematics, Swiss Federal Institute of Technology, CH-8092 Zurich, Switzerland.*

Received 4 October 2016; Accepted (in revised version) 4 May 2017

Abstract. We consider acoustic scattering of time-harmonic waves at objects composed of several homogeneous parts. Some of those may be impenetrable, giving rise to Dirichlet boundary conditions on their surfaces. We start from the recent second-kind boundary integral approach of [X. Claeys, and R. Hiptmair, and E. Spindler. *A second-kind Galerkin boundary element method for scattering at composite objects.* BIT Numerical Mathematics, 55(1):33-57, 2015] for pure transmission problems and extend it to settings with essential boundary conditions. Based on so-called global multi-potentials, we derive variational second-kind boundary integral equations posed in $L^2(\Sigma)$, where Σ denotes the union of material interfaces. To suppress spurious resonances, we introduce a combined-field version (CFIE) of our new method.

Thorough numerical tests highlight the low and mesh-independent condition numbers of Galerkin matrices obtained with discontinuous piecewise polynomial boundary element spaces. They also confirm competitive accuracy of the numerical solution in comparison with the widely used first-kind single-trace approach.

AMS subject classifications: 65N12, 65N38, 65R20
Key words: Acoustic scattering, second-kind boundary integral equations, Galerkin boundary element methods.

1 Introduction

1.1 Acoustic scattering boundary value problem

The governing equation for acoustic scattering of time-harmonic waves is the Helmholtz equation. In this article, we confine ourselves to the case of a globally constant principal part given by $-\Delta$.

*Corresponding author. *Email addresses:* `xavier.claeys@upmc.fr` (X. Claeys), `hiptmair@sam.math.ethz.ch` (R. Hiptmair), `elke.spindler@sam.math.ethz.ch` (E. Spindler)

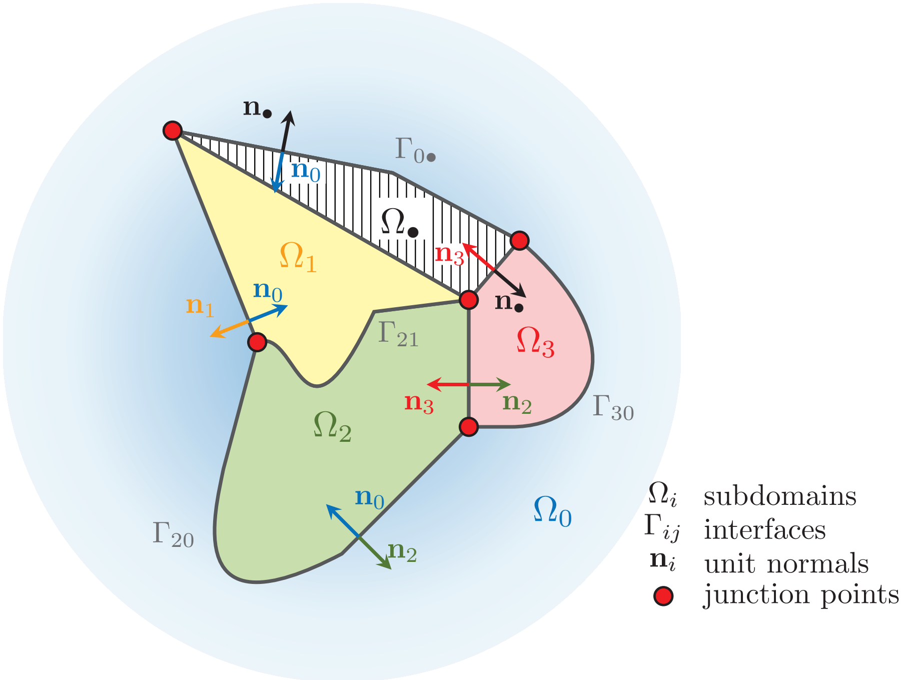

Figure 1: Two-dimensional illustration of a typical geometry of a composite scatterer for $L=3$.

The scatterer occupies a bounded domain $\Omega_* \subset \mathbb{R}^d$, $d=2,3$. We assume a partitioning of Ω_* into open Lipschitz subdomains, i.e. $\overline{\Omega}_* = \left(\bigcup_{i=1}^{L} \overline{\Omega}_i\right) \cup \overline{\Omega}_\bullet$, where $\overline{\Omega}$ denotes the closure of the domain Ω. The subdomains $\Omega_1,\cdots,\Omega_L$ represent the different homogeneous penetrable materials whereas the impenetrable object with Lipschitz curvilinear polygonal/polyhedral boundary is given by $\Omega_\bullet$. See Fig. 1 for a drawing of the scatterer in the case $d=2$. The unbounded exterior complement of the scatterer is given by the Lipschitz domain $\Omega_0 := \mathbb{R}^d \setminus \overline{\Omega}_*$. Like $\Omega_1,\cdots,\Omega_L$, also Ω_0 is filled with homogeneous penetrable material. We characterize the penetrable materials by their wave numbers $\kappa_i \in \mathbb{R}_+$, for $i \in \{0,1,\cdots,L\}$. They enter the piecewise constant coefficient function $\kappa \in L^\infty(\mathbb{R}^d)$, $\kappa\big|_{\Omega_i} \equiv \kappa_i$. The impenetrable object $\Omega_\bullet$ will be modeled by imposing Dirichlet boundary conditions at its boundary $\partial\Omega_\bullet$.

By construction, we observe that $\Omega_i \cap \Omega_j = \emptyset$ for $j \neq i$, for indices $i,j \in \{\bullet,0,1,\cdots,L\}$. The boundary of the subdomain Ω_i is given by $\partial\Omega_i$ for $i \in \{\bullet,0,1\cdots,L\}$. For Lipschitz domains, and in particular for each Ω_i, there exists a unit normal vector field $\mathbf{n}_i \in L^\infty(\partial\Omega_i)$, $\mathbf{n}_i : \partial\Omega_i \to \mathbb{R}^d$, pointing towards the exterior of Ω_i.

The interface between two subdomains Ω_i and Ω_j is denoted by $\Gamma_{ij} := \partial\Omega_i \cap \partial\Omega_j$. Moreover, we introduce the so-called skeleton $\Sigma := \bigcup_{i=0}^{L} \partial\Omega_i$, the union of all boundaries of subdomains.

In our scattering model sources are given through an *incident wave*, coming from infinity and impinging on the scattering obstacle. We assume that the source field $U_{\text{inc}} \in$