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Numerical Study on Statistical Behaviors of Two-Dimensional Dry Foam

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Abstract. We investigate the statistical behaviors of two-dimensional dry foam using numerical simulations based on the immersed boundary (IB) method. We model the liquid phase of a foam as a thin elastic boundary with surface tension and the gas phase as a viscous incompressible fluid which can go through the liquid boundary. We present evidence of the existence of a limiting scaling state of the dry foam dynamics in which the asymptotic value of μ_2, the second moment of the distribution of the numbers of cell sides, lies in the range of 1.3 ± 0.3. We also numerically verify some well-known formulas derived in the dynamics of two-dimensional dry foam such as von Neumann relation, Lewis law, and Aboav-Weaire law. Our simulation results are comparable to those of soap froth experiments and Potts model simulations. Furthermore, we investigate the statistical behaviors of two-dimensional dry foam in an oscillatory shear flow to show the applicability of our method to more general flow conditions.

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Key words: Foam dynamics, immersed boundary method, scaling state, von Neumann relation, Lewis law, Aboav-Weaire law.

1 Introduction

Foam or foam-like material is ubiquitous in daily life and in industry, appearing in such forms as a pint of beer, soda water, shaving cream, dish washer solution, and fire extinguisher. Liquid foam is a gas-filled space divided into cells with liquid boundaries. The thin liquid boundaries move under the influence of surface tension and allow the surrounding gas to flow through the boundaries. Thus the capillarity of the cell boundaries

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and gas exchange between cells together result in the evolution of the size of the cells and the topological structure of a foam [1]. This process is called diffusive coarsening.

The diffusive coarsening of a two-dimensional dry foam obeys a simple dynamical law called von Neumann relation [2,3] for the evolution of cells. Let A_n be the area of a cell with n sides, its rate of change can be determined as follows

$$\frac{dA_n}{dt} = k(n-6), \tag{1.1}$$

where k is the diffusion constant proportional to both the permeability and the surface tension. The von Neumann relation simply says that the area is constant for six-sided cells, a cell with fewer than six sides tends to shrink (and in fact reaches zero area in finite time), and a cell with more than six sides tends to grow. If the von Neumann relation alone governed the coarsening process of a foam, the problems related to its dynamics would be trivial; however, when a cell shrinks and finally disappears, its neighboring cells might increase or decrease their number of sides, resulting in topological changes in the foam structure, and thus the rates of area change of these cells increase or decrease.

The statistics of two-dimensional foams has been widely investigated using numerical simulations and experimental studies, see [1,4–7] and the references therein. A typical coarsening process begins with a nearly regular array of hexagonal cells and gradually evolves into a completely disordered pattern. The principal quantity used to characterize such disordered cellular structure is μ_2, as defined later, the second moment of the distribution $\rho(n)$ of the number of cells with n sides. It is known that there exists a scaling state in which the distribution $\rho(n)$ is almost time-invariant with a fixed range of the second moment $\mu_2 = 1.5 \pm 0.3$. The topological disorder of foam structure including the time-invariant $\rho(n)$ has been extensively studied experimentally [8,9] and computationally [10–12], just to name a few.

There have been many numerical modelings for the evolution and scaling state of a foam. In [1,6,13,14], the authors simulated the evolution of a two-dimensional dry foam within the framework of the following assumptions: Laplace-Young condition, Plateau's rule, and the von Neumann relation (1.1). Potts model uses a quasi-microscopic metallurgical view of foam structure in which the interior of each cell is composed of a collection of lattices [1]. Potts model has been widely used to simulate the cellular structure as an extension of the Ising model, and its applicability to the coarsening process of soap froth in a steady state has been well demonstrated in [15,16]. There are other foam simulations that take the fluid dynamics into account based on a boundary integral formulation [17,18]. More numerical works on two- and three-dimensional dry or wet foams can also be found in [19,20] and the references therein.

In [21], the authors introduced an immersed boundary (IB) method [22] to simulate the fluid dynamics of a two-dimensional dry foam by modeling the gas phase of the foam as a viscous incompressible fluid and the liquid phase as a massless network of permeable internal boundaries with surface tension. The gas diffusion through the liquid phase of the foam was modeled by allowing the internal boundaries to slip relative to the