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# A Hybrid Method with TENO Based Discontinuity Indicator for Hyperbolic Conservation Laws

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**Abstract.** With the observation that the TENO weighting strategy can explicitly distinguish smooth scales from nonsmooth scales in spectral space, in this paper, a new discontinuity indicator is proposed based on the high-order TENO paradigm [Fu et al., JCP 305(2016): 333-359]. The local flow structures are classified as smooth or nonsmooth scales, and the hybrid numerical discretization scheme is applied correspondingly, i.e. the high-order upwind linear scheme without characteristic decomposition is employed for resolving smooth scales while the nonlinear low-dissipation TENO scheme is adopted to capture discontinuities. Since the time-consuming characteristic decomposition and smoothness-indicator computation of TENO are avoided in smooth regions, the overall computational efficiency can be improved significantly. Moreover, the cut-off wavenumber separating smooth and nonsmooth scales is determined by the parameter $C_T$. In contrast to the thresholds of other discontinuity indicators, which are typically defined in physical space, $C_T$ takes effects in wavespace rendering its high generality. A set of benchmark cases with widespread length-scales is simulated to assess the performance of the proposed discontinuity indicator and the resulting hybrid shock-capturing scheme. Compared to the monotonicity-preserving discontinuity indicator and the TVB discontinuity indicator, the proposed algorithm delivers better performance with a fixed set of parameters for all considered benchmarks.

**AMS subject classifications**: 35L65, 65M06, 76J20, 76N15, 76F05

**Key words**: TENO, high order scheme, shock-capturing scheme, nonlinear scheme, finite volume method, finite difference method.

## 1 Introduction

For compressible fluid dynamics, it is well known that discontinuities, e.g. shock and contact waves, can develop after finite-time evolution even if the initial flow is smooth

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enough. In order to resolve these flow structures with widespread length-scales effectively, the high-order shock-capturing scheme is widely used in modern computational fluid dynamics. Among these, the popular methods include, e.g. the essentially non-oscillatory (ENO) [1] scheme, the weighted essentially non-oscillatory (WENO) [2, 3] scheme etc. In the light of ENO and WENO concept, lots of efforts focus on improving the accuracy order in critical points [4,5], the low-dissipation property [6,7], the nonlinear adaptation property [8, 9], and the numerical robustness [10, 11].

However, when applying these high-order shock-capturing schemes, which are typically devised based on the scalar prototype equation, to solve hyperbolic laws, the overall computational costs are rather expensive due to the local characteristic decomposition and the nonlinear-weights computing. In [3], the nonlinear weights of the fifth-order WENO-JS scheme are computed from the pressure and entropy to avoid the characteristic decomposition. Alternative solution is to introduce the hybrid concept, for which the efficient linear scheme without characteristic decomposition is applied in smooth regions while the expensive nonlinear scheme is adopted in the vicinity of discontinuities [12, 13]. Adams and Shariff [14] propose the high-resolution shock-capturing compact-ENO scheme. Pirozzoli [15] proposes a conservative hybrid scheme for resolving the compressible turbulent flows by combing the compact scheme with WENO. Ren et al. [16] develop the characteristic-wise hybrid compact-WENO scheme. More recently, a posteriori, efficient, high-spectral resolution hybrid finite-difference method for compressible flows is proposed [17]. A systematical study of different hybrid schemes has been conducted in [18]. However, the performance of these hybrid methods strongly depends on an effective discontinuity indicator [19–21]. Popular discontinuity indicators include, e.g. the minimod-based TVB limiter [22], Harten's subcell resolution [23], the average total variation of the solution (ATV) [19], etc. Unfortunately, the performances of these indicators are mostly case-sensitive due to the fact that only low-order information is examined and problem-dependent parameters are typically involved [17–19, 24, 25].

Recently, a family of high order targeted ENO (TENO) schemes is proposed by Fu et al. [26–30] for compressible fluid dynamics. The ENO-like stencil selection procedure renders its novelty and is validated to deliver better performances over classical WENO schemes [26, 31, 32]. In this paper, based on the idea that the TENO weighting strategy allows for distinguishing nonsmooth scales from low-wavenumber smooth regions in spectral space, i.e. the local scales are judged as smooth when the spectral property of the corresponding optimal linear scheme is recovered exactly, otherwise nonsmooth, a new discontinuity indicator is proposed. The built-in parameter $C_T$ determines the cut-off wavenumber separating the smooth and the nonsmooth scales. In combination with the TENO6 scheme, a new efficient hybrid TENO scheme is constructed for hyperbolic conservation laws. We demonstrate the performance and the generality by conducting a set of one-dimensional and two-dimensional benchmark simulations without tuning the built-in parameter $C_T$ case-by-case.

The remainder of this paper is organized as follows. In Section 2, the concept of high-order TENO reconstruction is reviewed and the fourth-order TENO6 scheme is