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An MRT Extension to the Multigrid Lattice Boltzmann Method

Charles Armstrong and Yan Peng*

Department of Mathematics and Statistics, Old Dominion University, Norfolk, VA, 23529, USA.

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Abstract. A full approximation storage multigrid method has been shown to improve the convergence properties of the D2Q9 BGK lattice Boltzmann method [14]. In this paper we extend the method to the multiple relaxation time collision model due to its improved stability. The new method is validated using Poiseuille flow. The method is then extended to three dimensions. The efficiency of the V-cycle and W-cycle multigrid schedules and the under-relaxed Jacobi and Gauss Seidel multigrid smoothers are reported for the two and three dimensional models for lid driven cavity flow.

AMS subject classifications: 76M25, 76D99

Key words: Lattice Boltzmann method, multiple relaxation time, multigrid, full approximation storage multigrid.

1 Introduction

The lattice Boltzmann method (LBM) has been shown to be a versatile CFD tool that can be implemented for problems involving complex physical phenomena [3,18,20]. Furthermore, the traditional LBM algorithm lends itself well to high performance paradigms due to its simple two-step implementation. On a uniform Cartesian grid, this two-step procedure consists of a collision step, involving local operations at each grid point, and a streaming step where values are shifted to adjacent grid points. One drawback of the LBM, however, is the slow convergence rate when solving for steady flows or time-dependent flows with large separations in relevant time and spatial scales. Multigrid methods, which have been used in traditional CFD methods to implement solvers that exhibit linear scaling with respect to the problem size, offer a potential remedy to this issue.

*Corresponding author. *Email addresses:* carms015@odu.edu (C. Armstrong), ypeng@odu.edu (Y. Peng)

In [19], Tölke et al. develop a multigrid method, which solved the discrete Boltzmann equation with a second order finite difference discretization. The method improved the convergence properties of the LBM for low Reynolds numbers ($Re=10$), however, the implementation detracted from the parallel friendly nature of the original LBM and the efficiency gains diminished for $Re\geq 100$. In [14], Mavriplis proposed a multigrid LBM, which retained the two-step procedure of the traditional LBM by developing a multigrid scheme based directly on the non-linear lattice Boltzmann equation. This method improved the computational efficiency of the LBM by multiple orders of magnitude up to $Re=1000$. In [16], Patil et al. implemented the method proposed by Mavriplis for the solution of elliptic PDEs, analyzed the efficiency gains and implemented a parallel version of the algorithm.

To the best of the present authors' knowledge there has not been any further study of the method proposed in [14] for steady fluid flows. In this work we extend the method proposed by Mavriplis to the MRT collision model and propose an implementation in three dimensions. This is followed by an extensive analysis of the method's efficiency for lid driven cavity flow in two and three dimensions.

The structure of this paper is as follows: Section 2 presents the FAS multigrid method in a general setting, Section 3 discusses the method proposed in [14] and incorporates the MRT collision model into the method, Section 4 contains a validation and order analysis of the method for Poiseuille flow, and Sections 5 and 6 contain the efficiency analysis of the method in two and three dimensions for lid driven cavity flow.

2 FAS multigrid method

First, we will develop the framework of the Full Approximation Storage (FAS) scheme in the context of solving a system of non-linear equations, $A(u)=f$, where u is the exact solution, A is a non-linear operator. In the discussion that follows h and H denote the grid spacing on the fine and coarse grids respectively. Similarly, variables with a subscripted h denote variables on the fine grid and variables with a subscripted H denote variables on the coarse grid. All iterative equations have been written so that the variables on the $n+1^{th}$ iteration are written in terms of the variables on the n^{th} iteration.

On the fine grid we attempt to solve the system $A_h(u_h)=f_h$, where A_h and f_h are the fine grid approximations of A and f, and u_h is the exact solution to the resulting system. Using an iterative scheme to solve the system we can obtain an approximate solution, v_h. The residual of this system is defined as:

$$r_h(v_h)=f_h-A_h(v_h)=A_h(u_h)-A_h(v_h). \tag{2.1}$$

Eq. (2.1) provides a posterior error estimate of our approximate solution v_h.

For linear problems, Eq. (2.1) can be re-written in terms of the error, $e_h=u_h-v_h$, as:

$$A_h(e_h)=r_h. \tag{2.2}$$