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An Approximate Second-Order Closure Model for Large-Eddy Simulation of Compressible Isotropic Turbulence

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Abstract. In this paper, the detailed dynamic characteristics of the subgrid scale (SGS) stress tensor and heat flux are investigated through Taylor series expansion in numerical simulations of compressible isotropic turbulence. A new approximate second-order closure (ASOC) model is introduced based on the transport equations of the first-order Taylor series approximation of SGS stress tensor and heat flux. The proposed model is implemented in large eddy simulation (LES) of compressible isotropic turbulence. Detailed comparisons with direct numerical simulation (DNS) dataset using both *a priori* and *a posteriori* approaches are carried out. *A priori* tests show that, SGS stress tensor and heat flux have high correlations with the first-order Taylor series approximation. Their root mean square (rms) values are close to those of the first-order Taylor series approximation. In *a posteriori* tests, the proposed ASOC model yields good agreement with DNS dataset. Compared with the results of the dynamic Smagorinsky model (DSM) and dynamic mixed model (DMM), the ASOC model predicts better energy spectra at high wavenumbers. The probability density function (PDF) and the structure functions of velocity and thermodynamic variables are further studied, demonstrating that the statistical properties of the simulated flows are improved by the ASOC model. The numerical results illustrate the ability of the model to improve the statistical properties of the simulated flows in the context of LES. Finally, a simplified ASOC model can be derived by neglecting the effect of density gradient for low turbulent Mach number turbulence.

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1 Introduction

Large-eddy simulation (LES) is an important approach to numerical simulations of compressible turbulence at high Reynolds numbers. LES has gained great success in simulating complex turbulent flows and astrophysics for its low computational cost and high accuracy [1–6]. The majority transport properties of momentum, energy, heat, and chemical species in a turbulent flow are dominated by the large-scale motions, while the effects of the small-scale dynamics are modelled. The small-scale motions tend to be more homogeneous and universal, and be less affected by the boundary than the large-scale motions over a wide range of turbulent flows; therefore, lots of applicable LES models can be simpler and require fewer adjustments when applied to different flows, as compared with the models for the Reynolds-averaged Navier-Stokes equations [7–10].

A huge amount of works have been devoted to LES of incompressible flows since the pioneering works on the LES by Smagorinsky, Lilly and Deardorff [9, 11–15]. The relationship between the SGS stress τ_{ij} and large-scale straining $\tilde{S}_{ij}$ is fundamental for SGS stress modelling, and different types of functional relationships between them have resulted in many SGS models, such as: eddy viscosity models [1,6,10,11,16]. Second-order closure model has got more and more attention since it is the most logical and physical model [6, 17–20]. It is possible for the second-order closure model to reproduce accurately the SGS stress field and heat flux in complex flows [21]. In order to obtain the correct first-, second-,and third-order statistics from the LES, the stresses need to obey several necessary conditions [20]. The transport equation of SGS stress is usually closed with the dissipation rate. Earlier SGS stress model was derived by considering the transport equation for the correlation of the subgrid-scale fluctuating velocities [17, 22]. The redistribution term can be modeled based on the analogy between the RANS and LES modeling [18, 23]. The diffusion term can be modeled by means of the gradient law hypothesis [24]. Algebraic stress models were derive, which were shown to perform better than the nonlinear eddy viscosity model [25]. The elliptic relaxation EVM of Durbin was also inspired and derived [26]. Chaouat and Schiestel have proposed the partially integrated transport modeling (PITM) method using a stress transport model based on second-moment closure [27–29]. The PITM framework was transposed to the frequency space by Fadai-Ghotbi et al. [30]. It should be emphasized that there are lots of the hybrid models of Reynolds averaged Navier-Stokes (RANS) and LES developed in recent years [31–41]. Besides, Heinz derived a transport equation for the SGS stress by using a stochastic model [42].