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Asymptotic Estimates for the Ruin Probability of a Multidimensional Delay-Claim Risk Model with Dependent Claims

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Abstract. This paper studies a multidimensional delay-claim risk model in which an insurance company operates d $(d \geq 2)$ lines of business exposed to a common renewal counting process. Each catastrophic event simultaneously produces main and delayed claims across all business lines, where the delayed claims are settled after random delay periods. The surplus process incorporates a geometric Lévy price process to describe investment returns. Assuming that the main and delayed claims follow subexponential distributions and satisfy a conditional linear dependence structure, we derive asymptotic estimates for the finite-time ruin probability. The obtained results extend existing findings on delay-claim models to the multidimensional framework and contribute to a deeper understanding of ruin behavior under dependence and heavy-tailed risks.

AMS subject classifications: 62P05, 62E20, 91G05

Key words: Asymptotics, multidimensional risk model, dependence structure, delayed claim, subexponential class.

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1 Introduction

Nowadays, insurance companies typically manage multiple lines of business concurrently, such as health, motor, and homeowner's insurance. Since a single

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catastrophic event can give rise to claims across several lines, risk models have been developed to capture such interdependencies. In particular, models incorporating both main and delayed claims have been extensively investigated. Suppose an insurer operates d ($d\geq 2$) lines of business simultaneously. A catastrophic event may generate a main claim in each line, which is settled immediately, while the associated delayed claim is processed after a random period. For instance, a car accident may result in property damage in multiple lines, leading to immediate claims, whereas compensation for personal injuries may be deferred, introducing dependencies between the main and delayed claims.

In this paper, we consider a multidimensional delay-claim risk model where the surplus process can be described as follows:

$$U(t)=\begin{pmatrix}U_1(t)\\U_2(t)\\\vdots\\U_d(t)\end{pmatrix}=\begin{pmatrix}x_1e^{L(t)}\\x_2e^{L(t)}\\\vdots\\x_de^{L(t)}\end{pmatrix}+\begin{pmatrix}\int_0^t c_1(s)e^{L(t-s)}ds\\\int_0^t c_2(s)e^{L(t-s)}ds\\\vdots\\\int_0^t c_d(s)e^{L(t-s)}ds\end{pmatrix}-\begin{pmatrix}\sum_{i=1}^{N(t)}X_{1,i}e^{L(t-\tau_i)}\\\sum_{i=1}^{N(t)}X_{2,i}e^{L(t-\tau_i)}\\\vdots\\\sum_{i=1}^{N(t)}X_{d,i}e^{L(t-\tau_i)}\end{pmatrix}$$

$$-\begin{pmatrix}\sum_{i=1}^{N(t)}Y_{1,i}e^{L(t-\tau_i-D_{1,i})}I_{\{\tau_i+D_{1,i}\leq t\}}\\\sum_{i=1}^{N(t)}Y_{2,i}e^{L(t-\tau_i-D_{2,i})}I_{\{\tau_i+D_{2,i}\leq t\}}\\\vdots\\\sum_{i=1}^{N(t)}Y_{d,i}e^{L(t-\tau_i-D_{d,i})}I_{\{\tau_i+D_{d,i}\leq t\}}\end{pmatrix},\quad t\geq 0, \tag{1.1}$$

where $(x_1,\ldots,x_d)^\top$ denotes the vector of the initial reserve and $c_k(t)\geq 0$ is the density function of premium income at time t for $k=1,\ldots,d$. Assume $\{(X_{1,i},\ldots,X_{d,i});$ $i\in\mathbb{N}_+\}$ denotes the i-th main claims of the business lines occurring simultaneously at time τ_i. For $k=1,\ldots,d$ and $i\in\mathbb{N}_+$, each main claim $X_{k,i}$ is associated with a delayed claim $Y_{k,i}$ occurring at time $\tau_i+D_{k,i}$, where $D_{k,i}$ denotes an uncertain delay time. Let $\{\theta_i;i\in\mathbb{N}_+\}$ be a sequence of nonnegative, independent, and identically distributed (i.i.d.) random variables representing claim inter-arrival times. The arrival times of the main claims $\tau_i=\sum_{j=1}^i\theta_j$, $i\in\mathbb{N}_+$ constitute a counting process $\{N(t);t\geq 0\}$ which is a renewal process with a finite renewal function

$$\lambda(t)=\mathbb{E}N(t)=\sum_{i=1}^\infty\mathbb{P}(\tau_i\leq t).$$

Additionally, for each $k=1,\ldots,d$, $\{D_{k,i};i\in\mathbb{N}_+\}$ is a sequence of nonnegative (possibly degenerate at 0) i.i.d. random variables with a generic random variable D_k with distribution H_k, and the price process of the investment portfolio is