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Observation-Driven INAR(1) Models with Novel and Flexible Links

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Abstract. Observation-driven integer-valued autoregressive models are widely used for modeling count time series exhibiting dynamic dependence, yet their performance critically depends on the way that thinning probabilities are linked to past observations. Most existing specifications rely on the logit link and may respond excessively to large counts. In this paper, we introduce a class of new observation-driven integer-valued autoregressive models using logarithmic and soft-clipping links that attenuate the influence of large observations. The proposed framework allows for stochastic covariates. Estimation is carried out using conditional maximum likelihood and conditional least squares methods. Simulation studies and two real data applications are used to illustrate the proposed models.

AMS subject classifications: 62M10, 62M20

Key words: Observation-driven model, covariate, logarithmic link, soft-clipping link, conditional maximum likelihood.

1 Introduction

To model integer-valued time series with values in $\mathbb{N}_0 = \{0,1,2,\ldots\}$, traditional autoregressive (AR) frameworks developed for continuous-valued data are often inadequate because they rely on Gaussian assumptions. To address this limitation, McKenzie [16] and Al-Osh and Alzaid [1] introduced the integer-valued autoregressive (INAR) model, in which the multiplicative structure of AR models is replaced by a binomial thinning operator. Numerous extensions of the INAR framework have been proposed. With respect to statistical characteristics of count

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time series, Kang *et al.* [12, 13] developed models that accommodated under-, equi-, and overdispersion; Yang *et al.* [30] considered a bivariate threshold Poisson INAR process; Kang *et al.* [14] proposed a zero-modified geometric specification; and Kang *et al.* [10] introduced a parsimonious specification designed to capture multiple empirical features. Serially dependent innovations were also considered; see [11, 26]. From a Bayesian perspective, Miao and Wang [17] developed Bayesian procedures for estimating the order of INAR(q) models.

An important extension was introduced by Zheng *et al.* [35], who proposed the random-coefficient INAR model with time-varying thinning probabilities. Zheng and Basawa [34] further introduced observation-driven INAR models in which the dependence structure evolves with past observations. Several subsequent studies incorporated covariates into this framework, including the empirical likelihood approach of Ding and Wang [6], the minification-based INAR process of Qian and Zhu [21], and the random-coefficient threshold INAR model of Yang *et al.* [29]. Other extensions have focused on alternative thinning operators. Yu *et al.* [32] introduced a class of observation-driven random-coefficient INAR processes based on negative binomial thinning, and Yu and Tao [31] further generalized observation-driven parameters under Poisson thinning. For recent comprehensive overviews of integer-valued time series models, see [3, 18, 20].

Most existing works on observation-driven INAR models treat covariates as fixed design sequences or conditionally exogenous inputs. When covariates are modeled as stochastic processes, such models can be viewed as Markov chains in random environments, as formalized by Kifer [15]. Fokianos and Truquet [9] consider finite-state Markov chains in random environments, whereas Truquet [24] develops a framework for general state spaces; in both cases, the observed dynamics are conditionally time-inhomogeneous given an exogenous process. A common strategy is to enlarge the state space by incorporating the shifted environment, thereby obtaining a time-homogeneous representation for the joint process and a convenient route to stationarity and ergodicity. Complementary to these results, Doukhan *et al.* [7] provide a coupling-based treatment of observat ion-driven models in random environments via backward iterations in Wasserstein distance, without relying on small-set or uniform contraction assumptions. In observation-driven INAR models with stochastic covariates, the thinning mechanism is specified through a conditional transition kernel rather than an explicit recursion. This makes drift or Lyapunov verification on the count state space nontrivial.

The choice of link plays a crucial role in observation-driven INAR models. In most existing formulations, the thinning probability is modeled as a logit function of past observations. While this specification ensures that the thinning prob-