CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2025-0071

Vol. **7**, No. 3, pp. 461-488
May 2026

Smooth Bijective Projection of Polygonal Meshes via a Cubic B-Spline Shell

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Received 1 July 2025; Accepted 28 July 2025

Abstract. Polygonal meshes are a fundamental surface representation, yet their resolution can vary significantly. Establishing a smooth, bijective projection between meshes of different resolutions is crucial for consistent attribute transfer but becomes challenging when handling sharp bends and complex geometric features.
This paper presents a novel approach to address this challenge by implicitly representing the shell enclosing two polygonal meshes using a cubic trivariate B-spline function, where the inner and outer bounding surfaces are formulated as level sets of a single cubic B-spline function. Our method enforces the bijective projection requirements on the cubic B-spline function, ensuring that the resulting gradient field naturally defines a robust bijective projection. Leveraging the favorable properties of cubic B-spline functions – namely, 1) sufficient smoothness while maintaining expressive representation, and 2) computational efficiency and ease of implementation, our approach efficiently computes a smooth and bijective projection even for challenging cases. Compared to existing shell-based bijective projection methods, our method consistently produces valid bijective projections, even in complex scenarios, outperforming state-of-the-art techniques. We further demonstrate its effectiveness in robust attribute transfer and precision-controlled shape manipulation.

AMS subject classifications: 68U05, 97R60

Key words: Cubic B-spline shell, attribute transfer, smooth bijective projection.

1 Introduction

Polygonal meshes are ubiquitous in digital geometry processing, offering a discrete representation for encoding 3D surfaces through vertices, edges, and facets. While they

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provide exceptional modeling flexibility, performing tasks such as mesh simplification [14, 42], real-time rendering [48], and collision detection [4, 43] consistently across different mesh triangulations remains challenging. Large-scale models or irregular tessellations often introduce unnecessary computational burdens, making efficient processing difficult. Research efforts have sought to address these limitations by exploring geometric proxies that balance computational efficiency with shape fidelity.

Recent research suggests that shell representations – volumetric domains bounded by two closely spaced surfaces with controlled separation – offer an effective solution to this challenge. Analogous to sandwich composites in continuum mechanics [24], shell structures provide several advantages, such as bounded geometric approximation errors [20], bijective attribute mappings [39], and unified surface-volume parametrizations [31]. These properties have led to widespread adoption across applications such as texturing [62], meshing [34], and physical simulation [7]. In these applications, attributes such as textures, displacements, and physical properties must be preserved during remeshing.

Existing explicit shell-based approaches, including linear shell [24] and high-order shell [39][†], employ piecewise bijective projections to maintain attribute integrity within the shell space. However, these methods struggle to ensure smooth bijectivity. As shown in Fig. 1, linear shells can introduce distortions. Furthermore, explicit shell representations require careful handling to prevent adjacent intervals from intersecting, as such intersections violate bijectivity and reduce smoothness.

To address these fundamental limitations, we propose a novel mathematical framework for constructing cubic B-spline shells (CBS) that ensures two key properties: 1) an analytic representation of sandwich-walled spaces using implicit functions, and 2) homeomorphically preserved projections within the shell domain. Our approach defines bounding surfaces as iso-surfaces of a carefully designed implicit function f, ensuring smoothness and continuity via a C^2-regular B-spline construction. The bijective projection requirement translates to a gradient constraint $|\nabla f| \geq \delta > 0$ throughout the shell region, which we enforce through analytic gradient bounding in our spline formulation. Furthermore, this can be efficiently solved using a sparse linear system. Compared to existing proxies, B-spline functions provide an optimal balance between geometric flexibility and computational efficiency, as their localized nature significantly reduces computational overhead by restricting function evaluations to a subset of control parameters. Additionally, we introduce a gradient-normal alignment term and prove that when the gradients of the implicit function align with the normal vectors at mesh edges, the surface-restricted gradient changes tend to be harmonic.

To demonstrate the effectiveness of CBS, we showcase two key applications: 1) mesh editing with global precision control, and 2) attribute transfer between surfaces within the shell. These applications highlight our framework's ability to maintain high-fidelity mappings while significantly reducing distortion compared to existing methods.

[†]Since the code is not open, we limit our comparison with linear shell.