Biharmonic Distance Driven Voronoi Diagrams Restricted on Curved Surfaces

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Abstract. Computing surface-based Voronoi diagrams is a fundamental operation in geometry processing, typically relying on either geodesic or straight-line distances as solvers. However, when the input is a 3D model containing thin-plate structures, geodesic distances incur significant computational overhead, whereas straight-line distances can result in ownerless regions. To address this issue, we propose integrating biharmonic embedding distances into the SurfaceVoronoi framework. Specifically, mesh vertices can be embedded into a high-dimensional spectral space, ensuring that the embedding distance closely approximates the straight-line distance between sufficiently close points. In contrast, when points reside on opposite sides of a thin plate, the embedding distance significantly exceeds the straight-line distance, effectively preventing dominance from penetrating through to the opposite side. Our proposed framework offers several advantages: 1) It operates efficiently, as the embedding distance can be rapidly evaluated as a straight-line distance in high-dimensional space. 2) It guarantees the "one site, one region" property, even for models consisting of thin-plate structures. 3) It enables high-quality triangulation through iterative repositioning of each site to the centroid of its dominant region. Extensive experiments conducted on benchmark meshes demonstrate these benefits.

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1 Introduction

Partitioning a 2-manifold surface into regions based on proximity to a set of sites is a fundamental problem in geometry processing. The resulting structure, known as a surface-

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based Voronoi diagram (VD), has been widely applied in tasks such as surface sampling [6, 25], remeshing [1, 3], parameterization [17], and shape abstraction [20].

Over the past decades, two primary types of distance metrics have been commonly adopted for constructing surface-based Voronoi diagrams: geodesic distances and Euclidean distances. Geodesic distances are intrinsic and faithfully reflect the shortest path along the surface, enabling accurate and natural Voronoi partitioning [4, 19]. However, they are computationally expensive and often tightly coupled with the diagram construction process [8], limiting scalability to large meshes. In contrast, Euclidean-based Voronoi diagrams [3] provide a more efficient alternative by utilizing straightforward Euclidean distances in three-dimensional space. While faster, these approaches are extrinsic and prone to topological inconsistencies such as fragmented or ownerless regions, particularly on highly curved or thin-sheet surfaces. Although the propagation-based SurfaceVoronoi framework [23] partially mitigates the ownerless region problem inherent to Euclidean distances, it still produces geometrically unreasonable Voronoi regions in certain critical areas, such as the boundaries of thin structures. As shown in Fig. 1 with Euclidean distance-based Voronoi diagrams (EDBVD), these limitations significantly restrict the practical application scope of this approach.

In this work, we extend the SurfaceVoronoi framework by introducing a distance metric based on biharmonic embedding. Our key insight is recognizing that this distance metric inherently reflects the intrinsic geometric properties of the surface, making it an ideal candidate for integration with the SurfaceVoronoi framework. By performing a one-time embedding of each mesh vertex into a high-dimensional spectral space constructed from the eigenvectors of the biharmonic operator [9], we enable distance calculations that are both computationally efficient and geometry-aware. Distances are simply evaluated as Euclidean norms between embedding vectors, avoiding the need for expensive geodesic computations while maintaining sensitivity to the surface's intrinsic structure. This innovative combination yields a distance field that is smooth, globally coherent, and fully compatible with the existing SurfaceVoronoi pipeline without requiring any structural modifications, thereby enhancing partition quality while preserving computational efficiency.

Our embedding-driven SurfaceVoronoi algorithm offers a simple yet effective approach that overcomes key limitations of traditional distance metrics. By leveraging biharmonic embedding, our method elegantly handles challenging cases such as thin structures that conventional approaches struggle with, while maintaining computational efficiency and geometric fidelity.

2 Related work

2.1 Surface-based Voronoi diagram

The construction of Voronoi diagrams on curved surfaces, particularly non-differentiable polyhedral surfaces, plays a critical role in digital geometry processing applications.