CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2025-0065

Vol. **7**, No. 3, pp. 576-599
May 2026

An Optimization Process for p-Curves in Surface-to-Surface Intersection to Improve Accuracy and Reduce Redundancy

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Received 30 June 2025; Accepted 28 September 2025

Abstract. The problem of surface-to-surface intersection (SSI) is among the most fundamental challenges encountered in computer-aided geometric design. In industrial applications, the execution of an SSI operation usually demands a high degree of precision while maintaining a concise representation of the intersection curve. We present a post-processing scheme for refining the intersection curve in the parameter domain (p-curves), achieving an improvement in accuracy without additional control points inserted. Moreover, by adjusting the parameterization, a one-to-one correspondence is created between a pair of p-curves. The proposed methodology is grounded on the minimization of an objective function that describes both accuracy and parameterization. The effectiveness of our method is illustrated through examples in comparison with the open-source NURBS library SISL and commercial software ACIS.

AMS subject classifications: 49Q10

Key words: CAGD, surface intersections, watertight, parameterization, optimization.

1 Introduction

1.1 Background and motivation

The foundational concept of computer aided geometric design (CAGD) is the creation and employment of geometric model representations that are interpretable by computers. A 3-D entity is typically represented as an assembly parametric surface patches. The accurate, efficient, and robust computation of surface-surface intersection curves is a fundamental requirement for the construction of complex geometric models and other advanced techniques.

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Numerous algorithms have been developed to solve the SSI problem, which are generally classified into three categories [12]: lattice evaluation, subdivision methods, and marching methods. Currently, a frequently employed approach is the integration of subdivision and marching methods. The basic strategy is to initially determine a starting point for each segment of the curve using subdivision methods, followed by marching methods to complete the curve. The SSI algorithms discussed above generally stop at calculating an ordered set of intersection points. As the number of intersection points approached infinity, the ordered set converges towards the real curve of intersection, paralleling the statistical concept of sampling. Therefore, intersection points in the ordered set are also referred to as sampling points.

For industrial requirements, representations of computer-interpretable geometric models should follow the ISO standard. According to “Automation systems and integration – Product data representation and exchange” (ISO 10303, also known informally as STEP) [7], to represent a curve on a surface, two representations are required: one in the model space and another in the parameter domain. We adopt the term “p-curve” to denote an intersection curve in the parameter domain, while it is referred to as “trimming curves” in certain literature. An example of intersecting surfaces and respective p-curves is illustrated in Fig. 1. ISO standard stipulates several forms that p-curves should take, among which the B-spline form is most widely used.

To construct curve representations from the sampling point set, a curve fitting scheme is applied. Curve fitting is a well-studied topic in CAGD, including both curve approximation and interpolation. In addition to conventional fitting methods such as least square fitting and Hermite interpolation [2,3,11], numerous innovative techniques have been developed to address challenges in particular contexts [5,6,8,13]. l fitting methods include least square fit. The SINTEF Spline Library (SISL) [16], for example, employs the Hermite interpolation to obtain the space intersection curve as well as two p-curves. Hermite interpolation simultaneously interpolates the set of intersection points and tangents at each

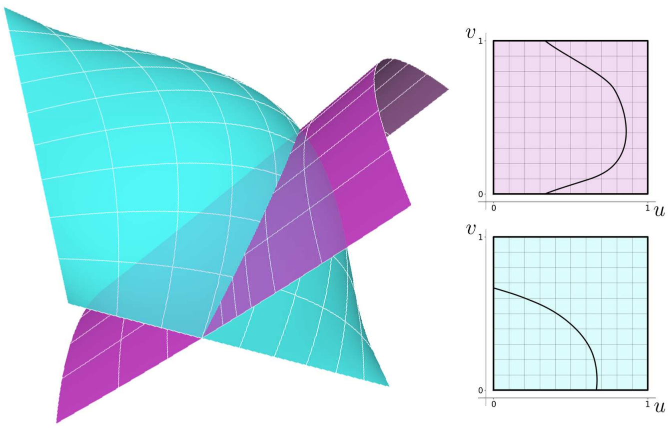

Figure 1: An example of intersecting surfaces in the model space and respective p-curves in the parameter space.