

2D High-Order Cage-Based Deformation by Polygonal Surface Patches

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Abstract. This paper presents a novel method for 2D high-order cage-based deformation using polygonal surface patches, providing a unified framework that supports various types of polygonal patches. The proposed deformation process comprises two main components: the contribution from the cage corner points and the influence of the middle control points along each boundary curve. First, a coarse deformation is reconstructed using generalized barycentric coordinates derived from the deformed cage corners. Then, displacement curves defined by the middle control points on each cage edge are employed to refine the deformation, precisely relocating the transition points to their final positions. Extensive experimental results demonstrate that our method produces high-quality results across diverse 2D deformation scenarios.

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Key words: Cage-based modeling, 2D shape deformation, polygonal surface patches, generalized barycentric coordinates.

1 Introduction

Cage-based deformation (CBD) [8] has emerged as a widely adopted and effective technique in geometric modeling and computer graphics, providing intuitive shape manipulation through coarse control structures [17]. The method utilizes a simple control mesh (cage) that encapsulates the target object, with deformation governed by generalized barycentric coordinates (GBCs) [2] to establish a mapping between cage vertices and interior points. Despite its effectiveness in various applications, traditional CBD method exhibits two notable limitations due to its linear boundary interpolation: first,

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the requirement for cage refinement and subsequent GBCs recomputation when additional degrees of freedom are needed [4]; second, the inherent constraint of maintaining piecewise-linear boundary representations [6, 16].

As an extension of traditional CBD, high-order CBD introduces the ability to represent curved cage boundaries, thereby greatly expanding the expressiveness of deformation techniques. Through degree elevation, linear edges of the cage can be transformed into higher-degree Bézier or spline curves by inserting middle control points. This process provides additional degrees of freedom along each boundary segment, allowing for more refined control over shape transformations. By enabling curved cage boundaries, high-order formulations significantly enhance the visual quality of the deformation process, addressing key limitations of linear models in both flexibility and shape fidelity [10]. However, existing high-order CBD methods typically require the surface patches used for deformation to satisfy the property of generalized barycentric reproduction (see Section 4), which limits the adoption of alternative patch types that may offer superior geometric or visual properties.

In this work, we introduce a novel framework for 2D high-order CBD that leverages a broad class of parametric polygonal surface patches. Unlike traditional approaches that are often limited to specific patch types, our method offers a generalized and unified formulation capable of incorporating various polygonal patch representations within a high-order deformation pipeline. For surface patches that inherently support direct deformation, our method yields results equivalent to conventional methods. More importantly, it enables the use of a wider variety of polygonal surface patches that would otherwise be unsuitable for direct deformation, potentially yielding superior results and expanding the applicability of high-order CBD.

The remainder of this paper is organized as follows. Section 2 provides a review of related work on high-order CBD. Section 3 introduces three curve deformation approaches, which are subsequently extended and analyzed for high-order CBD in Section 4. Section 5 presents experimental results and comparisons on various surface patches. Section 6 discusses the limitations of the proposed method. Finally, the paper concludes with a summary of the key contributions.

2 Related work

For high-order CBD through cage coordinates, Li *et al.* [10] first introduced the concept of curved cage boundaries and proposed the cubic mean-value coordinates (MVC). This method transforms linear cages into curved ones by replacing straight edges with cubic Bézier curves, thereby increasing the expressiveness of the deformation. However, as an extension of the classical MVC [3], cubic MVC inherits several inherent limitations. In particular, when applied to non-convex cages, it may generate negative weights, which can lead to visually undesirable artifacts during deformation.

In 2023, Michel and Thiery [14] proposed polynomial Green coordinates (PGC), which extends Green coordinates [12] to support cages with polynomial boundaries. However,