CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2025-0062

Vol. **7**, No. 3, pp. 556-575
May 2026

Efficient Two-Dimensional Randomized Progressive Iterative Approximation for Large-Scale B-Spline Fitting

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Received 27 June 2025; Accepted 10 September 2025

Abstract. The randomized progressive iterative approximation (RPIA) is a local and approximate geometric iteration method designed for large-scale data fitting. At each iteration, RPIA updates only the control points indexed by a specific set, leaving the others unchanged. In this work, we introduce a two-dimensional RPIA (D2RPIA) for fitting B-spline curves and surfaces. Unlike RPIA, D2RPIA updates the control points with an adaptive step-size, which is determined by imposing a constraint on the new control points. This adaptive step-size allows D2RPIA to achieve the current optimal result, thereby enhancing the convergence rate compared to RPIA. We prove that D2RPIA converges linearly in the mean square to the least-squares solution. Several numerical studies are presented to validate our theoretical results.

AMS subject classifications: 65D05, 65D10, 65D17, 65F10

Key words: Least-squares fitting, B-spline, randomized, progressive iterative approximation.

1 Introduction

1.1 Background

In the era of big data, the least-squares fitting is a vital tool that seeks the best function to describe a set of data points by minimizing the sum of the squares of the residuals. It

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has a wide range of applications such as curve and surface fitting in geometric design, regression analysis in machine learning, parameter estimation in signal denoising and signal recovery, and so on. The growth of big data poses a challenge for traditional least-squares algorithms, which require considerable computational power to process large datasets.

With the increasing volume of data and limitations in computational resources, researchers have developed and analyzed various numerical techniques to efficiently solve the least-squares problem. Classical approaches include direct solvers, such as QR decomposition and singular value decomposition [2]. These factorizations are fundamental in numerical linear algebra. Recent advances have focused on improving computational efficiency for large-scale problems. Iterative solvers, such as the Richardson method and gradient descent, offer alternatives by progressively refining solutions, making them suitable for large-scale data [16,18]. Nonlinear optimization techniques, including Newton's method and trust-region methods, are often used [6,17]. These numerical approaches differ in terms of scalability and efficiency, and depend on the specific application scenario, such as data characteristics, computational resources, and performance requirements. In practical applications, selecting the most appropriate method requires balancing computational complexity, solution accuracy, and data scale.

1.2 Related works

Let $\{\boldsymbol{q}_j\}_{j=0}^m$ in $\mathbb{R}^2$ or $\mathbb{R}^3$ be a point set designated for fitting. Each point is associated with a parameter u_j for $j\in[m]$. The objective is to find a least-squares B-spline fitting curve of the form

$$\mathcal{C}(u)=\sum_{i=0}^{n}N_i(u)\boldsymbol{p}_i,$$

where $\{\boldsymbol{p}_i\}_{i=0}^n$ is a control point sequence to be determined and $\{N_i(u)\}_{i=0}^n$ is the B-spline basis. In recent years, the least-squares progressive iterative approximation (LSPIA) method was proposed by Deng and Lin [3]. In the LSPIA procedure, a sequence of curves is generated by

$$\mathcal{C}^{(k+1)}(u)=\mathcal{C}^{(k)}(u)+\omega\sum_{i=0}^{n}\sum_{j=0}^{m}N_i(u)N_i(u_j)\boldsymbol{r}_j^{(k)}$$

for $k=0,1,2,\ldots$, where $\boldsymbol{r}_j^{(k)}=\boldsymbol{q}_j-\mathcal{C}^{(k)}(u_j)$ is the j-th difference vector and ω is a weight. This iteration method has clear geometric significance, i.e. the control point is progressively adjusted through a series of iterative steps to minimize the error between the data points and the fitting curve. That is, the control point is updated by

$$\boldsymbol{p}_i^{(k+1)}=\boldsymbol{p}_i^{(k)}+\omega\sum_{j=0}^{m}N_i(u_j)\boldsymbol{r}_j^{(k)},\quad i=0,1,\ldots,n. \tag{1.1}$$

Due to its stability, efficiency, and robustness in data fitting, the LSPIA method has attracted considerable attention from researchers over the last decade. Some variants of