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An Adaptive Finite Element DtN Method for the Open Cavity Scattering Problems

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Abstract. Consider the scattering of a time-harmonic electromagnetic plane wave by an open cavity which is embedded in a perfectly electrically conducting infinite ground plane. This paper concerns the numerical solutions of the open cavity scattering problems in both transverse magnetic and transverse electric polarizations. Based on the Dirichlet-to-Neumann (DtN) map for each polarization, a transparent boundary condition is imposed to reduce the scattering problem equivalently into a boundary value problem in a bounded domain. An a posteriori error estimate based adaptive finite element DtN method is proposed. The estimate consists of the finite element approximation error and the truncation error of the DtN operator, which is shown to decay exponentially with respect to the truncation parameter. Numerical experiments are presented for both polarizations to illustrate the competitive behavior of the adaptive method.

AMS subject classifications: 65M30, 78A45, 35Q60

Key words: Electromagnetic cavity scattering, TM and TE polarizations, adaptive finite element method, transparent boundary condition, a posteriori error estimates.

1 Introduction

Consider the electromagnetic scattering of a time-harmonic plane wave by an open cavity, which is referred to as a bounded domain embedded in the ground with its opening aligned with the ground surface. The open cavity scattering problems have significant applications in industry and military. In computational and applied electromagnetics, one of the physical parameter of interests is the radar cross section (RCS), which measures the detectability of a target by a radar system. It is crucial to have a deliberate control in the form of enhancement or reduction of the RCS of a target in the stealth

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technology. The cavity RCS caused by jet engine inlet ducts or cavity-backed patch or slot antennas can dominate the total RCS of an aircraft or a device. It is indispensable to have a thorough understanding of the electromagnetic scattering characteristic of a target, particularly a cavity, in order to successfully implement any desired control of its RCS.

Due to the important applications, the open cavity scattering problems have received much attention by many researchers in both of the engineering and mathematics communities. The time-harmonic problems of cavity-backed apertures with penetrable material filling the cavity interior were introduced and studied initially by researchers in the engineering community [20, 22, 28]. The mathematical analysis for the well-posedness of the variational problems can be found in [1–3], where the non-local transparent boundary conditions, based on the Fourier transform, were proposed on the open aperture of the cavity. It has been realized that the phenomena of electromagnetic scattering by cavities not only have striking physics but also give rise to many interesting mathematical problems. As more people work on this subject, there has been a rapid development of the mathematical theory and computational methods for the open cavity scattering problems. The stability estimates with explicit dependence on the wavenumber were obtained in [9, 10]. Various analytical and numerical methods have been proposed to solve the challenging large cavity problem [6, 8, 11, 23, 31]. The overfilled cavity problems, where the filling material inside the cavity may protrude into the space above the ground surface, were investigated in [15,16,26,30], where the transparent boundary conditions, based on the Fourier series, were introduced on a semi-circle enclosing the cavity and filling material. The multiple cavity scattering problem was examined in [25, 34], where the cavity is assumed to be composed of finitely many disjoint components. The mathematical analysis can be found in [7, 13] on the related scattering problems in a locally perturbed half-plane. We refer to the survey [24] and the references cited therein for a comprehensive account on the modeling, analysis, and computation of the open cavity scattering problems.

There are two challenges for the open cavity scattering problems: the problems are formulated in unbounded domains; the solutions may have singularities due to possible nonsmooth surfaces and discontinuous media. In this paper, we present an adaptive finite element method with transparent boundary condition to overcome the difficulties.

The first issue concerns the domain truncation. The unbounded physical domain needs to be truncated into a bounded computational domain. An appropriate boundary condition is required on the artificial boundary of the truncated domain to avoid unwanted wave reflection. Such a boundary condition is known as a transparent boundary condition (TBC). There are two different TBCs for the open cavity scattering problems. For a regular open cavity, where the filling material is inside the cavity, the Fourier transform based TBC is imposed on the open aperture of the cavity; for an overfilled cavity, where the filling material appears to protrude out of the cavity through the open aperture into the space above the ground surface, the Fourier series based TBC is imposed on the semi-circle enclosing the cavity and the protruding part. The latter is adopted