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Energy Stable Splitting Schemes for Maxwell's Equations in Lorentz Media

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Abstract. In this paper, we introduce energy-stable schemes based on operator splitting methods for Maxwell's equations in two-dimensional Lorentz dispersive media with transverse electric polarization, namely the sequential splitting scheme (SS-ML) and the Strang-Marchuk splitting scheme (SM-ML). Each splitting scheme involves two substages per time step, where 1D discrete sub-problems are solved using the Crank-Nicolson method for time discretization. Both schemes ensure energy decay and unconditional stability. The convergence analysis reveals that the SS-ML scheme exhibits first-order accuracy in time and second-order accuracy in space based on the energy technique, while the SM-ML scheme achieves second-order accuracy in both time and space. Additionally, numerical dispersion analysis yields two discrete numerical dispersion relation identities for each scheme. Theoretical results are supported by examples and numerical experiments.

AMS subject classifications: 65N06, 65N12, 65N15, 65M06, 65M12

Key words: Maxwell equation, Lorentz model, finite difference, stability, Yee scheme.

1. Introduction

The study of wave propagation in dispersive media has gained considerable attention, particularly in light of the advancements in electromagnetic meta-materials. A dispersive medium is a substance or material that exhibits dispersion, a phenomenon where different wavelengths of light or electromagnetic radiation traverse the material at varying speeds. One particular type of dispersive material that demonstrates distinct frequency-dependent behavior in response to electromagnetic fields is a Lorentz medium. In a Lorentz material, modifications are made to Maxwell's equations accounting for the specific characteristics of the material's permittivity and permeability as described by the Lorentz model. The utility of Lorentz media extends across diverse domains, encompassing optics, solid-state physics, and materials research [3, 23, 27].

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The Yee finite-difference time-domain (Yee-FDTD) method is a widely used technique for solving Maxwell's equations in the time domain [28, 29]. It discretizes the electric and magnetic fields on structured staggered space-time grids, enabling efficient simulations of electromagnetic wave propagation and interactions with various structures and materials. With its second-order accuracy in both space and time, the Yee-FDTD method is particularly suitable for electromagnetics, optics, and antenna design applications, delivering reliable and efficient solutions. Moreover, the Yee-FDTD method has been extended to handle Maxwell's equations in linear and nonlinear materials by combining it with other discrete methods [4, 5, 10, 28, 30]. The Yee-FDTD scheme has made significant extensions, particularly by accommodating nonuniform meshes within diverse material compositions [11, 20–22]. However, it exhibits a fundamental limitation as a conditionally stable numerical method. This conditionality dictates that both the time step and spatial step sizes must satisfy the Courant-Friedrichs-Lewy (CFL) stability condition during time-marching simulations [28]. Consequently, when spatial step sizes are reduced, the necessity for exceedingly minute time steps arises. This inherent demand for minuscule time steps places a significant computational burden on the conditionally stable Yee-FDTD method, particularly when addressing Maxwell's equations within extremely thin geometric structures [24].

To address the stability limitations and reduce computational expenses, the operator splitting method has been extensively studied for solving Maxwell's equations and is a widely adopted strategy for tackling complex time-dependent problems [7, 11, 13, 18]. This method involves breaking down complex time-dependent problems into simpler sub-problems, each focusing on a distinct physical process guided by a specific operator. These sub-problems are solved sequentially, and their solutions are combined using their initial conditions to solve the original problem. Commonly used operator splitting methods for time-dependent problems include sequential splitting, Strang-Marchuk splitting, and the alternating direction implicit (ADI-FDTD) approach. These methodology not only enhances computational efficiency compared with fully implicit techniques such as the Crank-Nicolson scheme but also maintain the critical attribute of unconditional stability. While the FDTD method is a powerful and widely used numerical technique in computational electromagnetics, it may not always preserve the energy conservation property, essential for ensuring physically meaningful simulations of electromagnetic phenomena. Chen *et al.* recently introduced energy-conserving splitting techniques for solving two- and three-dimensional Maxwell's equations [8, 9]. Additionally, extensions of the energy-based operator splitting method for Maxwell's equations in higher dimensions or with different materials have been proposed [2, 12, 14, 16, 25, 26].

In this paper, we introduce an operator splitting technique to solve the two-dimensional transverse electric Maxwell equations in Lorentz dispersive media. We formulate energy stable splitting methods based on the staggered Yee-grid structure, employing both sequential and Strang-Marchuk splitting schemes. These strategies entail two stages per time step, effectively reducing computational efforts. We present a rigorous analysis of the methods concerning stability and convergence of fully discrete splitting schemes using energy methods. Our analysis verifies that both splitting schemes satisfy the energy decay relation,