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Multi-View Data Clustering via Dynamical Optimization of Consensus Laplacian Matrix

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Abstract. Multi-view data analysis has gained increasing popularity, in particular multi-view spectral clustering has attracted much attention for its outstanding performance in mining heterogeneity information in multi-view data. However, most spectral clustering methods exhibit the following disadvantages: firstly, learning consensus representation directly from multi-view data that may contain noise renders a distorted description; secondly, the traditional two-step process may fall into a suboptimal solution. To overcome these disadvantages, a novel multi-view spectral clustering method is proposed by unifying the optimization of consensus Laplacian matrix and the learning and discretization of spectral embedding into one step. We consider that the optimal Laplacian matrix is in the neighborhood of view-specific Laplacian matrix, as the view-specific Laplacian matrix only contains partial information from multi-view data, resulting in certain deviation from the optimal Laplacian matrix. The consensus Laplacian matrix was obtained in a dynamic optimization way with the spectral rotation and embedding information simultaneously determined. Extensive experiments have been conducted to demonstrate the effectiveness and superiority of our proposed method.

AMS subject classifications: 65M10, 78A48

Key words: Multi-view, dynamical optimization, spectral clustering, spectral rotation, unified framework.

1. Introduction

Multi-view data can capture wealth of information from various perspectives. However, due to the diverse sources of data acquisition, the features of multi-view data are of redundancy, correlation, diversity [5]. A key challenge is how to integrate the information of each view [36]. Early fusion is the most widely used method in multi-view clustering [17,30,32]. It offers a unified consensus representation for the multi-view data, and in

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the later stage uses a single-view clustering method to capture heterogeneity in the data. Late fusion [22,23] is an alternative way to integrate the multi-sources of data. Each view is clustered separately to generate multiple base clustering results, which are then combined in the final stage to produce a consensus clustering outcome.

Although the existing multi-view spectral clustering methods can achieve promising clustering results, it is still challenging to fully extract information embedded in the multi-view data due to the following reasons. First, most multi-view spectral clustering methods are realized in two independent steps [19]. While each step can achieve its own optimization goal, the two-step strategy always yields a suboptimal result rather than the global optimal one. Second, most multi-view spectral clustering methods directly learn a consensus representation from original multi-view data, which would render a distorted description for the data, because the data obtained from various sources are usually noisy and correlated [39, 40]. For example, some methods linearly combine the Laplacian matrices of all views to obtain a consensus Laplacian matrix for the final multi-view spectral clustering [18,42]. Third, existing methods usually treat each view as equally important, ignoring the importance of different views [15, 33].

To effectively solve the above issues, we propose a novel one-step multi-view spectral clustering framework by combining the learning of consensus Laplacian matrix and the discretization of spectral embedding in a unified framework. For the Laplacian matrix of each view, we assume that it contains partial information of the whole multi-view data and regard it as a minor deviation of the optimal Laplacian matrix (as shown in Fig. 1). Our goal is to dynamically optimize the optimal Laplacian matrix provided that it is located in the neighborhood of view-specific Laplacian matrices. In addition, each view is adaptively weighted in search process to reduce the noise effect.

To demonstrate the effectiveness of our method, we conducted extensive experiments on a number of widely used real-world datasets, and used well-known metrics: ARI, NMI, ACC, F-score, Purity to evaluate clustering performances, visualized the aggregated embedding data using t-stochastic neighborhood embedding (t-SNE). The proposed model converges in a fast manner and shows extraordinary performance in multi-view data integration.

The contributions of this study are summarized as follows:

- We search for the optimal Laplacian matrix in the neighborhood of Laplacian matrix of each view, allowing that the information contained in the Laplacian matrix of each view is not adequately accurate, and may slightly deviate from the optimal Laplacian matrix. The optimal Laplacian matrix and low dimensional embedding information can be obtained through ingenious design of the optimization objective and framework.

- By putting the learning and discretization of spectral embedding into a unified framework, we obtain the clustering result of multi-view data directly in optimization process. This one-step strategy can avoid loss of information and obtain more reasonable clustering result.