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Quasi-Periodic Breathers of the Hirota-Maccari System

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Abstract. The Hirota-Maccari (HM) system generalizing the Hirota equation, serves as a generalized (2+1)-dimensional model in fluid dynamics, plasma physics, and optical fiber communication. In this paper, we obtain the quasi-periodic breathers to HM system by using the Hirota's bilinear method and the theta function. Asymptotic analysis demonstrates that the quasi-periodic breathers can be reduced to regular breathers under small amplitude limits. Moreover, we also classified solutions based on their asymptotic behavior. Numerical examples are given to confirm the theoretical analysis.

AMS subject classifications: 65M10, 78A48

Key words: Quasi-periodic breather, Hirota-Maccari system, Hirota's bilinear method, theta function.

1. Introduction

Integrable systems and soliton equations have garnered significant attention in both physics and mathematics. These nonlinear partial differential equations serve as powerful tools to model various nonlinear phenomena in physical sciences, including fluid mechanics, nonlinear optics, magnetic fluids, and many other areas. The solutions of soliton equations are of significant interest due to their diverse types, such as soliton solutions, breather solutions, lump solutions, and rogue wave solutions. These solutions provide deep insights into the behavior of nonlinear systems and have broad applications in both theoretical and applied research.

It is well known that the Kadomtsev-Petviashvili (KP) equation is regarded as the most fundamental integrable nonlinear dispersive wave equations in (2+1)-dimensions — cf. [1, 19, 20] and references therein. This is because many integrable systems can be derived as special reductions of the KP hierarchy, which comprises the KP equation along with its infinitely many symmetries. Starting from the KP equations, Maccari introduced a new type of coupled nonlinear evolution equation in (2+1)-dimensions, called the Hirota-Maccari

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system

$$\begin{aligned} &\mathrm{i}u_t + u_{xy} + \mathrm{i}\beta u_{xxx} + uv - \mathrm{i}\beta|u|^2u_x = 0, \\ &3v_x + (|u|^2)_y = 0, \end{aligned} \tag{1.1}$$

which was derived by using an asymptotically exact reduction method based on Fourier expansion and spatiotemporal rescaling in [26]. The Hirota-Maccari system (1.1) is integrable due to the existence of the corresponding Lax pairs, and it could be reduced to the (1+1)-dimensional integrable Hirota equation [13, 27, 28, 30, 38] when $x = y$. Its exact solutions including soliton solutions, periodic waves solutions, travelling wave solutions, have been obtained by a variety of methods, such as the unified algebraic method [9], the Weierstrass elliptic function expansion method [5], the complex hyperbolic function method [4], the direct algebraic method [39], the solitary wave ansatz method [33], the extended trial equation method [7] and the bilinear method [37]. Through the coordinate transformation $x \to \mathrm{i}x, y \to -y, t \to -\mathrm{i}t$, the HM system (1.1) can be rewritten as

$$\begin{aligned} &u_t - \mathrm{i}u_{xy} + \beta u_{xxx} + uv + \beta|u|^2u_x = 0, \\ &3\mathrm{i}v_x - \left(|u|^2\right)_y = 0, \end{aligned} \tag{1.2}$$

which can be further transformed into the bilinear form

$$\begin{aligned} &\left(D_t - \mathrm{i}D_xD_y + \beta D_x^3 + \beta D_x\right) g \cdot f = 0, \\ &\left(3D_x^2 + 1\right) f \cdot f = gg^* \end{aligned} \tag{1.3}$$

under the dependent variable transformation

$$u = \frac{g}{f}, \quad v = -2\mathrm{i}(\ln f)_{xy},$$

where f is real, g is complex, $*$ means the complex conjugation, and D is the Hirota operator [14] defined by

$$\begin{aligned} &D_t^m D_x^n a(t,x) \cdot b(t,x) = \frac{\partial^m}{\partial s^m}\frac{\partial^n}{\partial y^n} a(t+s, x+y) b(t-s, x-y)|_{s=0,y=0}, \\ &m, n = 0, 1, 2, \ldots. \end{aligned}$$

The bilinear HM equation (1.3) is a special case of the coupled DS-KP equation proposed by Hietarinta in [12]. The rational and semi-rational solutions to (1.3) were studied in [31,34] by use of the KP hierarchy reduction method.

Recently, the quasi-periodic solution (also called algebra-geometric solutions or finite-gap solutions) to the (2+1)-dimensional integrable systems have been investigated by different methods [6, 10, 11, 17, 18, 22, 24]. This kind of solutions is expressed by the theta function [8], a summation of an infinite number of exponential functions. Since breathers characterized by theta functions reduce to regular breathers described by elementary functions (e.g., trigonometric functions), they are termed quasi-periodic breathers. Based on Hirota's bilinear method and theta functions, one of the authors and her collaborators introduced a direct approach for efficiently and directly computing quasi-periodic breathers