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Energy-Based Adaptive Deep Unfitted Nitsche Method for Elliptic Interface Problems

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Abstract. The paper proposes an energy-based adaptive sampling strategy to enhance the performance of the deep unfitted Nitsche method for elliptic interface problems. Instead of relying on fixed or random training points, the proposed refinement indicator dynamically concentrates samples in high-energy regions, including sharp coefficient jumps and interface singularities. This targeted allocation improves both accuracy and efficiency compared to random sampling. Numerical experiments in both two- and three-dimensional settings demonstrate that the method achieves robust accuracy and efficiency across diverse scenarios, from standard geometries to highly irregular interfaces, while effectively handling high-contrast coefficients and multi-subdomain configurations. These results confirm that the proposed adaptive strategy not only reduces training cost but also ensures reliable performance in complex interface problems.

AMS subject classifications: 65M50, 65M60

Key words: Interface problem, deep learning, unfitted Nitsche method, adaptive method.

1. Introduction

Deep learning has recently demonstrated remarkable advancements in various scientific domains, including text or audio recognition — e.g. large language models, object detection, and scientific computing [22]. In the field of solving partial differential equations (PDEs), traditional numerical methods such as the finite element method (FEM) and the finite difference method ensure superior accuracy of the numerical solutions but face difficulties in resolving problems with complicated geometries or higher dimensions [2]. To overcome the curse of dimensionality, mesh-free deep learning approaches have been proposed as alternatives to enhance the performance of traditional methods, given its universal approximation theorem [6,21]. Among these, physics-informed neural networks (PINNs) have emerged as a prominent framework by incorporating physical laws through residual-based loss functions that embed initial and boundary conditions [34]. Hybrid strategies

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integrating PINNs frameworks with classical numerical methods have also gained significant attention due to their ability to leverage the strengths of both approaches. The deep Galerkin method trains a neural network to minimize a mesh-free loss function that enforces the differential operator, initial, and boundary conditions of PDEs [37]. The deep Ritz method (DRM) utilizes variational formulations to minimize loss functions weakly constructed using the Ritz method [7, 41]. Additionally, weak adversarial networks parameterize the weak solution and test function through the primal and adversarial neural networks, thereby extending the applicability of DRM to a broader range of PDEs [27,48]. From another angle, the deep Nitsche method handles the essential boundary conditions by incorporating the idea of Nitsche's method [28,49]. On the operator-learning side, neural operators have been developed to approximate solution operators directly in a single Banach space, such as the deep operator network (DeepONet) [30] and Fourier neural operator [25]. Combined with PINNs, the physics-informed neural operator [26] and physics-informed DeepONet [42] demonstrate exceptional performance in learning the solution operator of PDEs even without the paired input-output training data.

Beyond direct PDE solvers, deep learning also shows remarkable capabilities for solving indirect solutions of PDEs in high dimensions. For instance, deep neural networks (DNNs) based on temporal-difference methods are trained for forward-backward stochastic differential equations from the perspective of reinforcement learning [49]; the deep backward stochastic differential equation method exploits the reformulation of the nonlinear parabolic PDEs as backward stochastic differential equations [13,29]. Both approaches demonstrate favorable performance in accuracy and computational efficiency, effectively overcoming the curse of dimensionality and highlighting the expressive approximation capabilities of deep neural networks.

Elliptic interface problems, however, remain particularly challenging because they involve discontinuous coefficients and singularities across material interfaces [20]. Classical numerical methods for such problems can be broadly categorized into fitted and unfitted methods [2]. Fitted methods use body-fitted meshes that align with the interface, enabling the application of standard finite element methods [51]. Unfitted methods allow interfaces to cut through mesh elements, requiring special techniques to enforce jump conditions. Key approaches include immersed finite element methods based on Cartesian meshes [23, 24], body-fitted meshes combined with immersed finite element methods [10], a condensed generalized finite element method [50], and the unfitted finite element method based on Nitsche's method [14], which introduces additional basis functions to handle discontinuities and enforce jump conditions using the variation of Nitsche's approach. The unfitted Nitsche's method also shows its efficiency in computing band structures without fitting the interfaces of periodic inclusions [12].

Applications of deep learning methods on elliptic interface problems have recently been used with rising interest. Residual-based PINNs and the deep Ritz method often encounter low accuracy when addressing such issues due to the complex interface conditions [47]. The piecewise continuous property and significant jump across the interfaces thus require specific handling of interface conditions. Several hybrid approaches have been proposed, such as decomposing the domain into singular and regular parts for neural networks and the