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Heterogeneous Optimized Schwarz Methods for Heat Conduction in Composites with Thermal Contact Resistance

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Abstract. Heat transfer in composites is crucial in engineering, where imperfect layer contact induces thermal contact resistance (TCR), causing interfacial temperature jumps. We propose to solve this numerically using the optimized Schwarz method (OSM), which decouples the heterogeneous problem into homogeneous subproblems. This approach avoids ill-conditioned systems typical of monolithic methods under high contrast and interface discontinuities. Convergence of the algorithm with a standard Robin transmission condition is established via energy estimates and Fourier analysis. For accelerated convergence, a scaled Robin condition is introduced, with rigorous optimization of its free parameter. Key findings emerge due to TCR: first, larger TCR values speed up OSM convergence, achieving asymptotic mesh-independence. This contrasts with the mesh-dependent behavior observed without TCR. Second, greater heterogeneity contrast enhances convergence; third, unlike TCR-free cases, higher thermal conductivity also promotes convergence, similar to heterogeneity; finally, the scaled Robin condition outperforms the standard one in both theory and practice. Numerical tests validate the results and demonstrate the method's potential for nonlinear problems on irregular domains.

AMS subject classifications: 65M55

Key words: Optimized Schwarz method, heat conduction in composites, thermal contact resistance, domain decomposition, optimized transmission condition.

1. Introduction

Composites are materials typically composed of various continuous solid components and are widely utilized in engineering applications such as aerospace [36] and electron-

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ics [54] due to their superior physical properties and relatively low cost. The heat conduction processes within composites significantly influence their thermal conductivity, thermal stability, and mechanical properties. Consequently, thermal analysis of composites is crucial for simulating the manufacturing processes, thermal stresses, and welding of these materials, which have been extensively studied through detailed modeling to enhance both performance and cost-effectiveness.

Steady-state heat conduction in composites is typically modeled using elliptic interface problems, a concept with a long historical lineage tracing back to [40, 41]. Early research assumed that heat transfer between distinct solid materials was continuous; specifically, the temperature and heat flux at the interface were considered continuous [13], a condition now recognized as perfect thermal contact. Notably, a substantial body of existing studies on heat transfer in composites relies on the assumption of perfect thermal contact. However, experimental evidence has demonstrated that temperature discontinuities frequently occur across interfaces [23]. This phenomenon can be attributed to thermal contact resistance, also referred to as “Kapitza” resistance [28], arising from imperfect contact between materials. Such imperfect contact is inevitable; the formation of interstitial media at rough contact interfaces limits the effective contact area, thereby introducing additional heat transfer resistance [24] compared to an ideal (perfect contact) scenario. This issue of imperfect contact is pervasive, occurring even in interface problems involving coupled solid and liquid phases. Therefore, it is imperative to consider TCR when analyzing heat transfer in composites.

A considerable body of research has been conducted on solving elliptic interface problems, employing both analytical and numerical methods. Analytical methods primarily focus on the scenario of perfect contact, wherein both temperature and heat flux remain continuous. Notable examples include the separation of variables [25], finite integral transforms [44], Laplace transforms [1], Fourier transforms [9], and Green’s functions [8]. However, research on analytical techniques addressing TCR is relatively limited [21, 22]. The application of these analytical methods can be challenging due to their reliance on strict assumptions regarding the geometry of the domains and/or the boundary conditions. In contrast, numerical methods offer great adaptability in managing interface problems characterized by complex configurations, which have garnered significant attention [20,30,31,37,38,40]. When addressing problems with interfacial jump conditions, careful consideration is required to handle elements that intersect the interface. Recently, several variants of the finite element method (FEM) have been developed to address imperfect contact problems, including the extended FEM [3, 51] and unfitted FEM [26, 48]. Other numerical approaches, such as the finite volume method [6] and the discontinuous Galerkin method [5], have also been explored for tackling interface problems involving TCR. The aforementioned methods typically solve heat transfer problems in composite materials in a monolithic manner, which often lead to the formation of large, ill-conditioned algebraic systems due to heterogeneity contrasts. This issue complicates accurate problem resolution and frequently results in non-physical oscillations near the interface. In contrast, domain decomposition methods [42, 47] partition the computational domain into several homogeneous subdomains and reconstruct the solution through an iterative process of subdomain